

Classical point-like electron with finite electromagnetic and total masses

S. I. Manaenkov

High Energy Physics Division Seminar

Gatchina, March 28, 2017

- Gravitation and Particle Physics
- Reissner-Nordström Solution
- Energy-Momentum Tensors of Electromagnetic and Gravitational Fields
- Total Inert Mass of Classical Electron
- "Electromagnetic Mass" of Classical Electron
- Charge Conjugation
- Discussion and Conclusions

Gravitation and Particle Physics

- Ratio of electromagnetic to gravitational forces for two electrons

$$\frac{\mathcal{F}_{el}}{\mathcal{F}_{gr}} = \frac{e^2}{R^2} / \frac{km^2}{R^2} = 4.2 \cdot 10^{42},$$

where $e = 4.8 \cdot 10^{-10}$ CGSE, $m = 9.11 \cdot 10^{-28}$ g, $k = 6.67 \cdot 10^{-8}$ cm³ g⁻¹ c⁻².

- The gravitation field does not play any role in the elementary particle structure!

Nevertheless this statement could be wrong since the equation for the gravitational field is nonlinear.

- Infinite electromagnetic mass of the electron

$$m_{em} = \frac{\mathcal{E}_{em}}{c^2} = \frac{1}{c^2} \int \frac{\vec{E}^2}{8\pi} dV = \int \frac{e^2 dR}{2c^2 R^2} \rightarrow \infty,$$

$dV = 4\pi R^2 dR$. $m_{em} c^2 = e^2 / (2r_0)$, $r_0 = 1.4 \cdot 10^{-13}$ cm is the classical electron radius.

- Gravitational interaction in Newtonian physics

$$dU_{gr} = -\frac{k dm_1 dm_2}{R_{12}} = -\frac{k}{R_{12}} \frac{e^2 dV_1}{c^2 R_1^4} \frac{e^2 dV_2}{c^2 R_2^4} < 0.$$

$$U_{gr} / \mathcal{E}_{em} = -\frac{1}{12} \frac{r_e^2}{R_c^2}, \quad \text{where } r_e^2 = ke^2/c^4, r_e = 1.4 \cdot 10^{-34} \text{ cm.}$$

Gravitation can play important role at $r \leq r_e$.

● Riemannian geometry

Minkowsky space-time

$$ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2$$

$$= \sum_{i,j} g_{ik} dx^i dx^j \equiv g_{ij} dx^i dx^j,$$

where $dx^0 = cdt$, $dx^1 = dx$, $dx^2 = dy$, $dx^3 = dz$, $g_{ij} = \text{diag}(1, -1, -1, -1)$.

- If the component of the metric tensor $g_{ij} = g_{ji}$ are functions of coordinates x^l , the geometry is Riemannian.
 - The space is Euclidian (pseudo-Euclidian) if the Riemannian tensor $R^j_{inl} \equiv 0$.
 - Metric tensor components play a role of the gravitational potentials and can be found from the Einstein equations: $R_{ij} - \frac{1}{2}Rg_{ij} = \frac{8\pi k}{c^4}T_{ij}$.
- T_{ij} is the energy-momentum tensor of matter, $R_{ij} = R^n_{inj}$ is the Ricci tensor, $R = R_{ij}g^{ji}$.

• Electric and gravitational fields

$$F^{\mu 0} = F_{\mu 0} = -F_{0\mu} = \vec{E}^{\mu} = \frac{e}{r^2} \vec{n}^{\mu},$$

where $\vec{n}^{\mu} = \mathbf{x}^{\mu}/r$ is the unit vector, $\mu = 1, 2, 3$; $r^2 = (\mathbf{x}^1)^2 + (\mathbf{x}^2)^2 + (\mathbf{x}^3)^2$.

$$g_{00} = \Lambda, \quad g^{00} = 1/\Lambda,$$

$$\Lambda = 1 - \frac{2km}{c^2 r} + \frac{ke^2}{c^4 r^2} \equiv \frac{D(r)}{r^2} = \frac{r^2 - r_g r + (r_e)^2}{r^2},$$

where $r_g = \frac{2km}{c^2} = 1.35 \cdot 10^{-55} \text{cm}$ is Schwarzschild radius of electron,

$$r_e^2 = \frac{ke^2}{c^4}, \quad r_e = 1.38 \cdot 10^{-34} \text{cm}. \quad r_e \gg r_g \Rightarrow D(r) > 0.$$

$$g^{\alpha\beta} = -[\delta_{\alpha\beta} + (\Lambda - 1)\vec{n}^{\alpha}\vec{n}^{\beta}],$$

where $\delta_{\alpha\beta} = \text{diag}(1, 1, 1)$ is the Kronecker symbol. $\det(g^{ij}) = -1.$

Energy-Momentum Tensors of Electromagnetic and Gravitational Fields

- **General Definition of Pseudo-Tensor t^{ij} for the case $\det(g_{ij}) = -1$**

$$P^i = \frac{1}{c} \int (T^{i0} + t^{i0}) dV,$$

where T^{ij} is the energy-momentum tensor of the electromagnetic field, while t^{ij} denotes the energy-momentum pseudo-tensor of the gravitational field.

$$T^{ij} + t^{ij} = \frac{\partial h^{ijl}}{\partial x^l} \equiv \sum_{l=0}^3 \frac{\partial h^{ijl}}{\partial x^l},$$

where $h^{ijl} = -h^{ijl}$ is antisymmetric for j and l and is given by (L.D. Landau and E.M. Lifshitz, 1947)

$$h^{ijl} = \frac{c^4}{16\pi k} \frac{\partial}{\partial x^n} [g^{ij} g^{ln} - g^{il} g^{jn}].$$

- **Main property of $(T^{ij} + t^{ij})$**

$$\frac{\partial [T^{ij} + t^{ij}]}{\partial x^j} \equiv \frac{\partial [T^{i0} + t^{i0}]}{\partial x^0} + \frac{\partial [T^{i1} + t^{i1}]}{\partial x^1} + \frac{\partial [T^{i2} + t^{i2}]}{\partial x^2} + \frac{\partial [T^{i3} + t^{i3}]}{\partial x^3} = 0.$$

Total Inert Mass of Classical Electron

- Energy-Momentum Tensors for the Reissner-Nordström solution

The metric tensor g^{ij} depends on r only. Greek indexes $\alpha, \beta, \dots = 1, 2, 3$; $\det(g_{ij}) = -1$.

$$T^{00} + t^{00} = \frac{\partial h^{00\alpha}}{\partial x^\alpha} = \frac{c^4}{16\pi k} \frac{\partial^2}{\partial x^\alpha \partial x^\beta} [g^{00} g^{\alpha\beta}]$$

- Total energy of the system of electromagnetic and gravitational fields

$$\mathcal{E} = P^0 c = \int (T^{00} + t^{00}) dV =$$
$$\frac{1}{8\pi} \int_0^\infty \left\{ \frac{r^4}{D^2} \left[\frac{e^2}{r^4} - \frac{4km^2}{r^4} + \frac{4kme^2}{c^2 r^5} - \frac{ke^4}{c^4 r^6} \right] \right\} 4\pi r^2 dr,$$

$$\mathcal{E} = mc^2.$$

where $D = r^2 - r_g r + r_e^2 = r^2 - \frac{2km}{c^2} r + \frac{ke^2}{c^4}$ and $dV = 4\pi r^2 dr$.

THERE IS NO ANY POINT-LIKE PARTICLE WITH MASS m BUT mc^2 IS THE TOTAL ENERGY OF THE SYSTEM OF ELECTROMAGNETIC AND GRAVITATIONAL FIELDS.

"Electromagnetic Mass" of Classical Electron

- Large- and small- r behaviour of the integrand for the total energy \mathcal{E}

$$r \gg r_e, \quad D \approx r^2, \quad r^4/D^2 \approx 1,$$

$$T^{00} + t^{00} \approx \frac{e^2}{8\pi r^4}, \quad \int_{r_e}^{\infty} \left\{ \frac{e^2}{8\pi r^4} \right\} 4\pi r^2 dr = \frac{e^2}{2r_e}.$$

$$r \leq r_e, \quad D \sim r_e^2, \quad r^4/D^2 \sim (r/r_e)^4,$$

$$(T^{00} + t^{00})4\pi r^2 \rightarrow -e^2/(2r_e^2).$$

Contribution of large distances ($r > r_e$) is positive and is about e^2/r_e , while contribution of small distances ($r < r_e$) is negative and is also about $-e^2/r_e$.

The total integral is equal to $mc^2 \ll e^2/r_e$
since $mc^2 = 0.511$ MeV while $e^2/r_e \sim 10^{21}$ MeV.

"Electromagnetic Mass" of Classical Electron

- "Electromagnetic mass" (energy) of electron

$$\mathcal{E}_{em} = m_{em}c^2 = \int T^{00}dV = \int T_0^0 g^{00}dV,$$

where $T_0^0 = \tilde{\mathbf{E}}^2/(8\pi) = e^2/(8\pi r^4)$, $dV = 4\pi r^2$,

Since $g^{00} = r^2/D = r^2/(r^2 - r_g r + r_e^2)$,

then $g^{00} \rightarrow r^2/r_e^2$ when $r \rightarrow 0$

and $g^{00} \rightarrow 1$ when $r \rightarrow \infty$.

$D = (r - r_g/2)^2 + r_{eg}^2$ with $r_{eg}^2 = r_e^2 - r_g^2/4 > 0$, since $r_e \gg r_g$. Hence $D > 0$ for any r .

Therefore the integral is convergent.

$$\mathcal{E}_{em} = 1.3 \cdot 10^{15} \text{ erg} = 8.2 \cdot 10^{26} \text{ eV}; m_{em} = 1.5 \cdot 10^{-6} \text{ g.}$$

$$\mathcal{E}_{em}/mc^2 \approx 1.6 \cdot 10^{21}.$$

- "Electromagnetic mass" of positron is the same as for electron

Charge Conjugation

- What is the electron?

$$\vec{E} = \frac{e}{r^2} \vec{n}, \quad g_{00} = \Lambda, \quad g^{00} = 1/\Lambda,$$

$$\Lambda = 1 - \frac{2km}{c^2 r} + \frac{ke^2}{c^4 r^2}, \quad g^{\mu\nu} = g^{\mu\nu}(\Lambda)$$

- What is the positron?

$$\vec{E} = -\frac{e}{r^2} \vec{n} \equiv \frac{|e|}{r^2} \vec{n},$$

$$g_{00} = \Lambda, \quad g^{00} = 1/\Lambda, \quad g^{\mu\nu} = g^{\mu\nu}(\Lambda)$$

- Charge conjugation transformation

$$e \rightarrow -e, \quad m \rightarrow m, \quad \vec{E} \rightarrow -\vec{E}, \quad g^{ij} \rightarrow g^{ij}$$

Positron has the same mass as the electron. It is not the particle with the negative mass as in the Dirac equation.

- Particles with $m = 0$ could be "vacuum" particles

Discussion and Conclusions

- **Alteration of the space-time metric due to gravitation field can made the electron mass finite.** Metric is changed at distances $r_e \sim 10^{-34}$ cm.
- The electromagnetic field contribution to the total electron mass is positive and dominates at $r \gg r_e$. The "electromagnetic mass" is about 10^{21} MeV/c².
- The gravitational field contribution to the total electron mass is negative and dominates at $r < r_e$. This contribution is also about 10^{21} MeV/c².
- The parameter m of the Reissner-Nordström solution is both the inert and gravitational mass.

There are solutions with $m = m_e$ and even with masses $m = 0$ and $m < 0$.

- The solution with $m = 0$ can probably be used for the construction of the vacuum state.
- The total electron mass is a mass of the electromagnetic and gravitation fields.

There is no need in a point-like massive particle (usual electron).

- **Charge conjugation:** $e \rightarrow -e$, $m \rightarrow m$, $\tilde{E} \rightarrow -\tilde{E}$, $g_{ij} \rightarrow g_{ij}$. **Positron mass is positive.**
- The action corresponding to the Reissner-Nordström solution is infinite:

$$L = -\frac{1}{16\pi c} g^{ij} g^{nl} F_{in} F_{jl} - \frac{c^3}{16\pi k} R = \frac{1}{8\pi c} \frac{e^2}{r^4}.$$