QCD amplitudes in MRK

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Contents

- Multi-peripheral kinematics
- Reggeization hypothesis
- Vertices and trajectory
- Proof of the Gluon Reggeization in NLA
- Quark Reggeization in LLA
- Conclusion

Multi-peripheral kinematics

The importance of the multi-peripheral kinematics (MPK) was realized long time ago.
K.A. Ter-Martirosyan, Nucl.Phys. 68 (1965) 591
In high energy perturbative QCD this kinematics provides the dominant contributions.
In the process of multi-particle production $A + B \longrightarrow P_0 + P_1 + \ldots + P_n + P_{n+1}$ the multi-Regge kinematics (MRK) means that all final particles are well separated in rapidity space: $y_0 \gg y_1 \gg \ldots \gg y_n \gg y_{n+1}$, here rapidities $y_i = \frac{1}{2} \ln \frac{k_i^+}{k_i^-}$,
$k_i = k_i^+ n_1 + k_i^- n_2 + k_{i\perp}$, where $k_i^+ = (k_i n_2)$, $k_i^- = (k_i n_1)$, and $n_1^2 = n_2^2 = 0$, $(n_1, n_2) = 1$

In the next-to-leading logarithmic approximation (NLA) the contribution of the quasi-multi-Regge kinematics (QMRK) also must be taken into account. In QMRK instead of one particle P_i we have jet J_i, so as within it particles have close rapidities. We can consider both MRK and QMRK cases treating with jets J_i consisting of one or two particles.

Hypothesis of the gluon Reggeization

The ground of the BFKL approach is assertion that the MPK amplitudes with the gluon exchange and negative signature are dominant, and their real part acquires the form:

$$\Re \mathcal{A}_{2\to n+2} = \bar{\Gamma}^{R_1}_{J_0 A} \left(\prod_{i=1}^{n} \frac{e^{\omega(q_i)(y_{i-1}-y_i)}}{q_{i\perp}^2} \gamma^{J_i}_{R_i R_{i+1}} \right) \frac{e^{\omega(q_{n+1})(y_n - y_{n+1})}}{q_{(n+1)\perp}^2} \Gamma^{R_{n+1}}_{J_{n+1} B}.$$

The hypothesis is extremely powerful:

- It allows us to express scattering amplitudes only through several effective vertices and gluon trajectory.
- It creates the basis of the BFKL approach to the theoretical description of high energy scattering.
- The Pomeron and Odderon in QCD appear as the compound state of the Reggeized gluons.
- The effective action based on Reggeized gluons is the most general way of the solution of saturation and unitarization problems.
- It gives a link between QCD and the String Theory.

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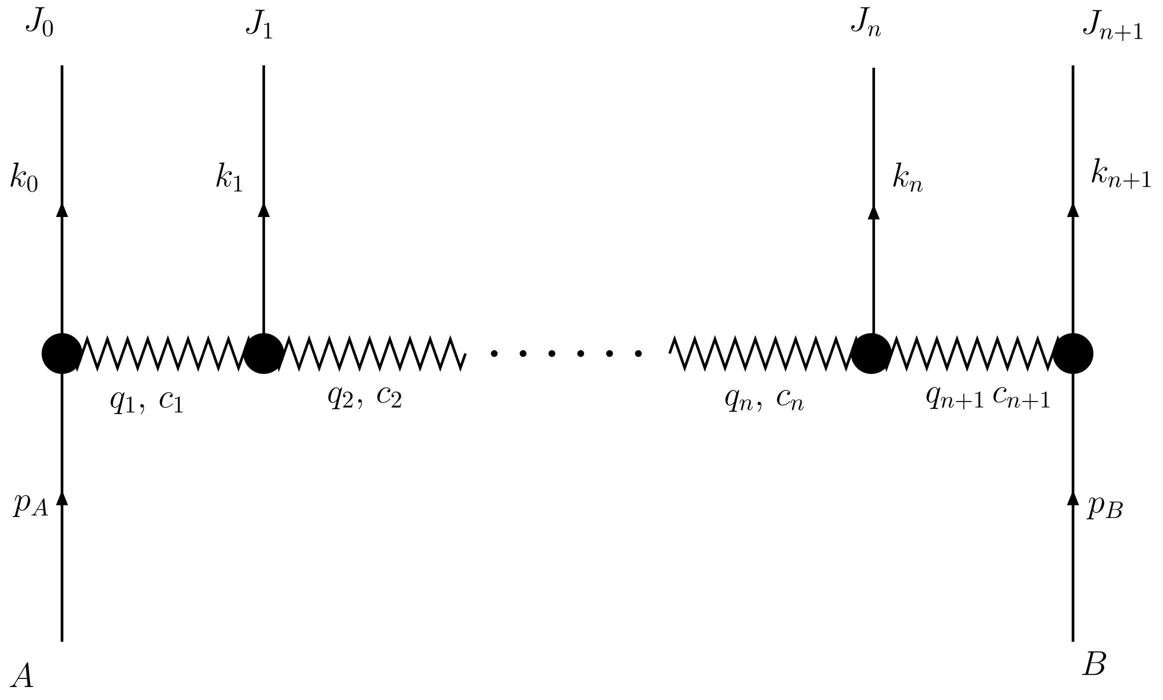

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The gluon Regge trajectory $\omega(t)$ was calculated up to two loops.

V.S. Fadin, R. Fiore and M.I. Kotsky, Phys. Lett. B387 (1996) 593

In the integral form the trajectory is known also for arbitrary space-time D.

V.S. Fadin, R. Fiore, M.I. Kotsky, Phys. Lett. B359 (1995) 181

In the limit $\epsilon \to 0$ we have in terms of Born trajectory

$\omega^{(1)}(t) = -g^2 \frac{N_c \Gamma(1-\epsilon)}{(4\pi)^{D/2}} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \left(-q_\perp^2\right)^\epsilon \propto \frac{\alpha_S}{\epsilon}$

explicit expression

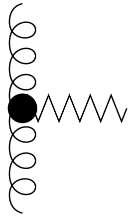
$$\omega(t) = \omega^{(1)}(t)\left(1+\frac{\omega^{(1)}(t)}{4}\left[\frac{11}{3}+\left(\frac{\pi^2}{3}-\frac{67}{9}\right)\epsilon+\left(\frac{404}{27}-2\zeta(3)\right)\epsilon^2+\frac{2n_f}{3N_c}\left(1-\frac{5}{3}\epsilon+\frac{28}{9}\epsilon^2\right)\right]\right)$$

J. Blumlein, V. Ravindran and W.L. van Neerven, Phys. Rev. D58 (1998) 091502

V. Del Duca and E.W.N. Glover, JHEP 0110 (2001) 035

Hypothesis of the gluon Reggeization

$$\Re\mathcal{A}_{2\to n+2} = \bar{\Gamma}^{R_1}_{J_0 A}\left(\prod_{i=1}^{n} \frac{e^{\omega(q_i)(y_{i-1}-y_i)}}{q_{i\perp}^2}\gamma^{J_i}_{R_i R_{i+1}}\right)\frac{e^{\omega(q_{n+1})(y_n-y_{n+1})}}{q_{(n+1)\perp}^2}\Gamma^{R_{n+1}}_{J_{n+1}B}$$



$\Gamma^R_{Q'Q}$ and $\Gamma^R_{G'G}$ are the vertices describing transitions $Q \to Q'$ and $G \to G'$ in collision with Reggeon R. Now they are known with NLO accuracy.

In light cone gauge the vertex of gluon transition can be written as:

$$\Gamma^{c(B)}_{G'G} = -g\big(e^*(p')e(p)\big)_\perp T^c_{G'G}$$

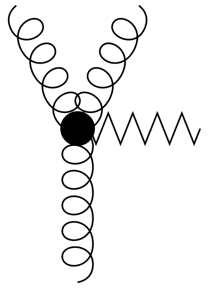
$$\Gamma^a_{G'G} = \Gamma^{a(B)}_{G'G}\Big\{1 + \frac{\omega^{(1)}(t)}{2}\Big[\frac{2}{\epsilon} + \psi(1) + \psi(1-\epsilon) - 2\psi(1+\epsilon) - \\ - \frac{9(1+\epsilon)^2+2}{2(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} + \frac{n_f}{N_c}\frac{(1+\epsilon)^3+\epsilon^2}{(1+\epsilon)^2(1+2\epsilon)(3+2\epsilon)}\Big]\Big\} + \\ + gT^a_{G'G}e'^{*}_{\perp\mu}e_{\perp\nu}\Big(g^{\mu\nu}_\perp - (D-2)\frac{q^\mu_\perp q^\nu_\perp}{q^2_\perp}\Big)\frac{\epsilon\,\omega^{(1)}(t)}{2(1+\epsilon)^2(1+2\epsilon)(3+2\epsilon)}\Big(1+\epsilon-\frac{n_f}{N_c}\Big),$$

V.S. Fadin, L.N. Lipatov, Nucl. Phys. B406 (1993) 259

For NLO $\Gamma^R_{Q'Q}$ see V.S. Fadin, R. Fiore, A. Quartarolo, Phys. Rev. D50 (1994) 2265

Hypothesis of the gluon Reggeization

$$\Re\mathcal{A}_{2\to n+2} = \bar\Gamma^{R_1}_{J_0 A}\left(\prod_{i=1}^{n}\frac{e^{\omega(q_i)(y_{i-1}-y_i)}}{q^2_{i\perp}}\gamma^{J_i}_{R_i R_{i+1}}\right)\frac{e^{\omega(q_{n+1})(y_n-y_{n+1})}}{q^2_{(n+1)\perp}}\Gamma^{R_{n+1}}_{J_{n+1}B}$$



$\Gamma^R_{\{Q'G'\}Q}$, $\Gamma^R_{\{Q\overline{Q}\}G}$ and $\Gamma^R_{\{G_1G_2\}G}$ are vertices describing the fragmentation of initial state particle in collision with Reggeon R.

$$\Gamma^c_{\{G_1G_2\}G} = \left(T^aT^c\right)_{i_1i_2}\Big(\mathcal{A}\big((k_1-x_1k)_\perp\big)-\mathcal{A}\big((x_2k_1-x_1k_2)_\perp\big)\Big)- \\ -\left(T^cT^a\right)_{i_1i_2}\Big(\mathcal{A}\big((-k_2-x_2k)_\perp\big)-\mathcal{A}\big((x_2k_1-x_1k_2)_\perp\big)\Big)$$

$$\mathcal{A}(p_\perp) = \frac{2g^2}{p^2_\perp}\Big[x_1x_2(e_1^*e_2^*)_\perp(ep)_\perp - x_1(e_1^*e)_\perp(e_2^*p)_\perp - x_2(e_2^*e)_\perp(e_1^*p)_\perp\Big]$$

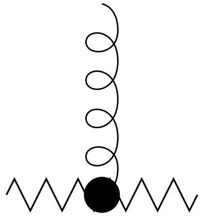
L.N. Lipatov, V.S. Fadin, Yad. Fiz. 50 (1989)

Hypothesis of the gluon Reggeization

$$\Re\mathcal{A}_{2\to n+2} = \bar{\Gamma}^{R_1}_{J_0 A}\left(\prod_{i=1}^{n}\frac{e^{\omega(q_i)(y_{i-1}-y_i)}}{q^2_{i\perp}}\gamma^{J_i}_{R_i R_{i+1}}\right)\frac{e^{\omega(q_{n+1})(y_n - y_{n+1})}}{q^2_{(n+1)\perp}}\Gamma^{R_{n+1}}_{J_{n+1}B}$$

$\gamma^{G}_{R_1 R_2}$ is the vertex of one gluon production.

$$\gamma^{G(B)}_{c_1 c_2}(q_1,q_2) = gT^a_{c_1 c_2} e^*_\mu(k) C^\mu(q_2,q_1)$$



$$C^\mu(q_2,q_1) = -q^\mu_1 - q^\mu_2 + n^\mu_1\left(\frac{q_1^2}{k^-}+2k^+\right) - n^\mu_2\left(\frac{q_2^2}{k^+}+2k^-\right),$$

L.N. Lipatov, Yad. Fiz. 23 (1976) 642

Different parts of this NLO vertex were calculated in some works:

V.S. Fadin, L.N. Lipatov, Nucl. Phys. B406 (1993) 259

V.S. Fadin, R. Fiore, A. Quartarolo, Phys. Rev. D50 (1994) 5893

V.S. Fadin, R. Fiore, M.I. Kotsky, Phys. Lett. B389 (1996) 737

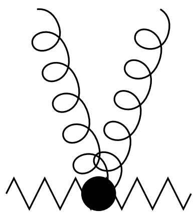
Now it is known in the NLO for arbitrary $D = 4 + 2\epsilon$

V.S. Fadin, R. Fiore, A. Papa, Phys. Rev. D63 (2001) 034001, hep-ph/0008006

Hypothesis of the gluon Reggeization

$$\Re \mathcal{A}_{2\to n+2} = \bar{\Gamma}^{R_1}_{J_0 A} \left(\prod_{i=1}^{n} \frac{e^{\omega(q_i)(y_{i-1}-y_i)}}{q_{i\perp}^2} \gamma^{J_i}_{R_i R_{i+1}} \right) \frac{e^{\omega(q_{n+1})(y_n - y_{n+1})}}{q_{(n+1)\perp}^2} \Gamma^{R_{n+1}}_{J_{n+1} B}$$

The vertices for two-gluon $\gamma^{G_1 G_2}_{R_1 R_2}$ and quark-antiquark production $\gamma^{Q\overline{Q}}_{R_1 R_2}$ in Reggeon-Reggeon collision were found for the first time in the work:



L.N. Lipatov, V.S. Fadin, Yad. Phys. 50 (1989)

Recently they were obtained together with vertices RPP, RRP, RPPP, RRPPP, RPPPP

E.A. Antonov, L.N. Lipatov, E.A. Kuraev, I.O. Cherednikov, Nucl. Phys. B721 (2005) 111, hep-ph/0411185

from the effective action:

L.N. Lipatov, Nucl.Phys. B452 (1995) 369; Phys.Rep. 286 (1997) 131

The method of the Reggeization proof

The proof of the gluon Reggeization in LLA was performed 30 years ago by
Ya. Ya. Balitskii, V.S. Fadin, E.A. Kuraev and L.N. Lipatov.
In NLA the gluon Reggeization proof is grounded now on the bootstrap relations (b.r.):

$$\frac{1}{-\pi i}\left(\sum_{l=j+1}^{n+1} \mathrm{disc}_{s_{j,l}} - \sum_{l=0}^{j-1} \mathrm{disc}_{s_{l,j}}\right) \mathcal{A}^{\mathcal{S}}_{2\to n+2}/(p_A^+ p_B^-) \;=\; \frac{\partial}{\partial y_j}\, \mathcal{A}^{\mathcal{S}}_{2\to n+2}(y_i)/(p_A^+ p_B^-)$$

that allow us to express partial derivatives $\partial/\partial y_j$ of the amplitudes, through the certain combination of discontinuities of the signaturized amplitudes:
V.S. Fadin, *Diffraction 2002*, Ed. by R. Fiore *et al.*, NATO Science Series, Vol. 101, p.235.
$\mathcal{S}$ means symmetrization with respect to simultaneous change of signs of all $s_{i,j}$ with $i < k \leq j$, performed independently for each number of channel $k = 1, \ldots, n+1$.
One of the methods for the b.r. derivation is based on the Steinmann theorem in conjunction with general analytical properties of the MRK amplitudes
O. Steinmann, Helv. Phys. Acta, **33**, 33(1960); J. Bartels, Nucl. Phys. B175, 365 (1980)

Bootstrap relations and gluon Reggeization in NLA

It is sufficient to prove b.r. with NLO accuracy only for symmetrized production

$$SP = \hat{\mathcal{S}} \prod_{i<j=1}^{n+1} \left(\frac{s_{i,j}}{|k_{i\perp}|\,|k_{j\perp}|} \right)^{\alpha_{ij}} = e^{\sum_{i<j=1}^{n+1} \alpha_{ij}(y_i - y_j)} \left(1 + \mathcal{O}(\alpha_S^2)\right),$$

where arbitrary $\alpha_{ij} \sim \alpha_S$ are only non-zero for some set of non-overlapping channels.

The following formulae complete the ground of b.r. (the second is valid only in NLO!).

$$\frac{1}{-\pi i} \left(\sum_{l=j+1}^{n+1} \mathrm{disc}_{s_{j,l}} - \sum_{l=0}^{j-1} \mathrm{disc}_{s_{l,j}} \right) SP = \left(\sum_{l=j+1}^{n+1} \alpha_{jl} - \sum_{l=0}^{j-1} \alpha_{lj} \right) SP = \frac{\partial}{\partial y_j} SP.$$

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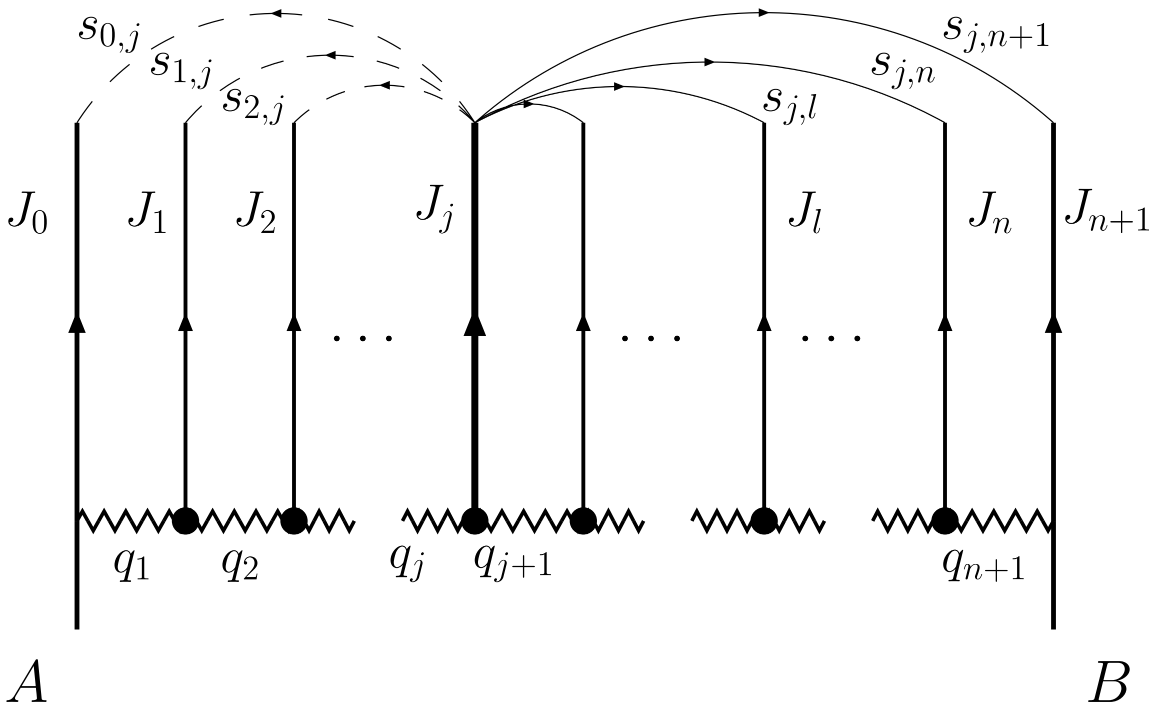
where arbitrary $\alpha_{ij} \sim \alpha_S$ are only non-zero for some set of non-overlapping channels.

If we prove the b.r. in perturbative calculation, it will means the proof of the Regge form in NLA, since one can recursively calculate Regge amplitudes loop-by-loop in all orders of coupling constant using MRK amplitudes only in the one loop approximation for every n as an input. Indeed, b.r. express all partial derivatives of the real parts at some number of loops through the discontinuities, calculated using the s-channel unitarity in terms of amplitudes with a smaller number of loops. In the NLA only real parts of the amplitudes do contribute in the unitarity relations.

Bootstrap relations and gluon Reggeization in NLA

In order to verify that Regge form of the amplitude is the solution of b.r. equation, we insert the Regge form of the amplitude and arrive at the ultimate form of b.r.

$$\frac{1}{-\pi i}\left(\sum_{l=j+1}^{n+1}\mathrm{disc}_{s_{j,l}}-\sum_{l=0}^{j-1}\mathrm{disc}_{s_{l,j}}\right)\mathcal{A}^{\mathcal{S}}_{2\to n+2} = \left(\omega(t_{j+1})-\omega(t_j)\right)\Re\mathcal{A}_{2\to n+2}$$



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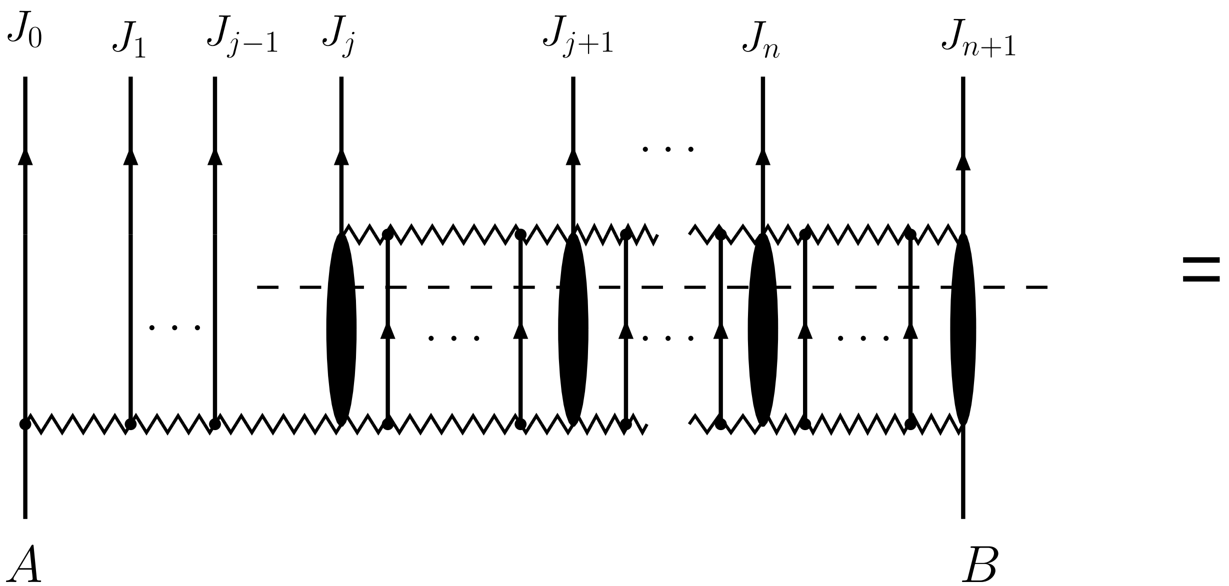
The verification of b.r. fulfilment has some remarkable features:

- It is possible to reduce all infi nite set of b.r. to limited numberof restrictions, named as bootstrap conditions, on the gluon trajectory and the Reggeon vertices.
- All bootstrap conditions are demonstrated to be satisfi ed by the known NLO vertices and the trajectory.
- Calculated separately discontinuities in the l.h.s. of the b.r. hold all the representations of colour group, but their sum contains only colour octets in every q_i–channel.

Calculation of discontinuities

Calculation of discontinuities in the l.h.s. of the b.r. is performed by the unitarity relation.



$$= -4i(2\pi)^{D-2}\delta\Big(q_{(j+1)\perp} - q_{j\perp} - \sum_{l=j}^{n+1} k_{l\perp}\Big)\mathrm{disc}_{s_{ij}}\mathcal{A}^{\mathcal{S}}_{2\to n+2} = \overline{\Gamma}^{R_1}_{J_0 A}\frac{e^{\omega(q_1)(y_0-y_1)}}{q^2_{1\perp}}\times$$

$$\times\left(\prod_{l=2}^{j}\gamma^{J_{l-1}}_{R_{l-1}R_l}\frac{e^{\omega(q_l)(y_{l-1}-y_l)}}{q^2_{l\perp}}\right)\langle J_j R_j|\left(\prod_{l=j+1}^{n} e^{\widehat{\mathcal{K}}(y_{l-1}-y_l)}\widehat{\mathcal{J}}_l\right)e^{\widehat{\mathcal{K}}(y_n-y_{n+1})}|J_{n+1}B\rangle$$

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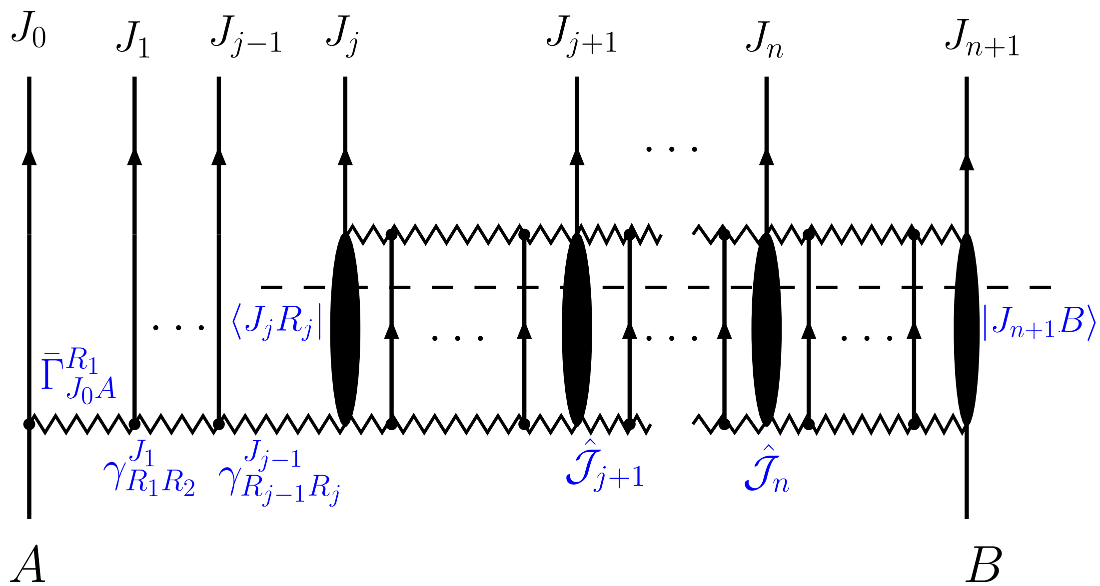

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$$\times\left(\prod_{l=2}^{j}\gamma^{J_{l-1}}_{R_{l-1}R_l}\frac{e^{\omega(q_l)(y_{l-1}-y_l)}}{q^2_{l\perp}}\right)\langle J_j R_j|\left(\prod_{l=j+1}^{n} e^{\widehat{\mathcal{K}}(y_{l-1}-y_l)}\widehat{\mathcal{J}}_l\right)e^{\widehat{\mathcal{K}}(y_n-y_{n+1})}|J_{n+1}B\rangle$$

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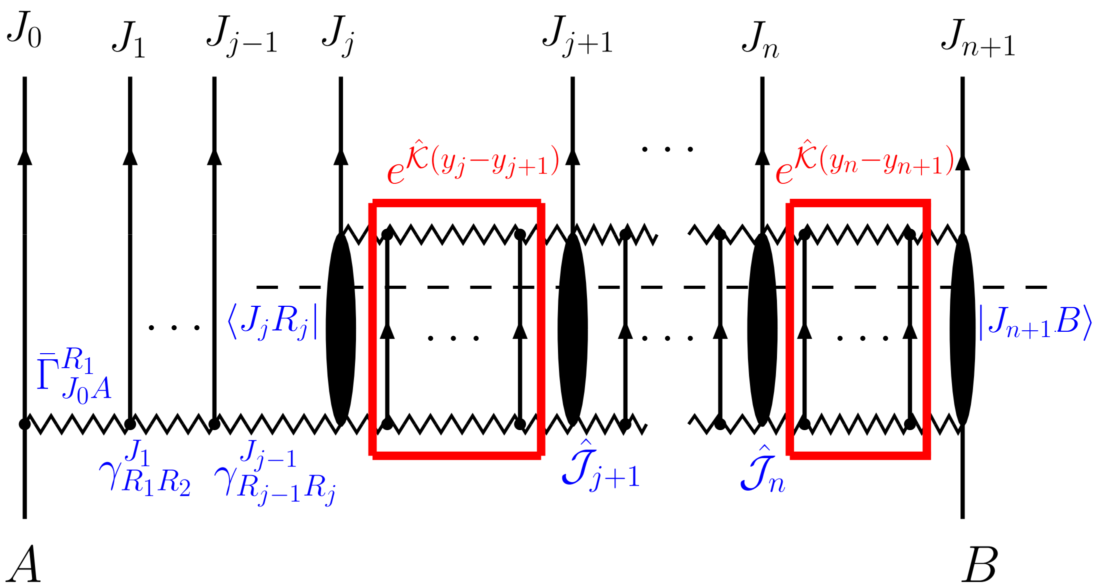

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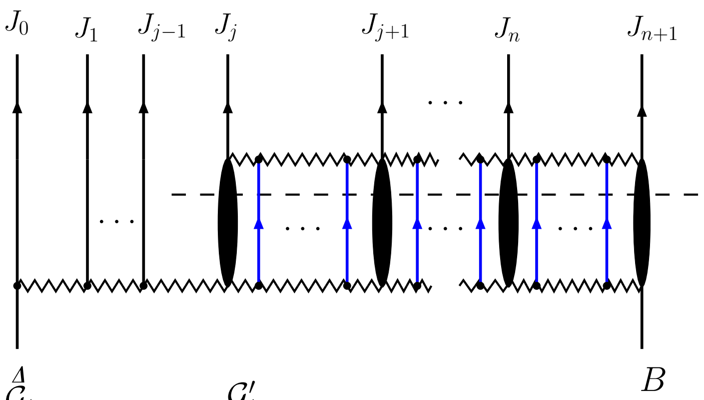
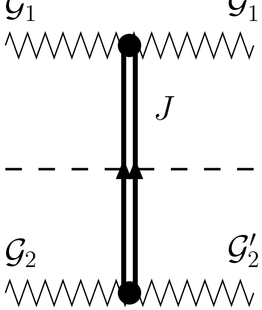
$$\times\left(\prod_{l=2}^{j}\gamma^{J_{l-1}}_{R_{l-1}R_l}\frac{e^{\omega(q_l)(y_{l-1}-y_l)}}{q^2_{l\perp}}\right)\langle J_j R_j|\left(\prod_{l=j+1}^{n} e^{\widehat{\mathcal{K}}(y_{l-1}-y_l)}\widehat{\mathcal{J}}_l\right)e^{\widehat{\mathcal{K}}(y_n-y_{n+1})}|J_{n+1}B\rangle$$

V.S. Fadin, R. Fiore, M.G. Kozlov, A.V. Reznichenko, hep-ph/0602006

Elements of the bootstrap conditions



BFKL kernel



$= \widehat{\mathcal{K}}_r^{\Delta}$

$$\widehat{\mathcal{K}} = \widehat{\Omega} + \widehat{\mathcal{K}}_r, \quad \widehat{\Omega} = \omega(\widehat{r}_1) + \omega(\widehat{r}_2), \quad \widehat{\mathcal{K}}_r = \widehat{\mathcal{K}}_r^{\Delta} - \widehat{\mathcal{K}}_r^{B} \widehat{\mathcal{K}}_r^{B} \Delta$$

$$\langle \mathcal{G}_1 \mathcal{G}_2 | \widehat{\mathcal{K}}_r^{\Delta} | \mathcal{G}'_1 \mathcal{G}'_2 \rangle \sim \sum_J \int \gamma^J_{\mathcal{G}_1 \mathcal{G}'_1} \gamma_J^{\mathcal{G}_2 \mathcal{G}'_2} \frac{d\phi_J}{2(2\pi)^{D-1}}$$

In our consideration we need only NLO octet non-forward BFKL kernel

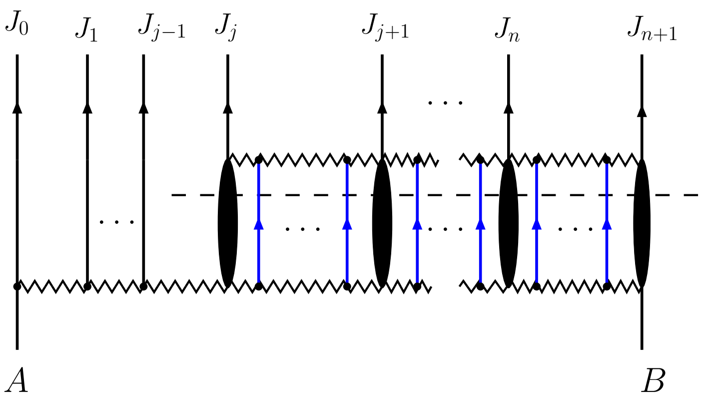
V.S. Fadin, D.A. Gorbachev, Yad. Fiz. 63, 2253 (2000); JETP Lett., 71, 222, (2000)

NLO forward singlet BFKL kernel is the most interesting for physical applications:

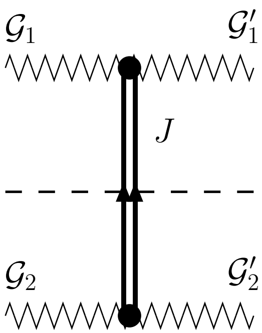
V.S. Fadin, L.N. Lipatov, Phys. Lett. B429 (1998) 127

M. Ciafaloni, G. Camici, Phys. Lett. B430 (1998) 349

Elements of the bootstrap conditions



BFKL kernel



$= \widehat{\mathcal{K}}_r^{\Delta}$

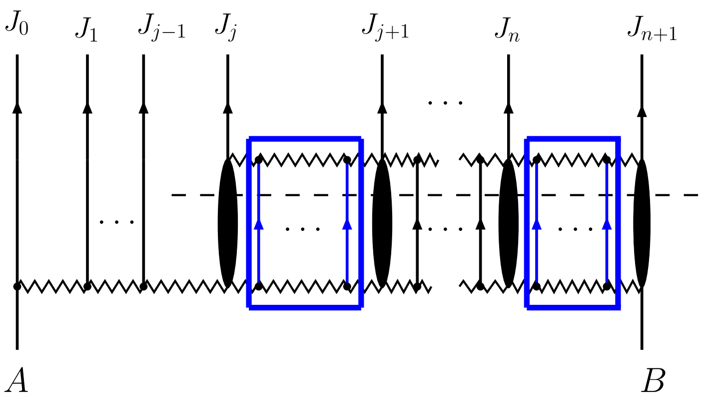
$$\widehat{\mathcal{K}} = \widehat{\Omega} + \widehat{\mathcal{K}}_r, \quad \widehat{\Omega} = \omega(\widehat{r}_1) + \omega(\widehat{r}_2), \quad \widehat{\mathcal{K}}_r = \widehat{\mathcal{K}}_r^{\Delta} - \widehat{\mathcal{K}}_r^{B}\widehat{\mathcal{K}}_r^{B}\Delta$$

$$\langle \mathcal{G}_1\mathcal{G}_2|\widehat{\mathcal{K}}_r^{\Delta}|\mathcal{G}_1'\mathcal{G}_2'\rangle \sim \sum_J \int \gamma^J_{\mathcal{G}_1\mathcal{G}_1'}\gamma_J^{\mathcal{G}_2\mathcal{G}_2'}\frac{d\phi_J}{2(2\pi)^{D-1}}$$

In contrast to the singlet kernel our quantity is non-physical, being singular as $\frac{1}{\epsilon}$ in $D = 4 + 2\epsilon$ regularization. The complete calculation of non-forward BFKL kernel within NLO for arbitrary colour representation has been accomplished only recently. The non-divergent form of its physical part has been found as well.

V.S. Fadin, R. Fiore, Phys. Rev. D 72 (2005) 014018

Elements of the bootstrap conditions

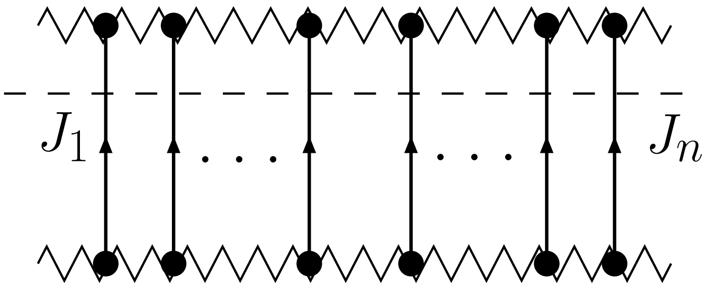


Green function

$Y_j = y_{j-1} - y_j$, Δ is the collinear divergence regularizator

$$\widehat{G}(Y_j)^{\Delta} = \sum_{n=0}^{\infty} \int\limits_{y_j+n\Delta}^{y_{j-1}-\Delta} e^{\hat{\Omega}(y_{j-1}-z_1)} \hat{\mathcal{K}}_r^{\Delta} dz_1 \int\limits_{y_j+(n-1)\Delta}^{z_1-\Delta} dz_2 e^{\hat{\Omega}(z_1-z_2)} \hat{\mathcal{K}}_r^{\Delta} \ldots \int\limits_{y_j+\Delta}^{z_{n-1}-\Delta} dz_n e^{\hat{\Omega}(z_n-y_j)} \hat{\mathcal{K}}_r^{\Delta} =$$

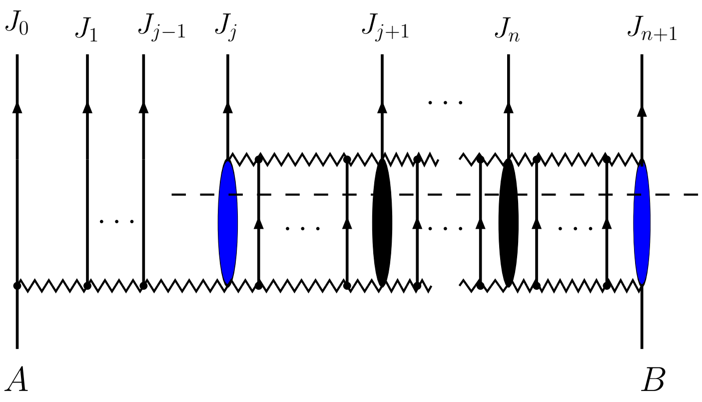
$$= \sum_{n=0}^{\infty}$$



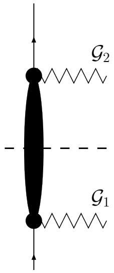
$$= (1 - \widehat{\mathcal{K}}_r^B \Delta) \widehat{G}(Y_j) (1 - \widehat{\mathcal{K}}_r^B \Delta)$$

$$\frac{d\widehat{G}(Y)}{dY} = \widehat{\mathcal{K}} \widehat{G}(Y) = \widehat{G}(Y) \widehat{\mathcal{K}}, \quad \widehat{G}(0) = 1 \Rightarrow \widehat{G}(Y) = e^{\widehat{\mathcal{K}} Y}$$

Elements of the bootstrap conditions

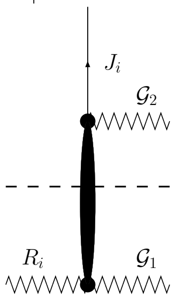


Impact-factors



$$\langle J_0 \bar{A}| = \langle J_0 \bar{A}|^{\Delta} \left(1 - \widehat{\mathcal{K}}_r^B \Delta\right)$$

$$\langle J_0 \bar{A}|\mathcal{G}_1 \mathcal{G}_2\rangle^{\Delta} \sim \sum_{\tilde{A}} \int \left(\Gamma^{\mathcal{G}_1}_{\tilde{A}A} \Gamma^{\mathcal{G}_2}_{J_0 \tilde{A}} - \Gamma^{\mathcal{G}_1}_{J_0 \tilde{A}} \Gamma^{\mathcal{G}_2}_{\tilde{A}A}\right) d\phi_{\tilde{A}}$$



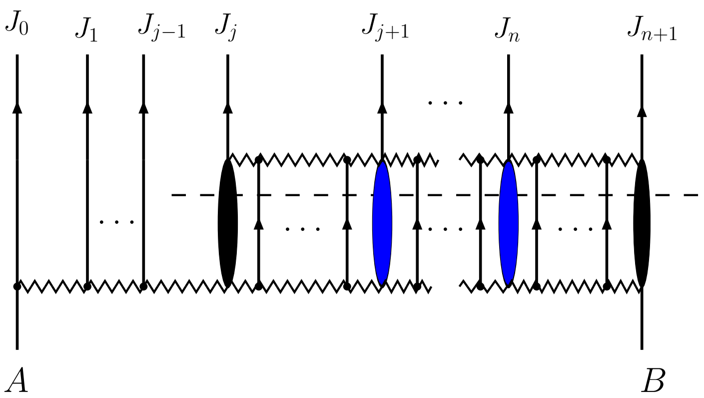
$$\langle J_i R_i| = \langle J_i R_i|^{\Delta} (1 - \widehat{\mathcal{K}}_r^B \Delta)$$

$$\langle J_i R_i|\mathcal{G}_1 \mathcal{G}_2\rangle^{\Delta} \sim \sum_{J} \int \left(\gamma^{J}_{R_i \mathcal{G}_1} \Gamma^{\mathcal{G}_2}_{J_i J} - \gamma^{J}_{R_i \mathcal{G}_2} \Gamma^{\mathcal{G}_1}_{J_i J}\right) d\phi_J$$

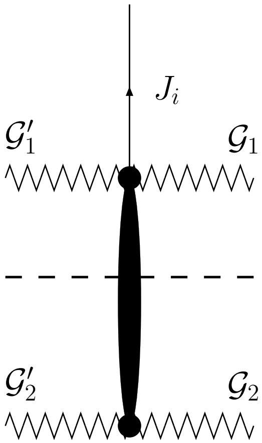
V.S. Fadin, *Diffraction 2002*, Ed. by R. Fiore *et al.*, NATO Science Series, Vol. 101, p.235.

J. Bartels, V.S. Fadin, R. Fiore, Nucl.Phys. B672 (2003) 329–356

Elements of the bootstrap conditions



Jet production operator



$$\widehat{\mathcal{J}}_i = \widehat{\mathcal{J}}_i^{\Delta} - \left(\widehat{\mathcal{K}}_r^B \widehat{\mathcal{J}}_i^B + \widehat{\mathcal{J}}_i^B \widehat{\mathcal{K}}_r^B\right)\Delta$$

$$\langle \mathcal{G}_1' \mathcal{G}_2' | \widehat{\mathcal{J}}_i^{\Delta} | \mathcal{G}_1 \mathcal{G}_2 \rangle \sim \Big[\gamma^{J_i}_{\mathcal{G}_1 \mathcal{G}_1'} \delta(r_{2\perp} - r'_{2\perp}) r^2_{2\perp} \delta_{\mathcal{G}_2 \mathcal{G}_2'} + \gamma^{J_i}_{\mathcal{G}_2 \mathcal{G}_2'} \delta(r_{1\perp} - r'_{1\perp}) r^2_{1\perp} \delta_{\mathcal{G}_1 \mathcal{G}_1'} +$$

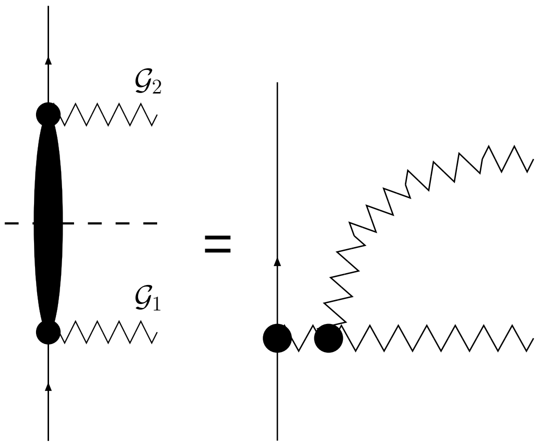
$$+ \sum_G \int_{y_i - \Delta}^{y_i + \Delta} \frac{dz_G}{2(2\pi)^{D-1}} \left(\gamma^{\{J_i G\}}_{\mathcal{G}_1 \mathcal{G}_1'} \gamma^{\mathcal{G}_2 \mathcal{G}_2'}_{G} + \gamma^{G}_{\mathcal{G}_1 \mathcal{G}_1'} \gamma^{\mathcal{G}_2 \mathcal{G}_2'}_{\{J_i G\}} \right) \Big]$$

V.S. Fadin, *Diffraction 2002*, Ed. by R. Fiore *et al.*, NATO Science Series, Vol. 101, p.235.
J. Bartels, V.S. Fadin, R. Fiore, Nucl.Phys. B672 (2003) 329–356

Bootstrap conditions for elastic case

$$\langle J_0 \bar{A}| = g\overline{\Gamma}^{R_1}_{J_0 A}\langle R_\omega(q_{A\perp})|$$

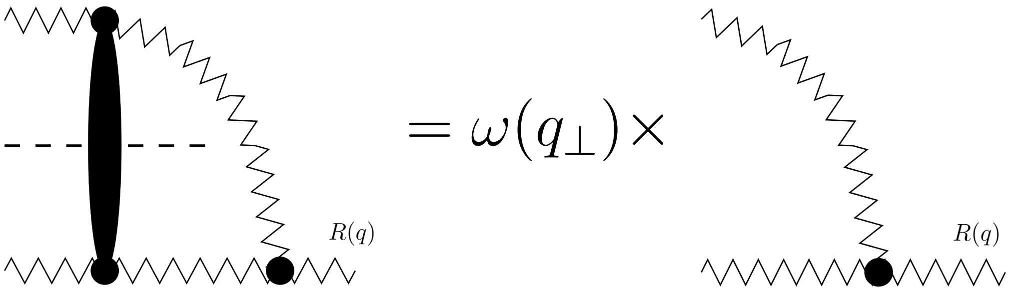
$$|\bar{J}_{n+1} B\rangle = g\Gamma^{R_{n+1}}_{J_{n+1} B}|R_\omega(q_{B\perp})\rangle$$



M. Braun, G.P. Vacca, Phys. Lett. B477 (2000) 156

V.S. Fadin, R. Fiore, M.I. Kotsky and A. Papa, Phys. Rev. D61 (2000) 094005, 094006

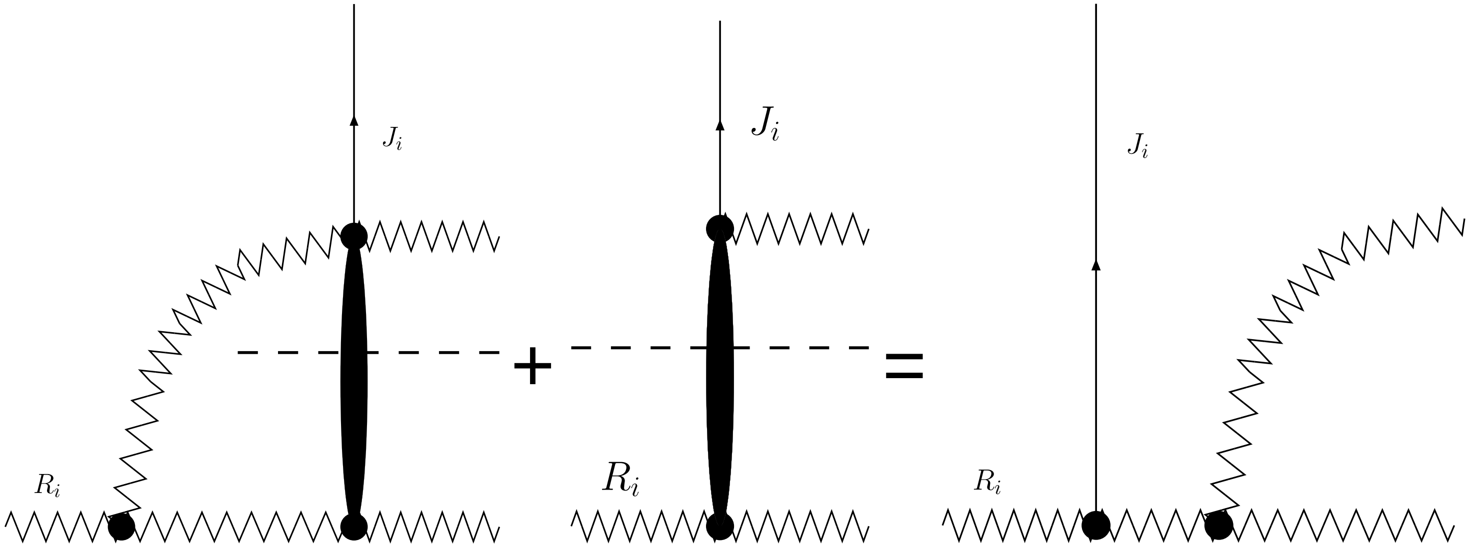
$$\widehat{\mathcal{K}}|R_\omega(q_\perp)\rangle = \omega(q_\perp)|R_\omega(q_\perp)\rangle$$



The following normalization $\frac{g^2 q_\perp^2}{2(2\pi)^{D-1}}\langle R_\omega(q'_\perp)|R_\omega(q_\perp)\rangle = -\delta(q'_\perp - q_\perp)\omega(q_\perp)$ is adopted.

V.S. Fadin, A. Papa, Nucl.Phys. B649 (2002) 309, hep-ph/0206079

Bootstrap conditions for inelastic case



$$g q_{i\perp}^2 \langle R_\omega(q_{i\perp})|\widehat{\mathcal{J}}_i + \langle J_i R_i| = g\gamma_{R_i R_i+1}^{J_i} \langle R_\omega(q_{i+1\perp})|; \quad J_i = \{G_1 G_2,\, Q\overline{Q}\}$$

V.S. Fadin, *Diffraction 2002*, Ed. by R. Fiore *et al.*, NATO Science Series, Vol. 101, p.235.

V.S. Fadin, M.G. Kozlov, A.V. Reznichenko, Yad. Fiz. 67 (2004) 377–393

$$g q_{i\perp}^2 \langle R_\omega(q_{i\perp})|\widehat{\mathcal{G}}_i + \langle G_i R_i| = g\gamma_{R_i R_i+1}^{G_i} \langle R_\omega(q_{i+1\perp})|$$

V.S. Fadin, M.G. Kozlov, A.V. Reznichenko, (2006) to be published

Quark Reggeization in LLA

$$\mathcal{A}^{\mathcal{R}}_{2\to n+2} = \overline{\Gamma}^{R_1}_{A'A} \frac{s_1^{\omega_1}}{d_1} \gamma^{P_1}_{R_1 R_2} \frac{s_2^{\omega_2}}{d_2} \cdots \gamma^{P_n}_{R_n R_{n+1}} \frac{s_{n+1}^{\omega_{n+1}}}{d_{n+1}} \Gamma^{R_{n+1}}_{B'B}$$

$$d_i = \begin{cases} q^2_{i\perp} \\ m - \not{q}_{i\perp} \end{cases} \qquad \omega_i = \begin{cases} \omega_{\mathcal{G}}(q_i), \text{ in } \mathcal{G} \text{ channel} \\ \omega_{\mathcal{Q}}(q_i) = \dfrac{g^2 C_F}{(2\pi)^{D-1}} \displaystyle\int \frac{(m - \not{q}_{i\perp}) d^{D-2} k_\perp}{(m - \not{k}_\perp)(q_i - k)^2_\perp}, \quad \text{in } \mathcal{Q} \text{ channel} \end{cases}$$

All engaged vertices and trajectory were calculated with necessary LO accuracy:

V.S. Fadin, V.E. Sherman, Zh. Eksp. Teor. Fiz. 23 (1976), 599; 72 (1977), 1640

$\omega_{\mathcal{Q}}$ is known now up to two loops for arbitrary space-time dimension D:

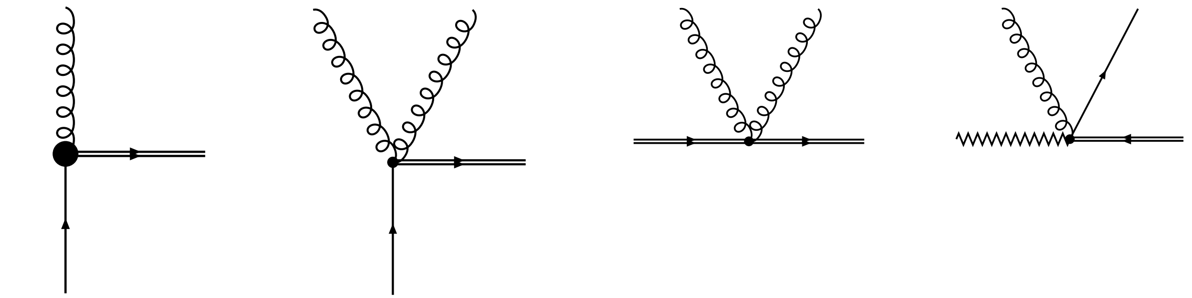
A.V. Bogdan, V. Del Duca, V.S. Fadin and E.W.N. Glover, JHEP 203 (2002), 32
A.V. Bogdan, V.S. Fadin, Yad. Fiz. 68 (2005), 1659–1675

The proof is performed by treating LO bootstrap conditions to bootstrap relations:
A.V. Bogdan, V.S. Fadin, Nucl. Phys. B, to be published (2006), hep-ph/0601117

Quark Reggeization in LLA

$$\mathcal{A}_{2\to n+2}^{\mathcal{R}} = \overline{\Gamma}_{A'A}^{R_1} \frac{s_1^{\omega_1}}{d_1} \gamma_{R_1 R_2}^{P_1} \frac{s_2^{\omega_2}}{d_2} \cdots \gamma_{R_n R_{n+1}}^{P_n} \frac{s_{n+1}^{\omega_{n+1}}}{d_{n+1}} \Gamma_{B'B}^{R_{n+1}}$$

Following vertices are calculated in NLO now:

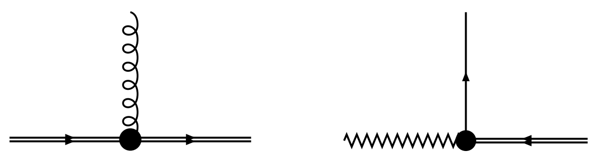


L.N. Lipatov, M.I. Vyazovsky, Nucl. Phys. B597 (2001) 399

V.S. Fadin, R. Fiore, Phys. Rev. D64 (2001) 114012

M.I. Kotsky, L.N. Lipatov, A. Principe, Vyazovsky, Nucl. Phys. B648 (2003) 277

but two residual effective vertices $R_Q G R_Q$ and $R_G Q R_Q$ have not been obtained yet:



Their calculation will allow us to perform the proof of quark Reggeization within NLA.

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- Quark Reggeization is required by the hadron phenomenology to construct Reggeons as a colourless states of Reggeized quarks.
  - This phenomenon has been recently proved in the LLA in the same bootstrap scheme.
  - Now the calculation of NLO vertices $R_Q G R_Q$ and $R_G Q R_Q$ is the primary task.