ОКТУПОЛЬНАЯ ДЕФОРМАЦИЯ ЯДЕР

1. Как октупольная деформация проявляется в свойствах ядер?

2. Результаты экспериментов на ISOLDE и TRIUMF:
октупольная деформация в $^{217-219}At$ и $^{225-229}Ac$

3. Что дают октупольно деформированные ядра для физики элементарных частиц?

Octupole deformation

$$R(\theta) = c(\beta_\lambda) R_0 \left[1 + \sum_{\lambda=2}^{\infty} \sqrt{\frac{2\lambda+1}{4\pi}} \beta_\lambda P_{\lambda 0}(\cos\theta) \right]$$

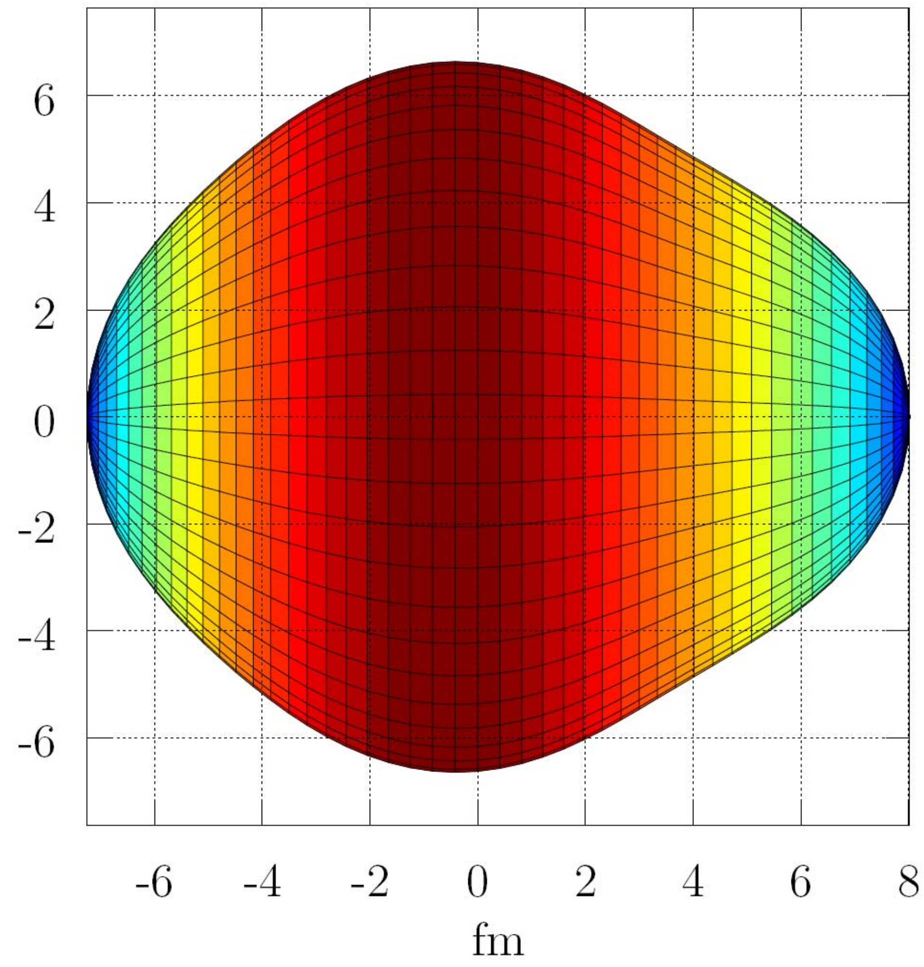
β_3 — octupole deformation parameter

$$Q_2(\text{intrinsic}) \approx \frac{3}{\sqrt{5\pi}} Z R_0^2 \beta_2$$

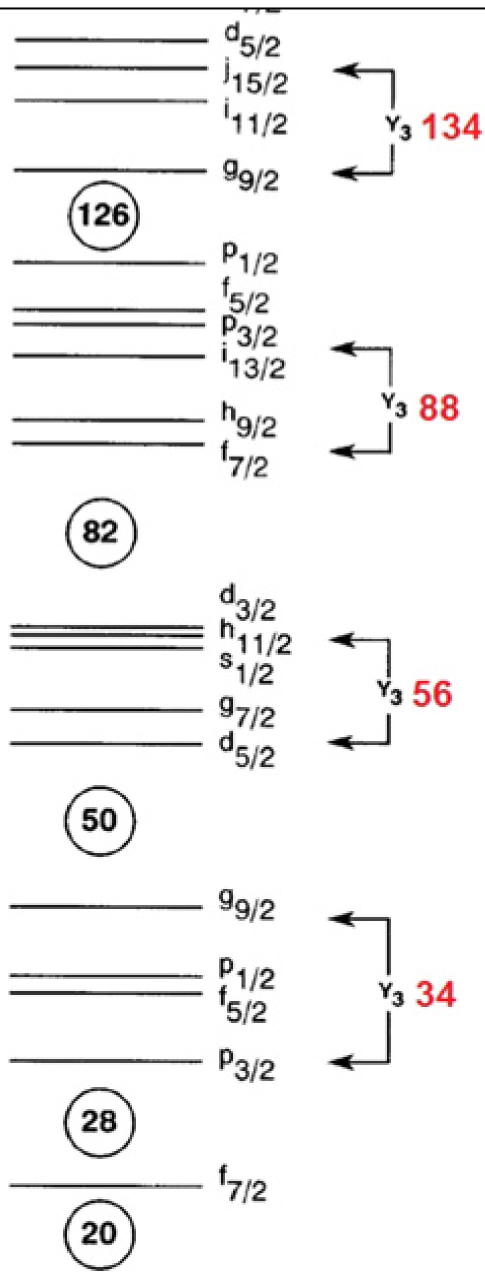
$$Q_3(\text{intrinsic}) \approx \frac{3}{\sqrt{7\pi}} Z R_0^3 \beta_3$$

$$Q_2^S = \frac{3K^2 - I\cdot(I+1)}{(I+1)\cdot(2I+3)} \cdot Q_2(\text{intrinsic})$$

^{224}Ra



Where are the possible regions of octupolarity?



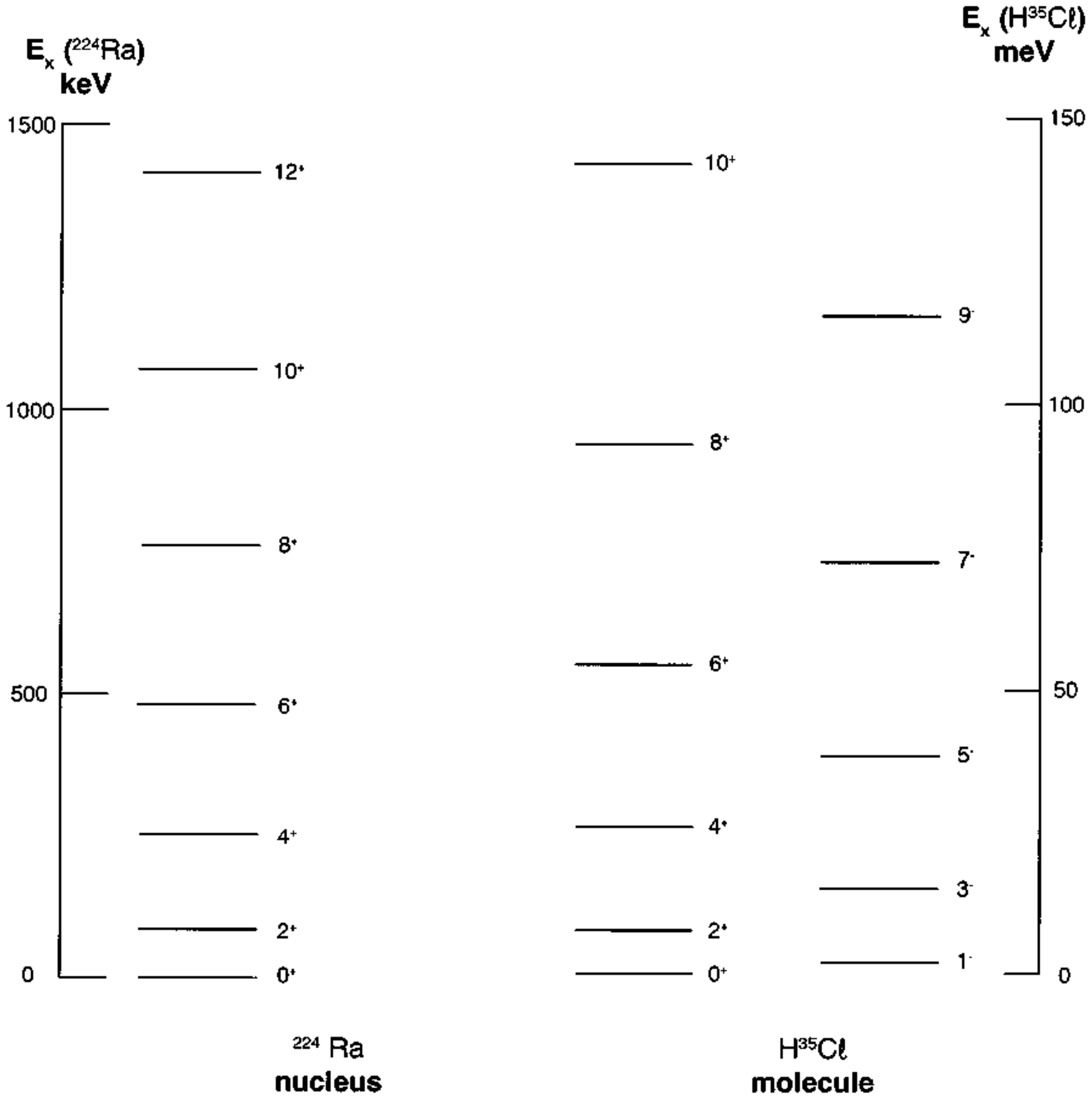
Strong octupole correlations leading to pear shapes can arise when nucleons near the Fermi surface occupy states of opposite parity with orbital and total angular momentum differing by 3.

Region of octupole deformation

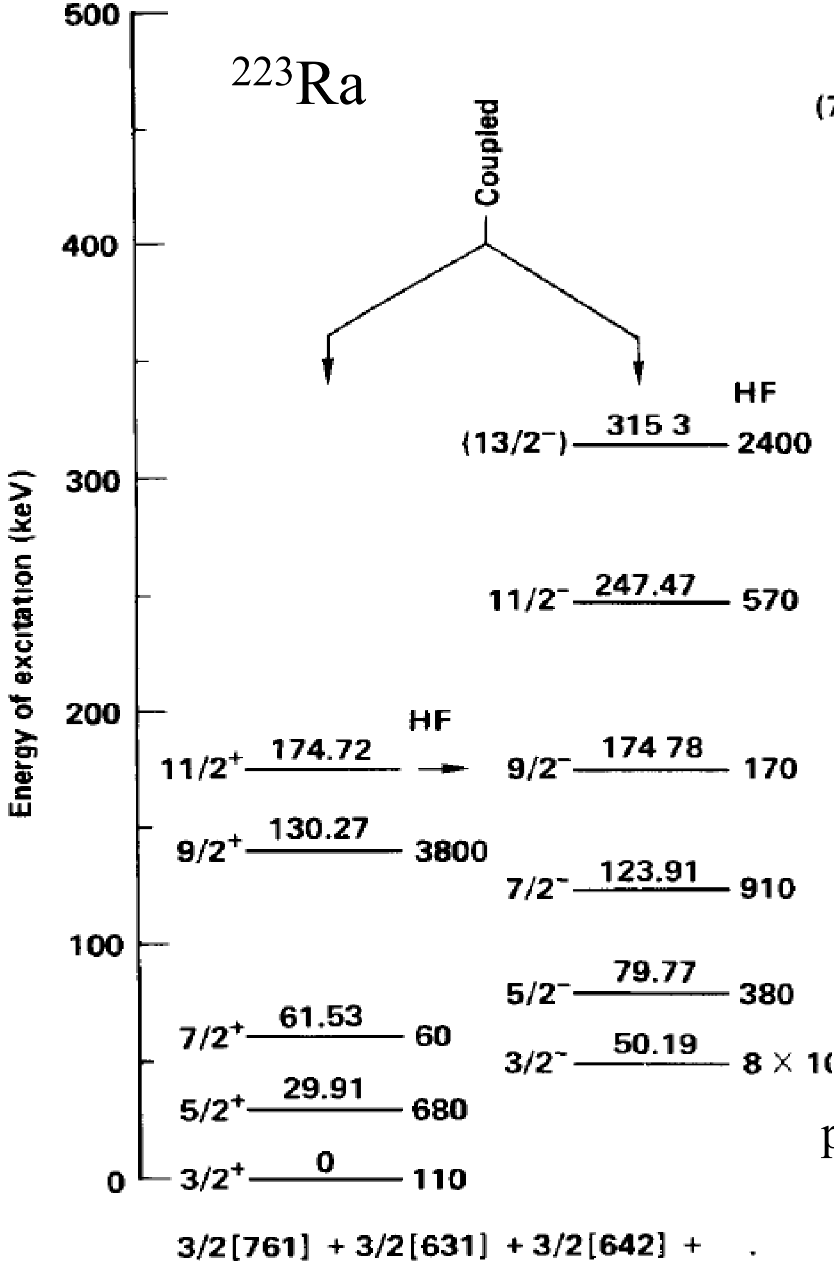
← Neutron No. →

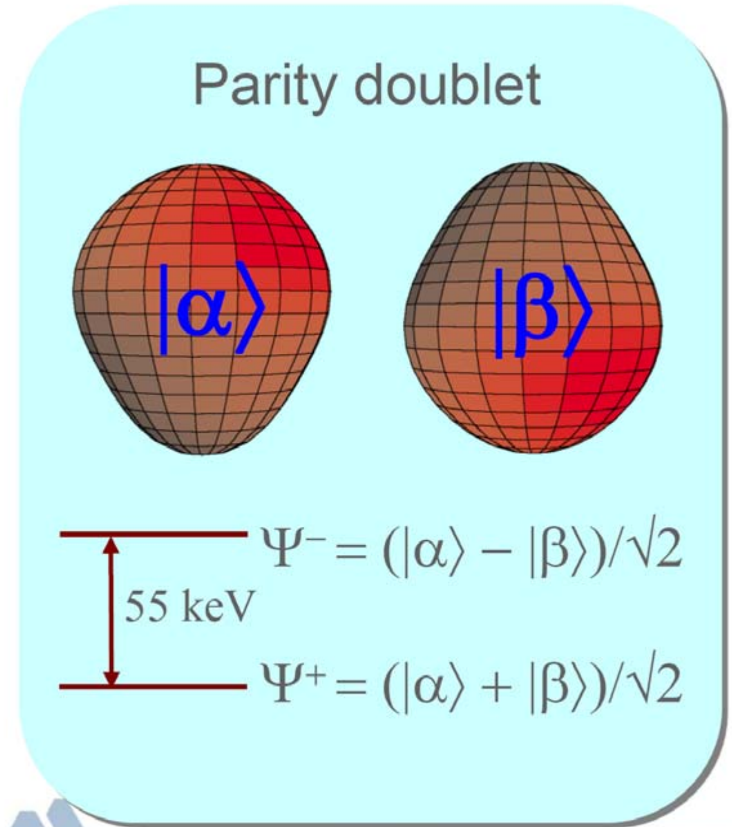
	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146
$_{82}$Pb		$9/2^+$		$9/2^+$																	
$_{83}$Bi	$9/2^-$		$(9/2)^-$		$9/2^-$																
$_{84}$Po		$(9/2^+)$		$9/2^+$		$(9/2)^+$															
$_{85}$At	$9/2^-$		$9/2^-$		$9/2^-$		$9/2^-$														
$_{86}$Rn		$9/2^+$		$(9/2^+)$		$9/2^+$		$5/2^+$		$7/2$											
$_{87}$Fr	$9/2^-$		$9/2^-$		$9/2^-$		$(9/2)^-$		$5/2^-$ $K=1/2^-$		$3/2^{(-)}$		$3/2^{(-)}$		$1/2$						
$_{88}$Ra		$9/2^+$		$9/2^+$		not $9/2^+$		$5/2^+$		$3/2^+$		$1/2^+$		$3/2^+$		$5/2$					
$_{89}$Ac	$9/2^-$		$9/2^-$		$9/2^-$				$5/2^-$		$3/2^-$		$3/2^-$		$(3/2^+)$		$(1/2^+)$				
$_{90}$Th				$(9/2^+)$		not $9/2^+$		$(5/2^+)$		$3/2^+$		$1/2^+$		$5/2^+$		$5/2^+$		$1/2^+$			
$_{91}$Pa											$(5/2^-)$		$(5/2^+)$		$3/2^-$ $K=1/2^-$		$3/2^-$ $K=1/2^-$		$(3/2^-)$ $(K=1/2^-)$		$(1/2^+)$
$_{92}$U												$3/2^+$		$(5/2)$		$5/2^+$		$7/2^-$		$1/2^+$	
$_{93}$Np													$(5/2)$		$(5/2^+)$		$5/2^+$		$5/2^+$		$5/2^+$

Low-lying negative parity bands in even-even nuclei



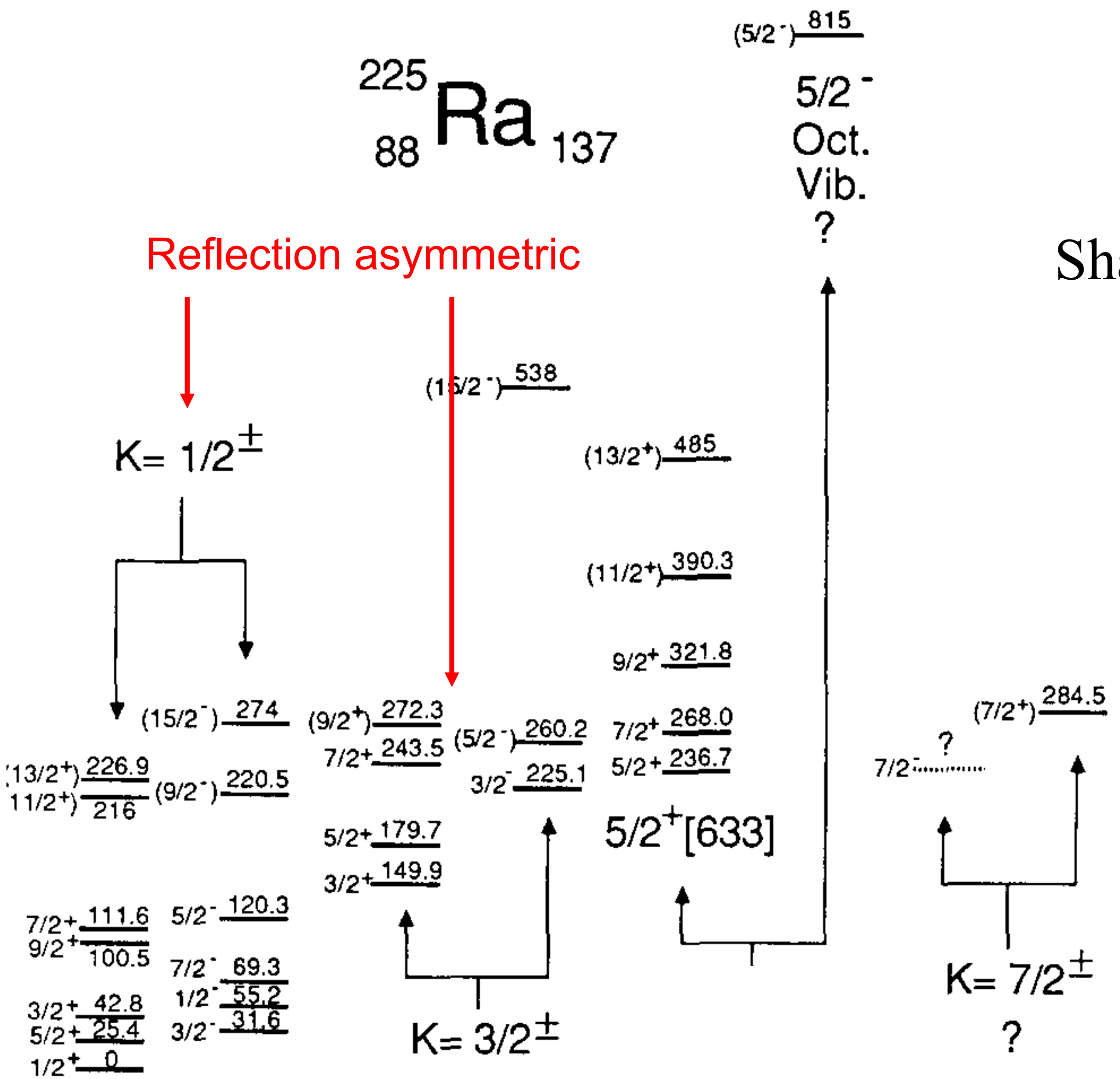
Parity doublets in odd nuclei





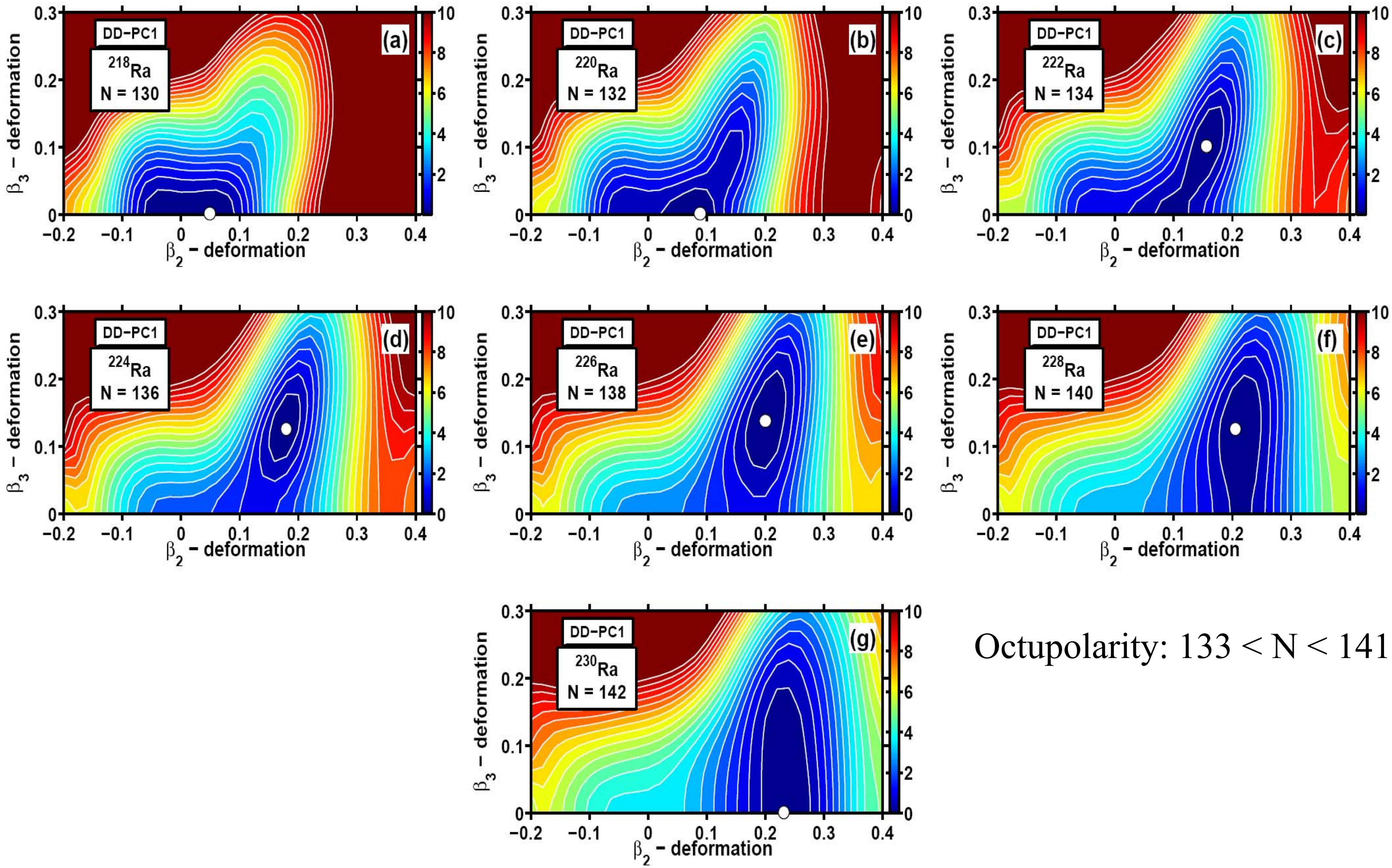
parity partner with the same intrinsic structure

Parity doublets in odd nuclei



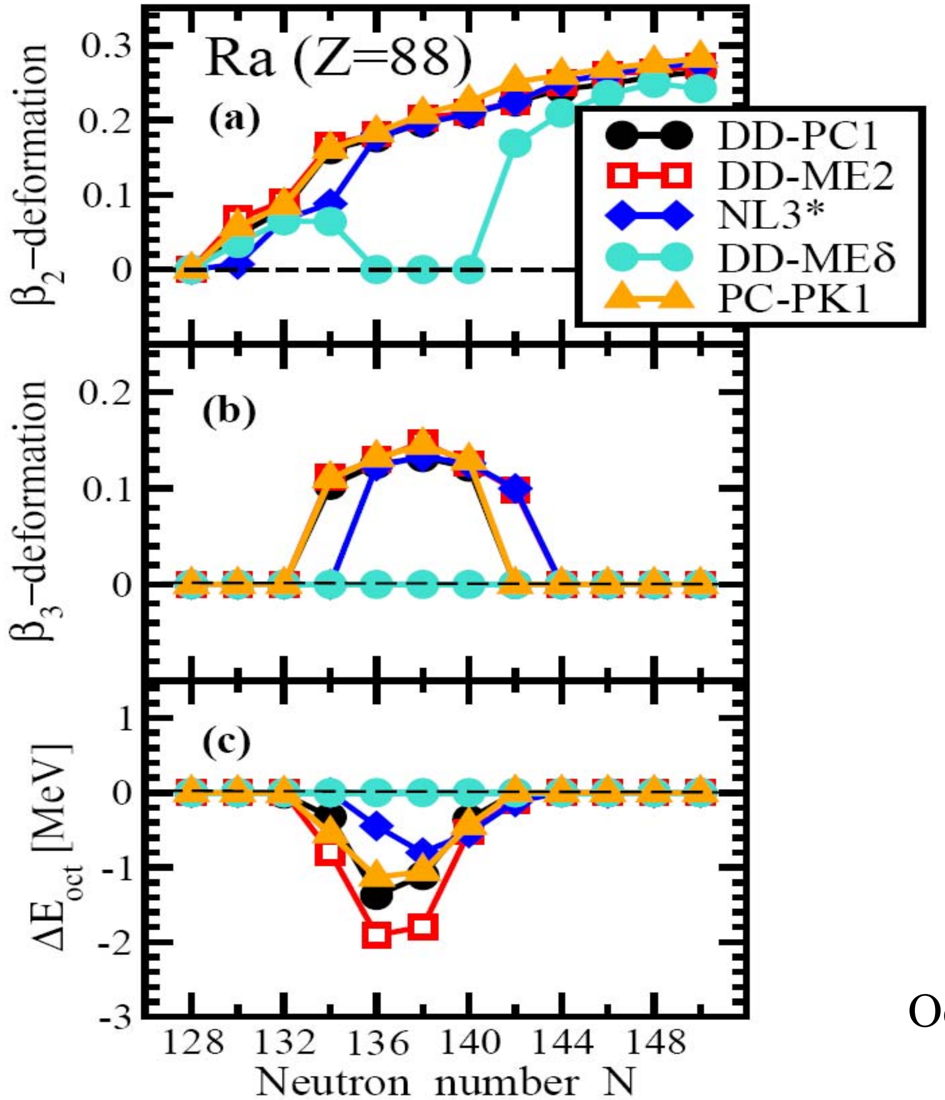
Shape coexistence

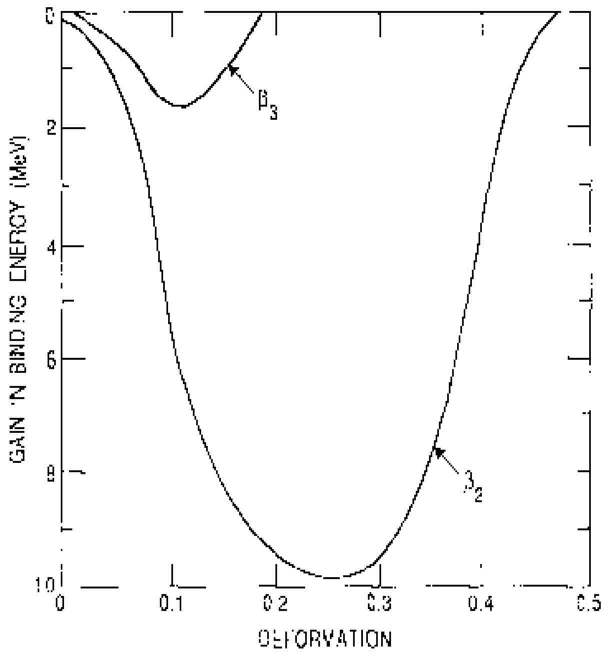
Minima in the potential energy surfaces at $\beta_3 > 0$



Octupolarity: $133 < N < 141$

Minima in the potential energy surfaces at $\beta_3 > 0$





Octupolarity: 133 < N < 142

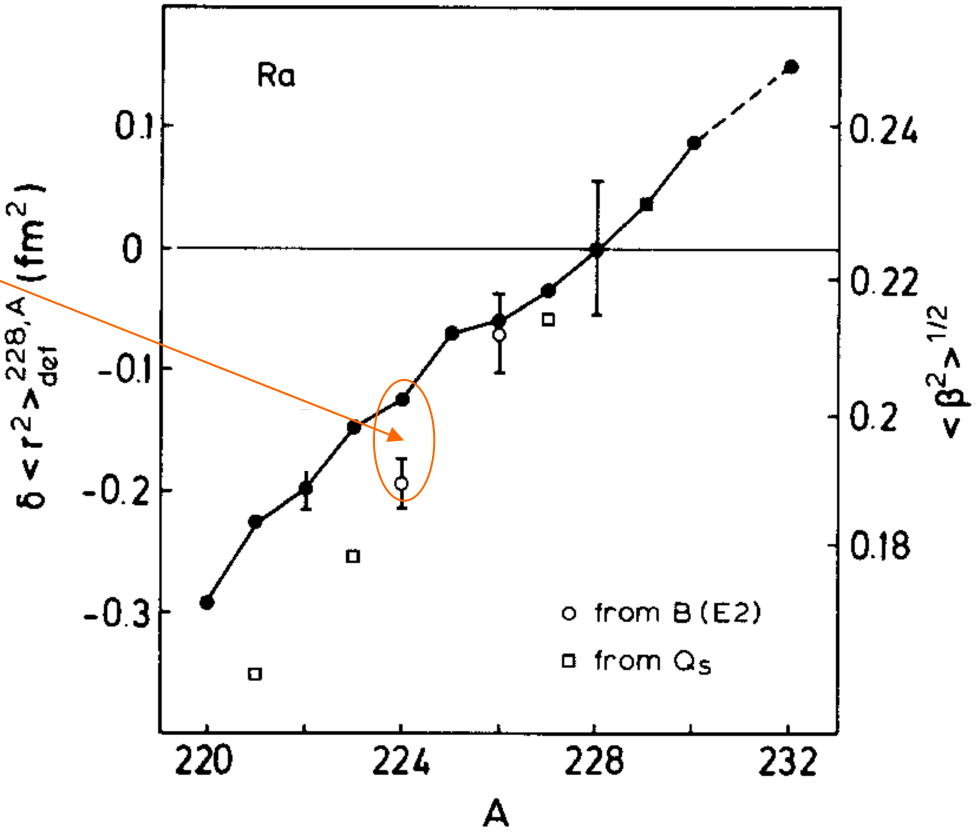
Octupole deformation and radii

$$\langle r^2 \rangle = \langle r^2 \rangle_0 \left[1 + \frac{5}{4\pi} \left(\langle \beta_2^2 \rangle + \langle \beta_3^2 \rangle \right) \right]$$

$$Q_0 = \frac{3}{\sqrt{5\pi}} Z R_0^2 \beta_2 \left(1 + \sqrt{\frac{5}{4\pi}} \frac{4}{7} \beta_2 \right)$$

$$\sum_i \boldsymbol{B}(\mathrm{E2}; 0^+ \rightarrow 2_i^+) = \left(\frac{3}{4\pi} ZR^2 \right)^2 \langle \beta_2^2 \rangle$$

$\beta_3(^{224}\mathrm{Ra}) \approx 0.1$
will close the gap



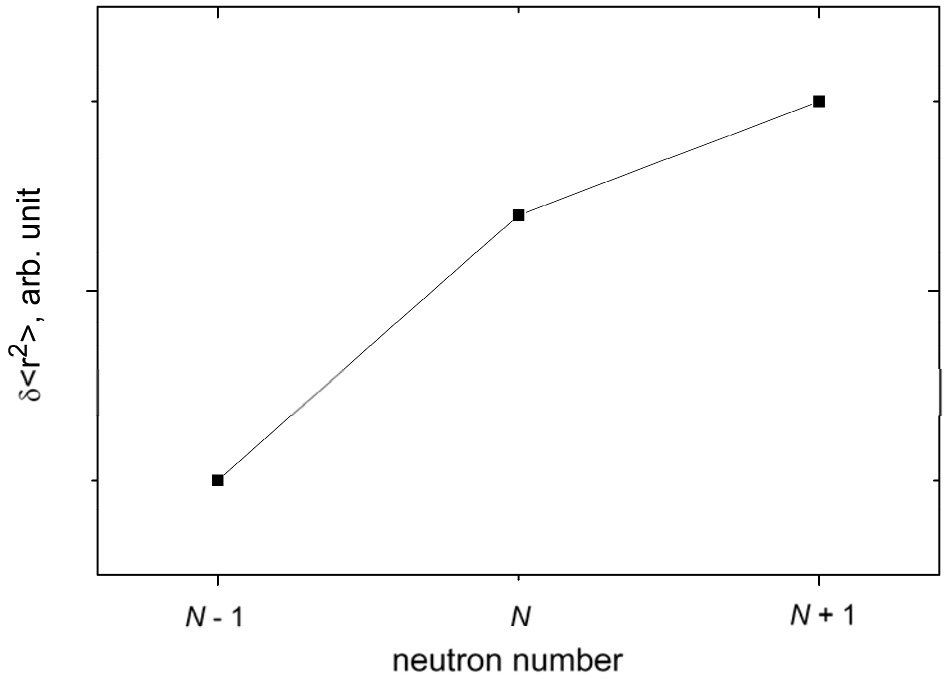
Odd-even staggering in radii

staggering parameter: $\gamma\left(N\right)=\dfrac{2\cdot\delta\left\langle r^2_{N,\,N-1}\right\rangle}{\delta\left\langle r^2_{N+1,\,N-1}\right\rangle}$ $\quad N$ — odd

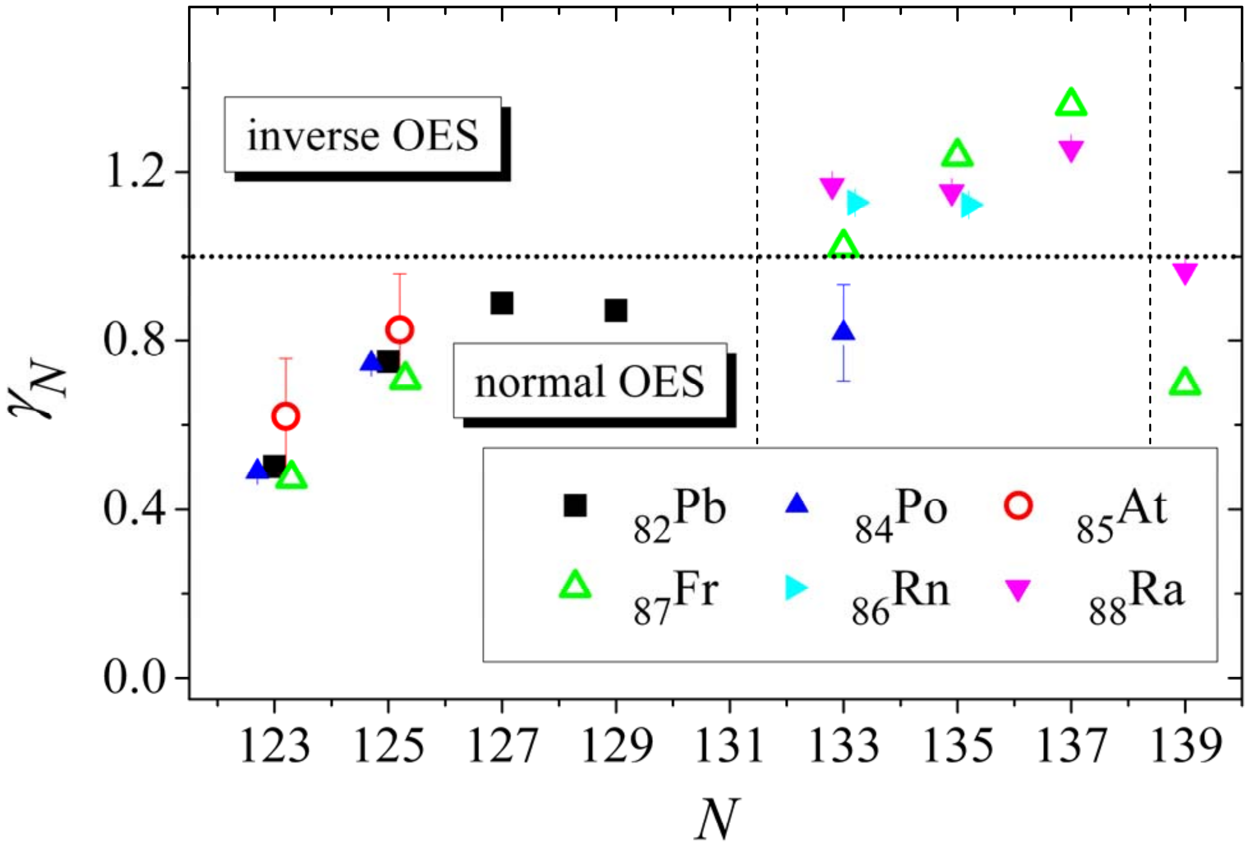
$\gamma = 1$ — no staggering

$\gamma < 1$ — normal staggering

$\gamma > 1$ — inverse staggering



Inverse radii staggering and octupole deformation



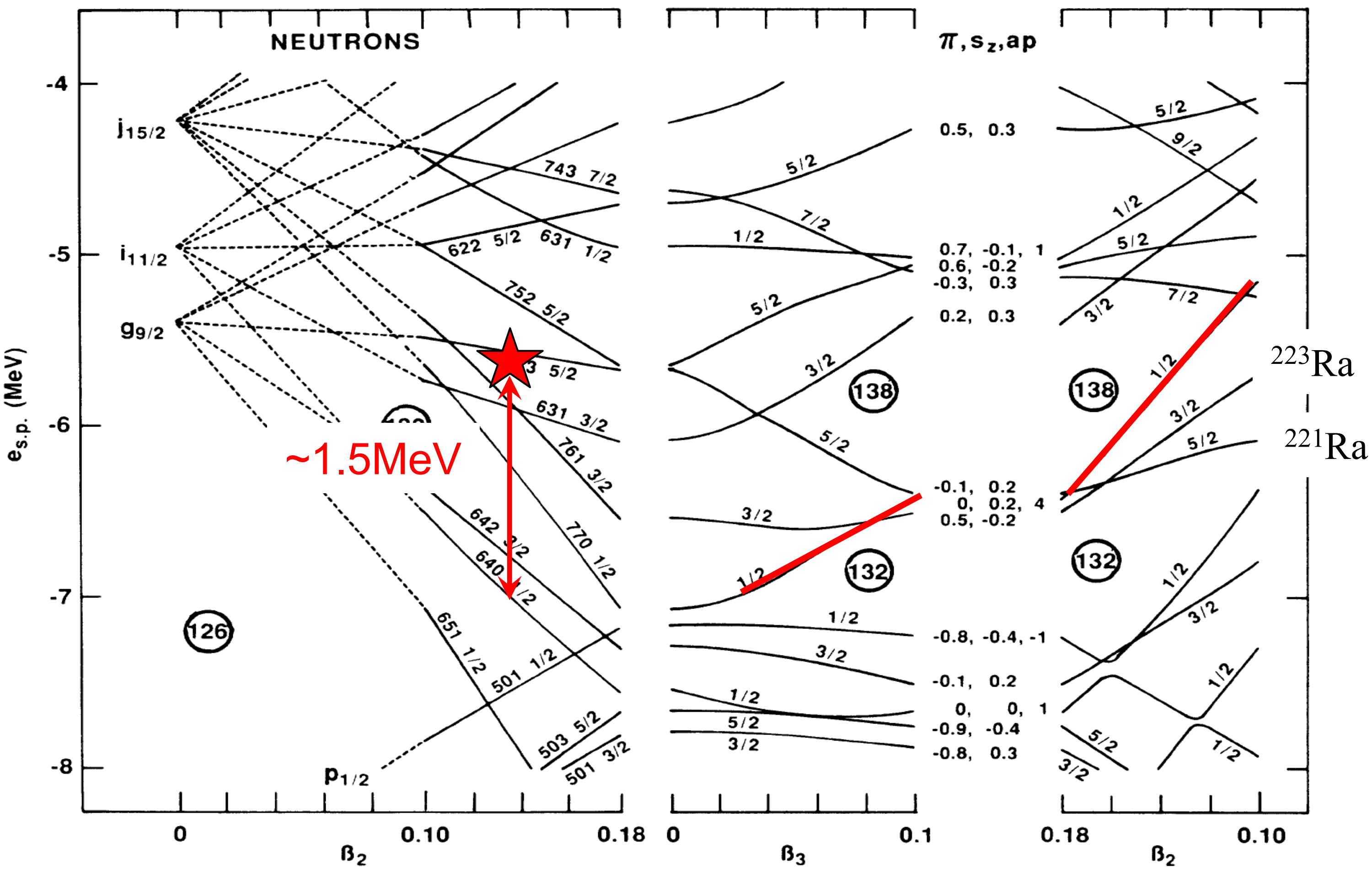
$$\langle r^2 \rangle = \langle r^2 \rangle_0 \left[1 + \frac{5}{4\pi} \left(\langle \beta_2^2 \rangle + \langle \beta_3^2 \rangle \right) \right]$$

The parity doublets are experimentally found to be more closely spaced in the odd nuclei than in their even neighbours.

Octupolarity: $132 < N < 139$ (inverse OES)

Spins and octupole deformation

^{225}Ra, $I = 1/2^{+}$ (N =137)



Magnetic moments and octupole deformation

Whether the parity doubled bands arise from single parity-mixed hybrid Nilsson orbitals generated from a stable octupole potential or from two different sets of Nilsson orbitals with different intrinsic properties.

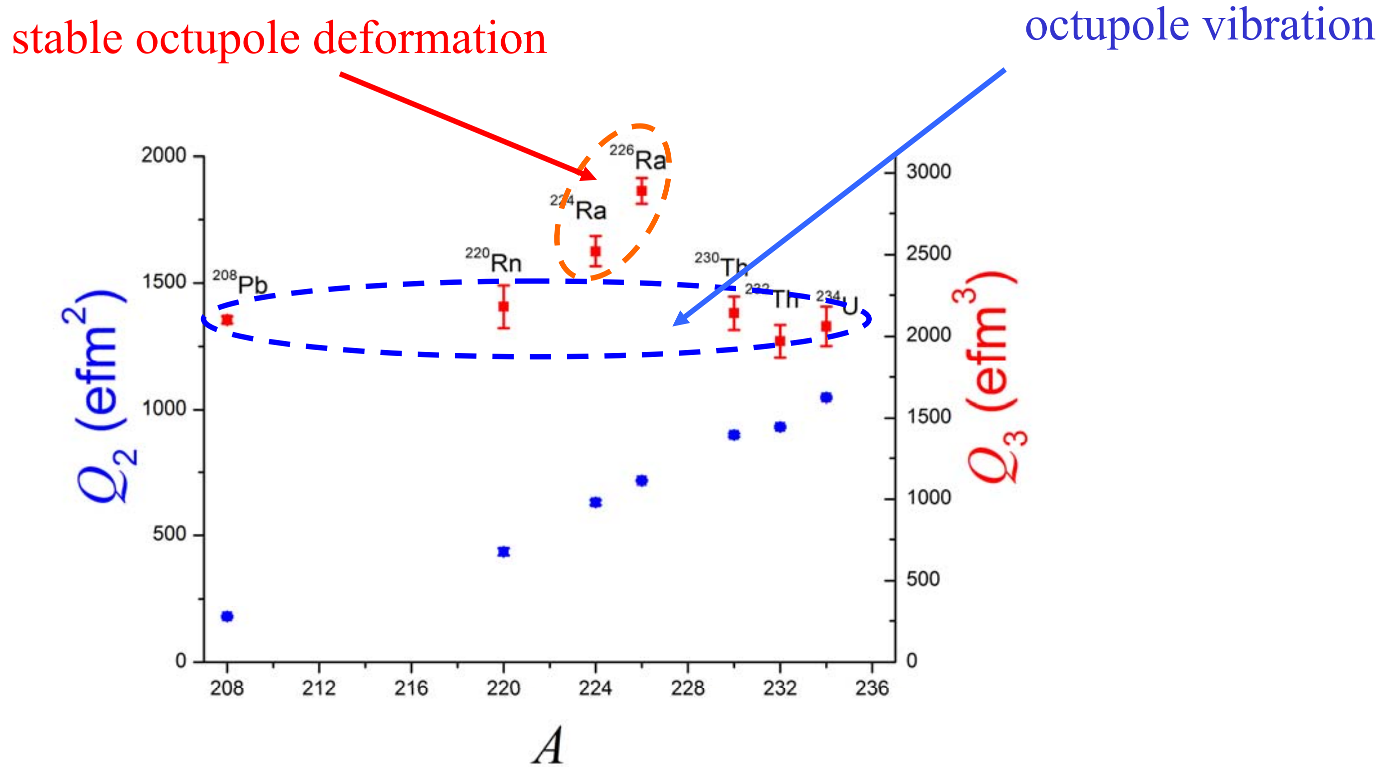
$|\pm\rangle = \frac{1}{\sqrt{2}}\big(|$  $\rangle \pm |$  $\rangle \big)$

$\langle + | O | + \rangle = \langle - | O | - \rangle = \langle$  $| O |$  $\rangle = \langle O \rangle_{\mathrm{intr.}}$

$\mu_{\mathrm{expt}}(^{223}\mathrm{Ra}, 3/2^-) = 0.4$ n.m.
$\mu_{\mathrm{expt}}(^{223}\mathrm{Ra}, 3/2^+) = 0.3$ n.m.

Parity mixed: $\mu_{\mathrm{theor}}(^{223}\mathrm{Ra}, 3/2^+) = \mu_{\mathrm{theor}}(^{223}\mathrm{Ra}, 3/2^-) = 0.5$ n.m.
Reflection symmetric: $\mu_{\mathrm{theor}}(^{223}\mathrm{Ra}, 3/2^+) = 0.03$ n.m.; $\mu_{\mathrm{theor}}(^{223}\mathrm{Ra}, 3/2^-) = -0.06$ n.m.

$B(E3)$ and octupole deformation

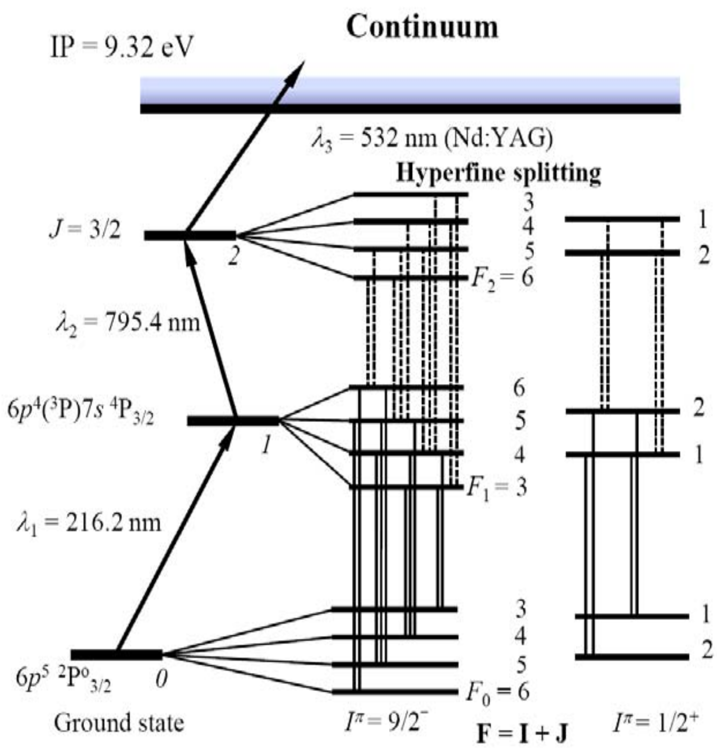


For octupole-vibrations it is expected that all E3 matrix elements between states other than those coupled via an octupole phonon, i.e. $<(I-3)^+||E3||I^->$ vanish.

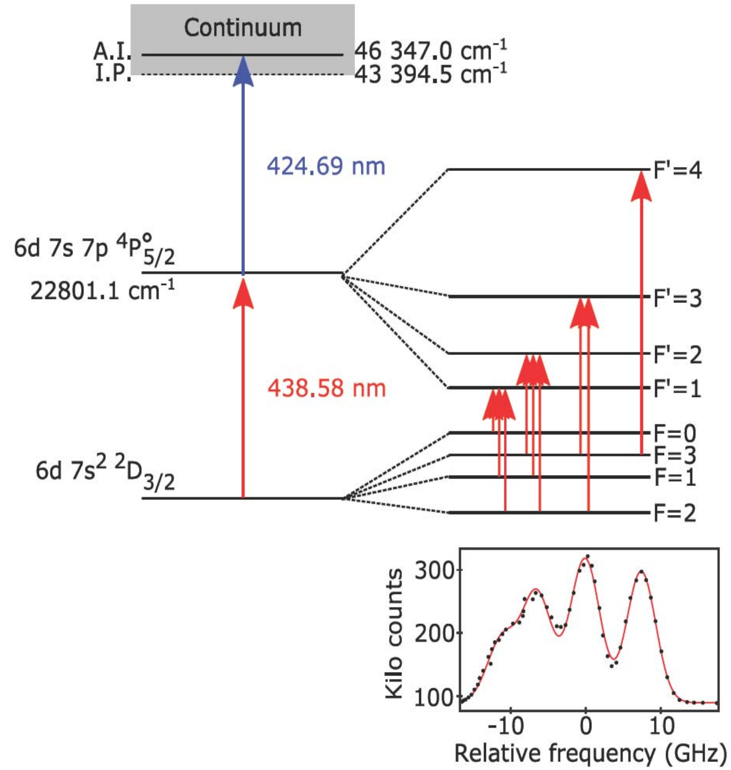
^{224}Ra: $<2^+||E3||1^-> = 210(40)$ W. u.

Ionization schemes

Astatine

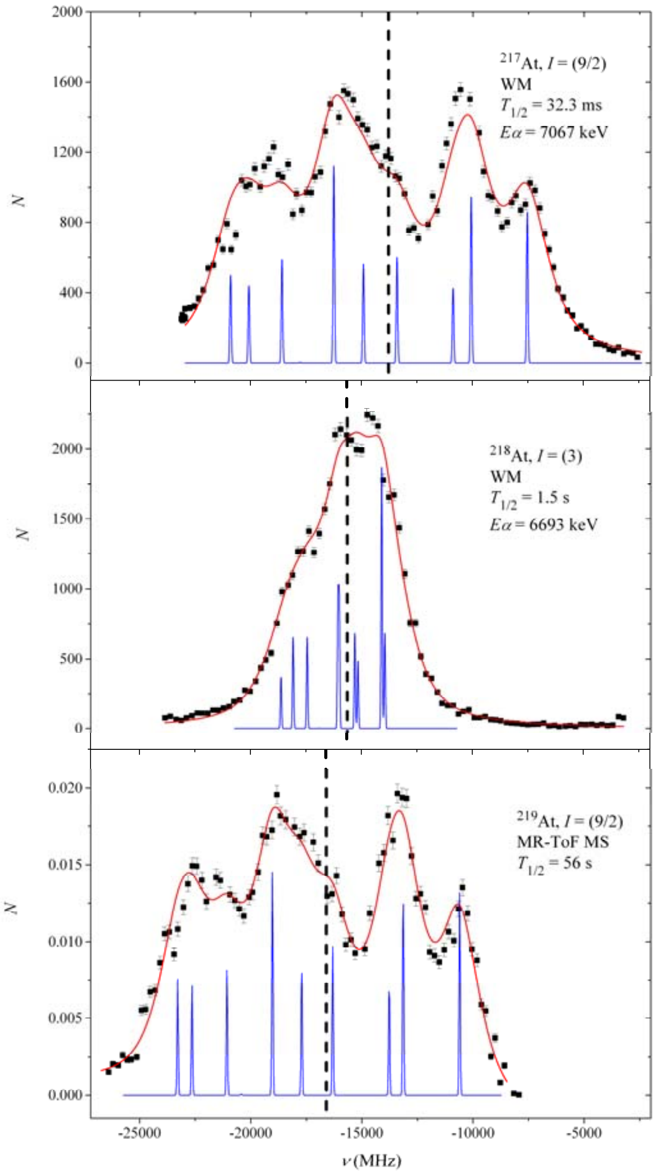


Actinium

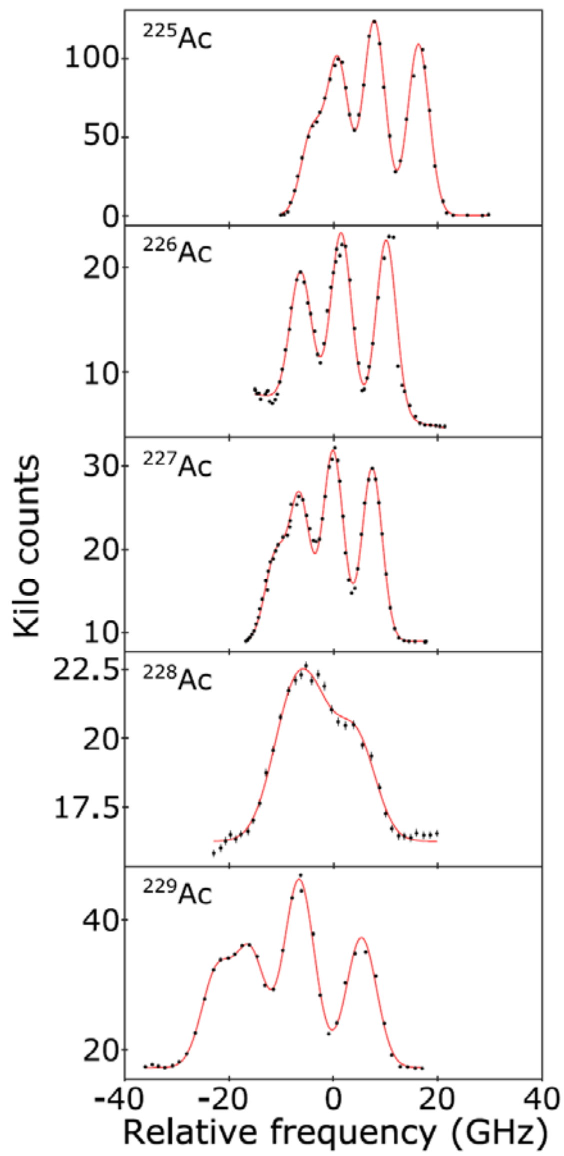


Experimental hfs spectra

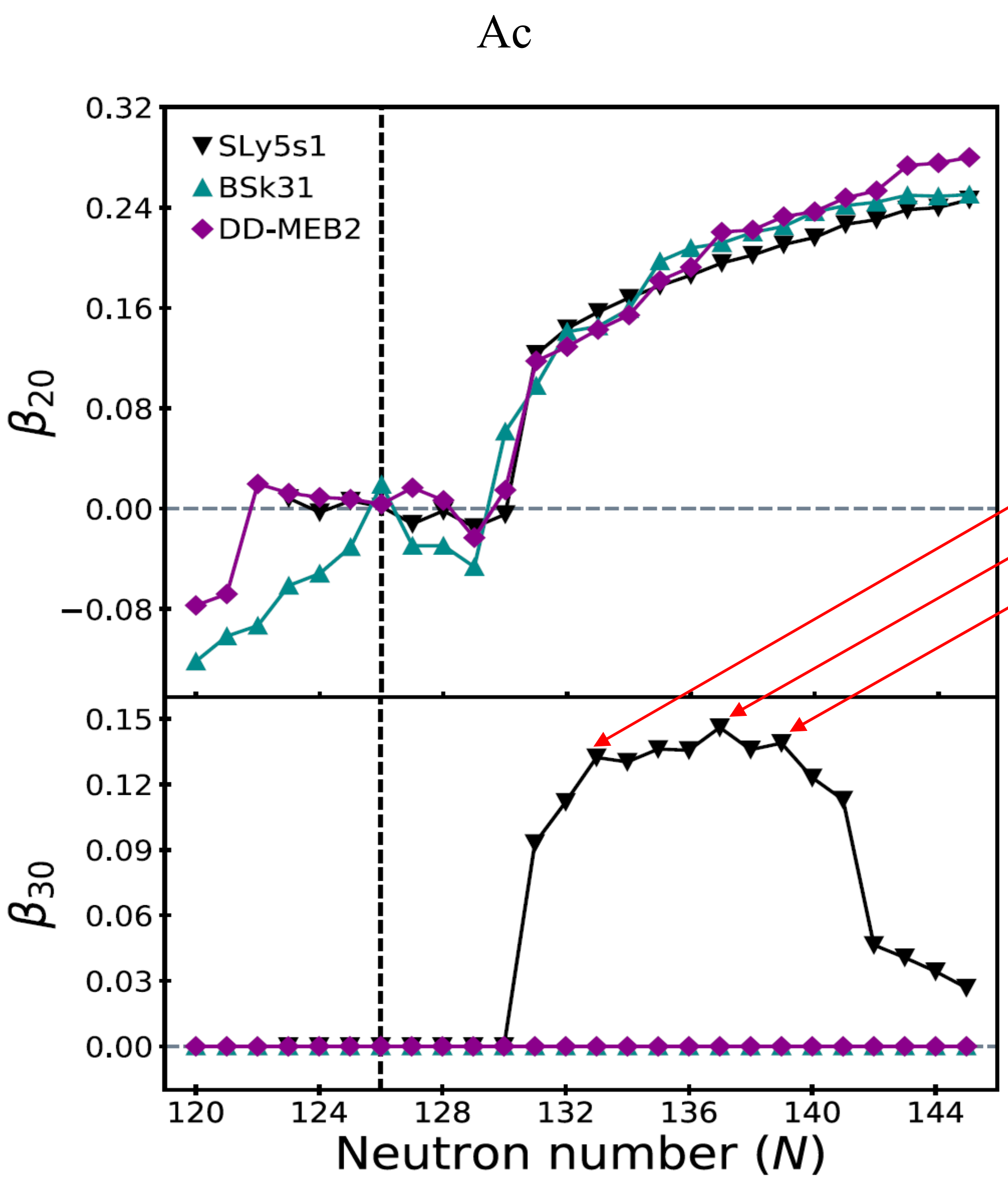
$^{217\text{–}219}$At



$^{225\text{–}229}$Ac



HF calculations: octupole region



Octupolarity: $132 < N < 142$ (Ac)

Increase of β_3 for odd-N nuclei produces the inverse OES

Octupole deformation and radii

$Q_S(^{217}\mathrm{Ac}) = 1.74(10)\ \mathrm{b}$

$$Q_S = \frac{3}{\sqrt{5\pi}} \frac{I\cdot(2I-1)}{(I+1)\cdot(2I+3)} Z R_0^2 \beta_2 \left(1+\sqrt{\frac{5}{4\pi}}\frac{4}{7}\beta_2\right) \Longrightarrow \beta_2 = 0.223(12)$$

$$\left\langle r^2 \right\rangle = \left\langle r^2 \right\rangle_0 \left[1+\frac{5}{4\pi}\left\langle \beta_2^2 \right\rangle\right] \Longrightarrow \delta\left\langle r^2 \right\rangle_{\mathrm{calc}}^{227,\,215} = 1.29\ \mathrm{fm}^2 \quad \delta\left\langle r^2 \right\rangle_{\mathrm{exp}}^{227,\,215} = 1.50\ \mathrm{fm}^2$$

influence of β_3? $\left\langle r^2 \right\rangle = \left\langle r^2 \right\rangle_0 \left[1+\frac{5}{4\pi}\left(\left\langle \beta_2^2 \right\rangle + \left\langle \beta_3^2 \right\rangle\right)\right]$

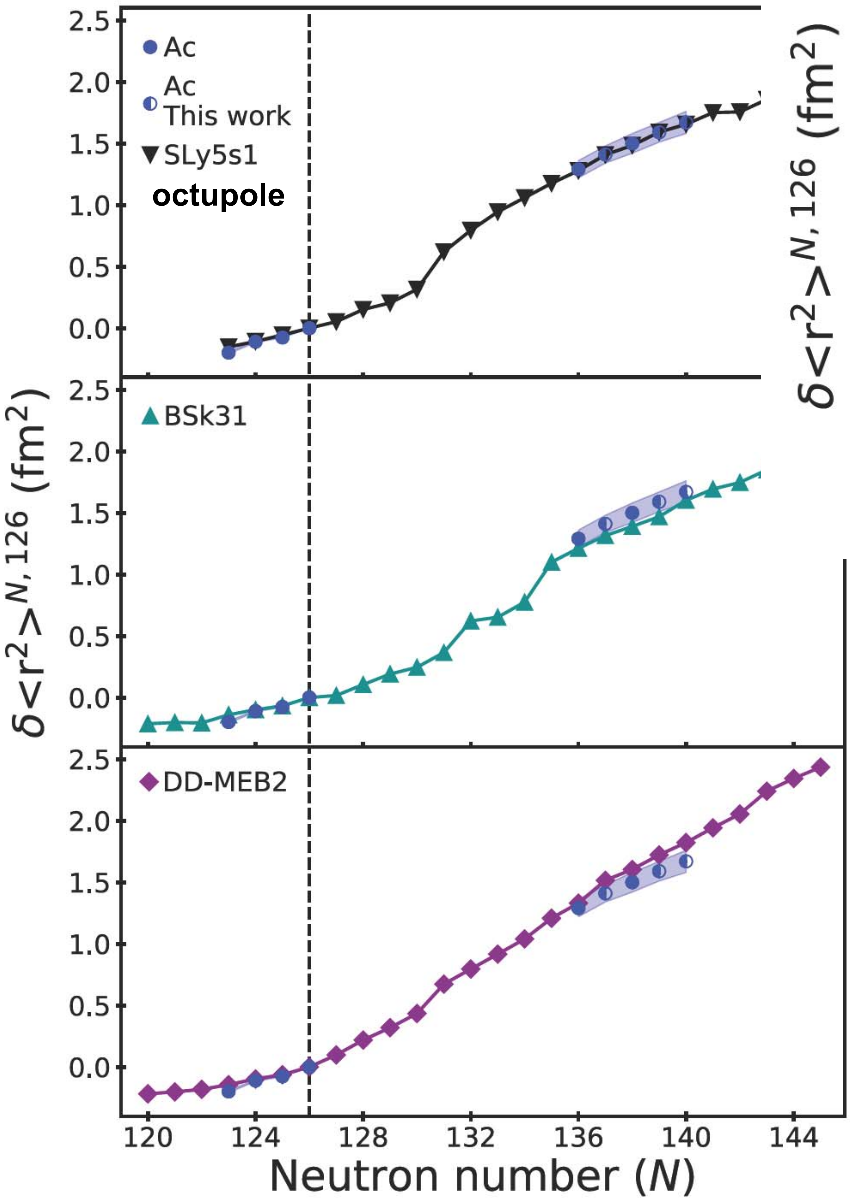
$$Q_S = \frac{I\cdot(2I-1)}{(I+1)\cdot(2I+3)} \frac{3}{\sqrt{5\pi}} Z R_0^2 f_1(\beta_2, \beta_3, \beta_4)$$

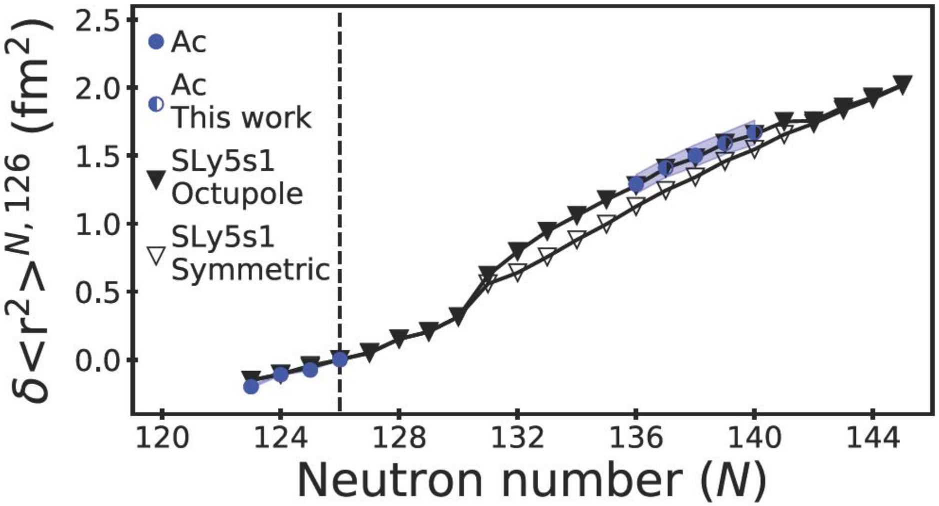
$$\left\langle r^2 \right\rangle = \left\langle r^2 \right\rangle_0 \left[1+f_2(\beta_2, \beta_3, \beta_4)\right]$$

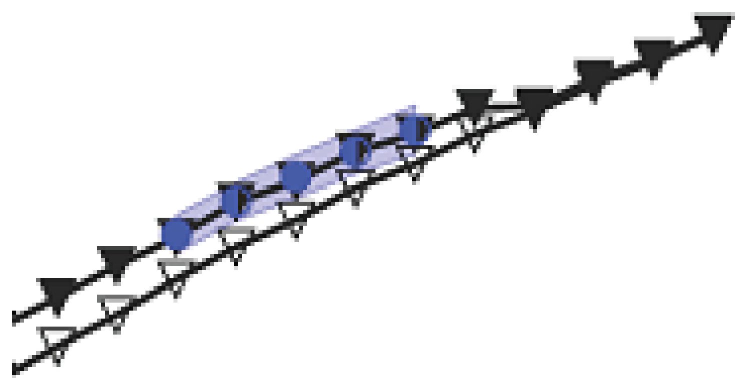
$\beta_3(^{217}\mathrm{Ac}) = 0.13(3)$

$[\beta_3(^{217}\mathrm{Ac})_{\mathrm{theor}} = 0.014]$

HF calculations

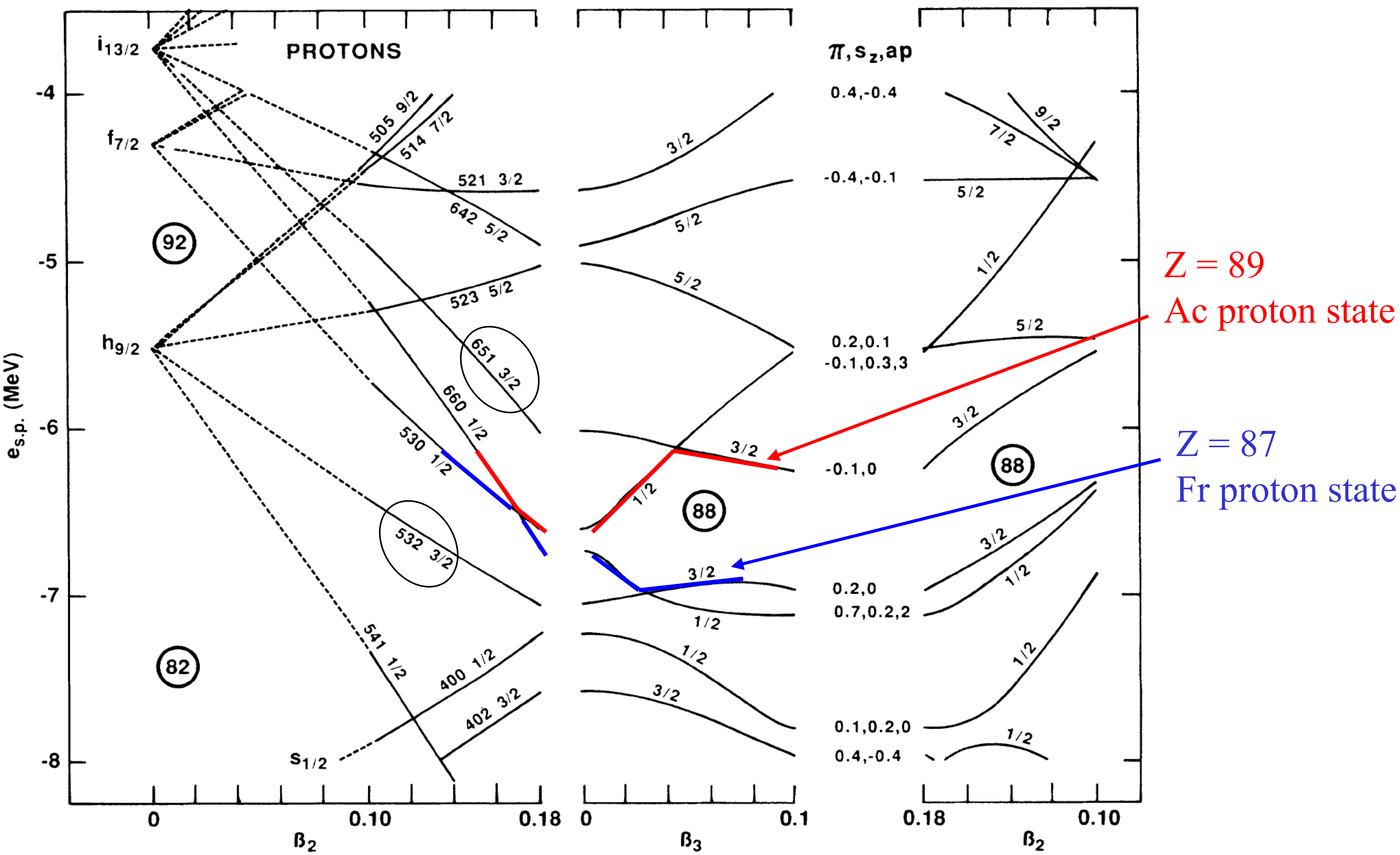




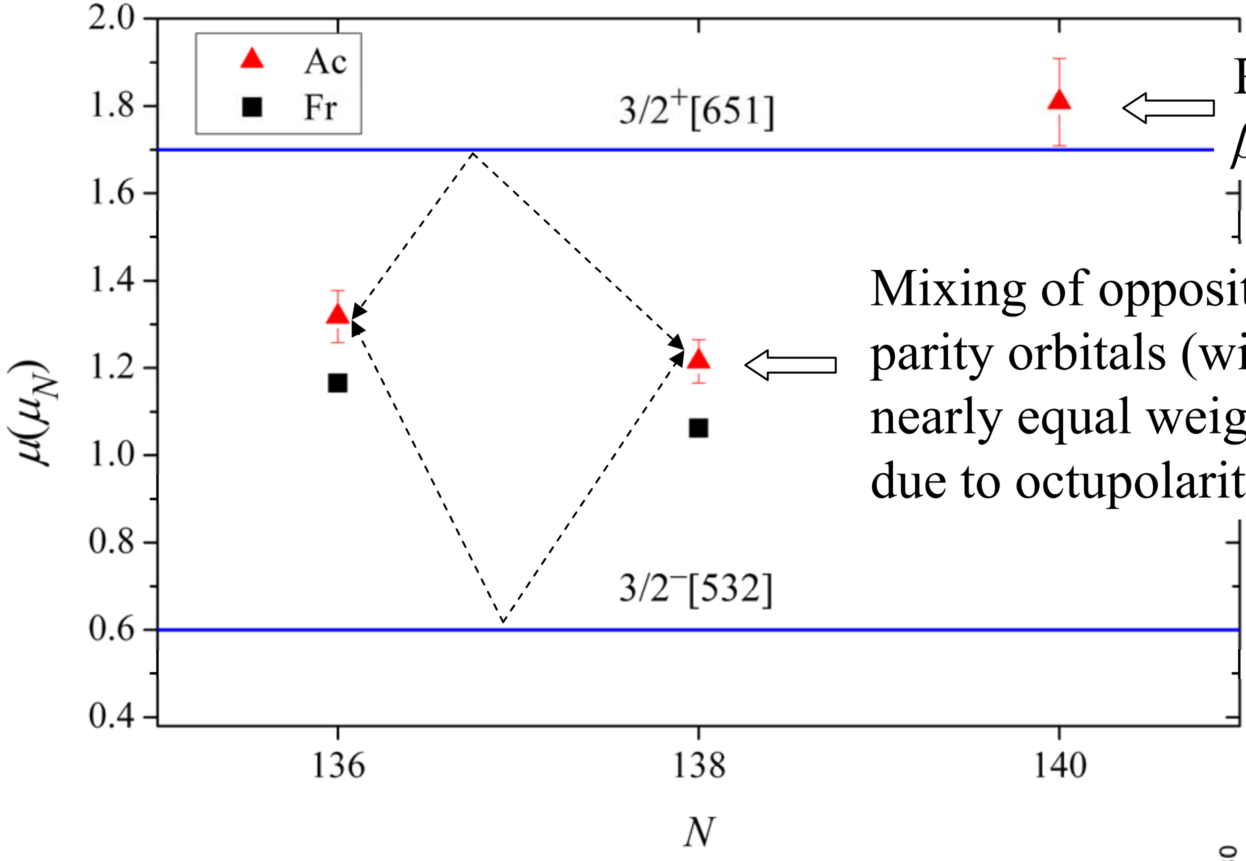


Mixed-parity single-particle states

$I^{\pi}(^{225,227}Ac) = 3/2^{-}$

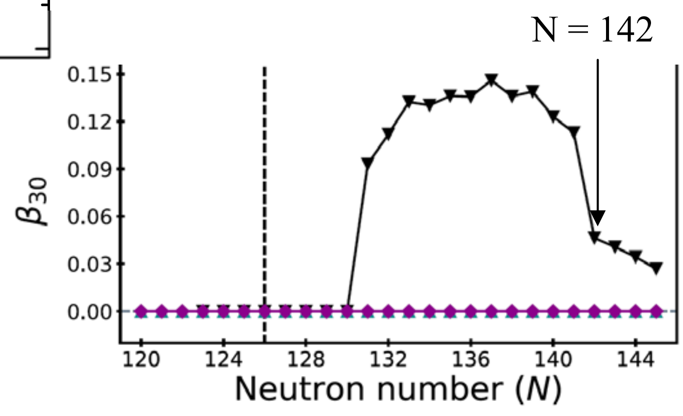


Magnetic moment of odd Ac isotopes as indicator of the octupolarity



Pure $3/2^+[651]$ state at $\beta_3 \approx 0$?

Mixing of opposite parity orbitals (with nearly equal weights) due to octupolarity



Magnetic moment of ^{218}At: a sign of the octupolarity?

$\mu_{\mathrm{expt}}(^{218}\mathrm{At};\, 3^-) = 1.25(12)\mu_N$.

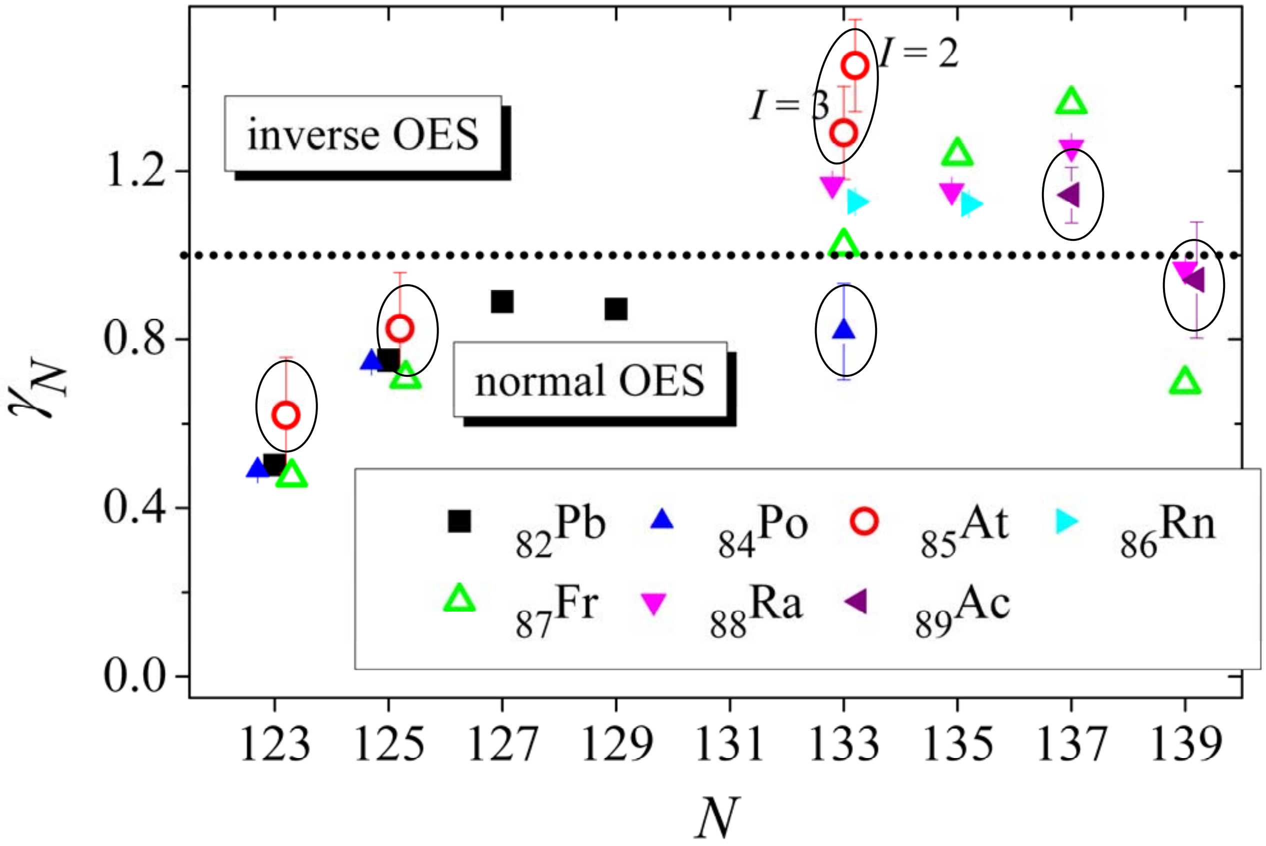
Additivity relation:

$$\mu_I(i_p, i_n) = \left(\frac{\mu(i_p)}{i_p} + \frac{\mu(i_n)}{i_n}\right)\cdot\left(I/2\right) + \left(\frac{\mu(i_p)}{i_p} - \frac{\mu(i_n)}{i_n}\right)\cdot\frac{i_p\cdot(i_p+1) - i_n\cdot(i_n+1)}{2\cdot(I+1)}$$

$\mu_{\mathrm{add}}(^{218}\mathrm{At};\, 3^-) = 0.87(9)\mu_N$ for configuration $(\pi 1h_{9/2} \otimes \nu 2g_{9/2})_{3-}$ (additivity relation)

Even a small admixture from $(\boldsymbol{\pi i_{13/2}} \otimes \nu\, g_{9/2})_{3+}$ or $(\boldsymbol{\pi i_{13/2}} \otimes \nu\, i_{11/2})_{3+}$ configurations would ensure an agreement between the additivity-rule estimations and the experimental results. Such an admixture is only possible in cases with mixing between opposite-parity states at nonzero octupole deformation. The use of other configurations decreases μ

Inverse radii staggering: Ac and At

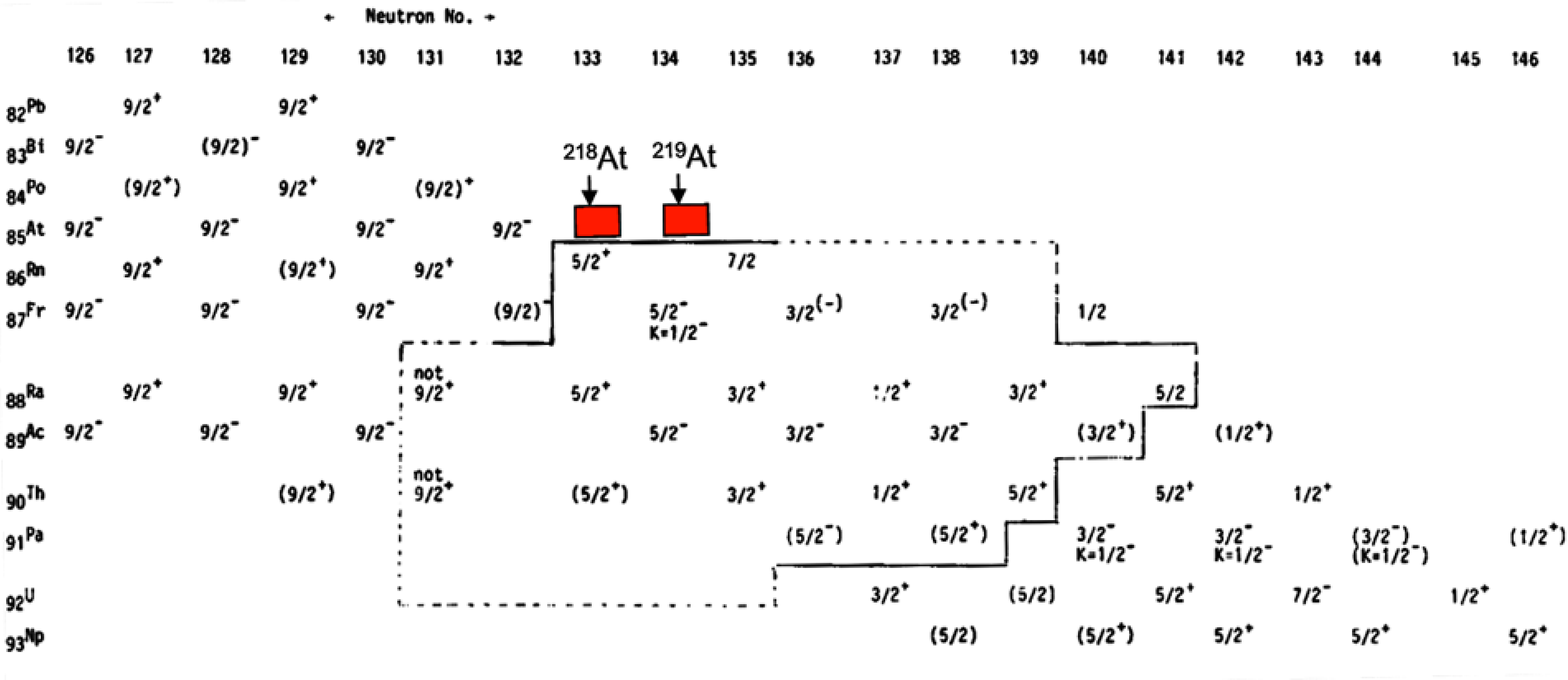


$Q_S\,(^{218}\mathrm{At}) = 0.55(33)$ b $\Longrightarrow$ $\beta_2 = 0.04(2)$

Octupole deformation without quadrupole one? — cf. ^{216}Fr.

Qualitative explanation by Otten is questioned (β_3 on top of β_2)

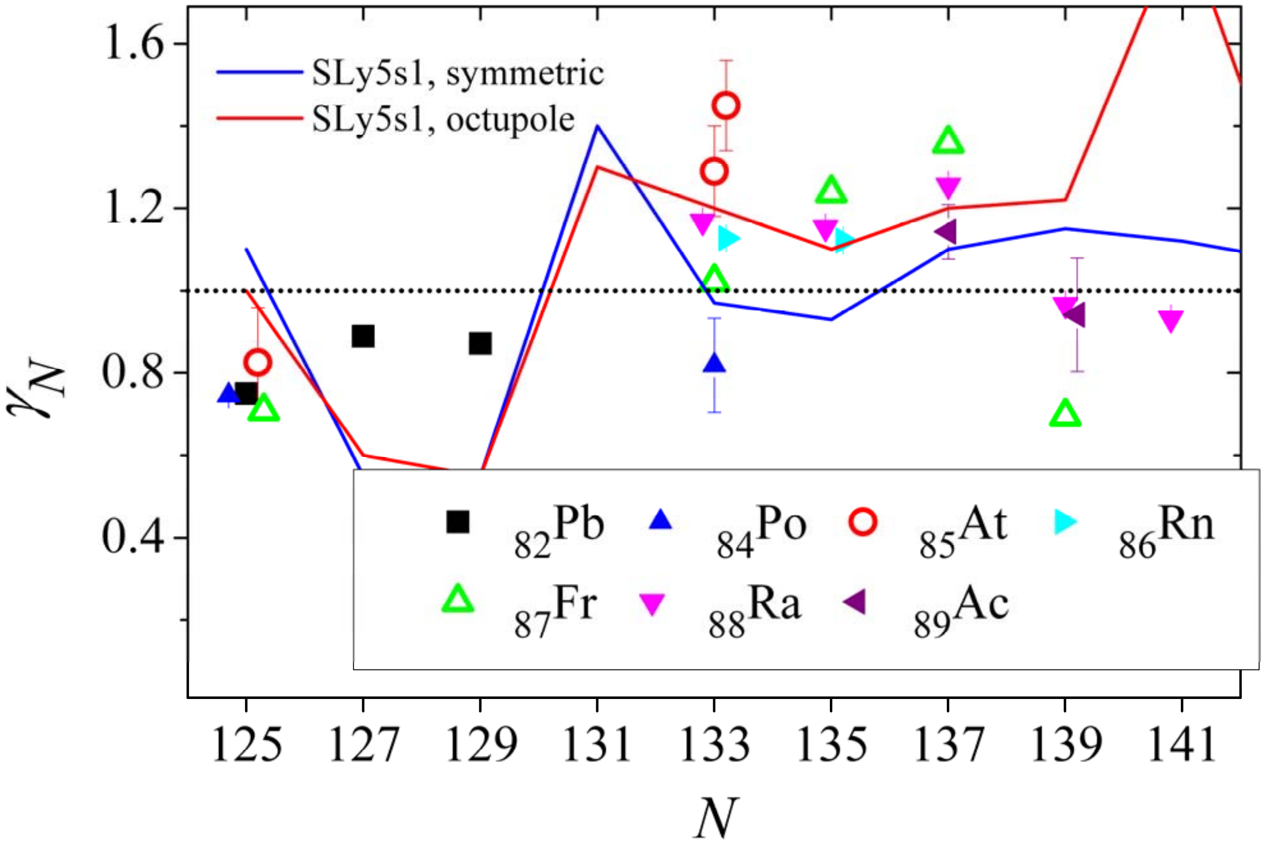
Region of octupole deformation



octupole collectivity: $86 \leq Z \leq 92$ (Rn, Fr, Ra, Ac, Th, Pa, U) — At?

$131 \leq N \leq 141$

Radii staggering: HF calculation for Ac



EDM search: Schiff moment

Theorem (Schiff)

The nuclear dipole moment causes the atomic electrons to rearrange themselves so that they develop a dipole moment opposite that of the nucleus. In the limit of nonrelativistic electrons and a point nucleus the electrons' dipole moment exactly cancels the nuclear moment, so that the net atomic dipole moment vanishes.

Post-screening nucleus-electron interaction is proportional to Schiff moment

$$\hat{S}_0 = \frac{e}{10}\sqrt{\frac{4\pi}{3}}\sum_i \left(r_i^3 - \frac{5}{3}\overline{r_{\text{ch}}^2} r_i \right) Y_0^1(\Omega_i) + \ldots .$$

$$S \equiv \langle \Psi_0 | \hat{S}_0 | \Psi_0 \rangle \approx \sum_{i \neq 0} \frac{\langle \Psi_0 | \hat{S}_0 | \Psi_i \rangle \langle \Psi_i | \hat{V}_{PT} | \Psi_0 \rangle}{E_0 - E_i} + \text{c.c.}$$

Ψ_i — another parity state

Electrostatic potential produced by the Schiff moment $\varphi(\mathbf{R}) = 4\pi \mathbf{S} \cdot \boldsymbol{\nabla} \delta(\mathbf{R})$

Atomic EDM induced by nucleus in an atom with a single electron in state *ns*

$$\mathbf{d}_{\text{atom}} = 2 \sum_m \frac{\langle ns | -e\varphi(\mathbf{R}) | mp \rangle \langle mp | -e\mathbf{R} | ns \rangle}{E_{ns} - E_{mp}}$$

EDM search: the best experimental limit

B. Graner, Y. Chen, E. G. Lindahl, and B. R. Heckel,
PRL 119, 119901 (2017) (University of Washington)

$d(^{199}\mathrm{Hg}) = 2.20\ (2.75)\ (1.48)\ 10^{-30}\ e\ \mathrm{cm}$
$|d(^{199}\mathrm{Hg})| < 7.4\ 10^{-30}\ e\ \mathrm{cm}$
$|S_{\mathrm{Hg}}| < 3.1\ 10^{-13}\ e\ \mathrm{fm}^3$

EDM search: enhancement at octupole deformation

the presence of the parity partner $|\overline{\Psi_0}\rangle$ of g.s. $|\Psi_0\rangle$

in ^{225}Ra $1/2^{+}$ ground state has a $1/2^{-}$ partner at 55 keV

$$S \equiv \langle\Psi_0|\hat{S}_0|\Psi_0\rangle \approx \sum_{i\neq 0} \frac{\langle\Psi_0|\hat{S}_0|\Psi_i\rangle\langle\Psi_i|\hat{V}_{PT}|\Psi_0\rangle}{E_0 - E_i} + \text{c.c.}$$

$$S \approx -2\frac{\langle\Psi_0|\hat{S}_0|\bar{\Psi}_0\rangle\langle\bar{\Psi}_0|\hat{V}_{PT}|\Psi_0\rangle}{\Delta E}$$

Enhancement Factor: EDM (^{225}Ra) / EDM (^{199}Hg)

Skyrme Model	Isoscalar	Isovector
SIII	300	4000
SkM*	300	2000
SLy4	700	8000

Schiff moment of ^{225}Ra, Dobaczewski, Engel (2005)
Schiff moment of ^{199}Hg, Ban, Dobaczewski, Engel, Shukla (2010)

Atomic physics enhances any EDM in Ra by another factor of 3 over that in Hg

Schiff moment: calculations

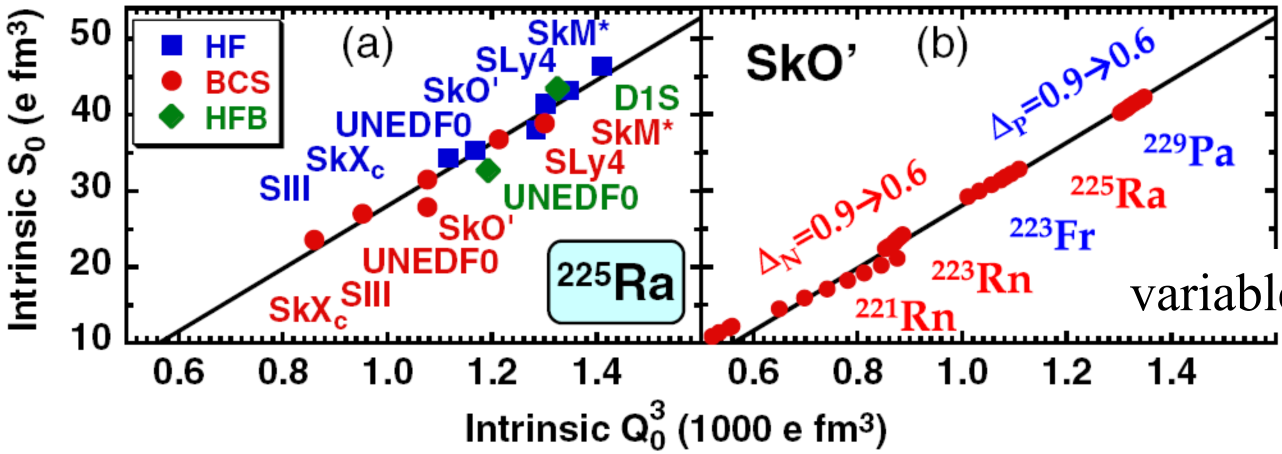
Intrinsic Schiff moments S_0 (e fm^3)

EDF	method	^{221}Rn	^{223}Rn	^{223}Fr	^{225}Ra	^{229}Pa
SIII	BCS	13.9	6.9	24.0	27.0	37.6
SkM*	BCS	21.6	30.7	32.5	38.9	46.4
SkO′	BCS	11.4	18.3	23.2	31.5	40.9
SkX$_c$	BCS	6.4	13.8	18.3	23.6	34.6
SLy4	BCS	20.4	26.2	28.8	36.8	46.0
UNEDF0	BCS	10.6	17.2	19.9	27.9	33.4
D1S	HFB	27.6	30.9	34.3	43.4	52.0
UNEDF0	HFB	20.5	23.5	25.1	32.7	32.9

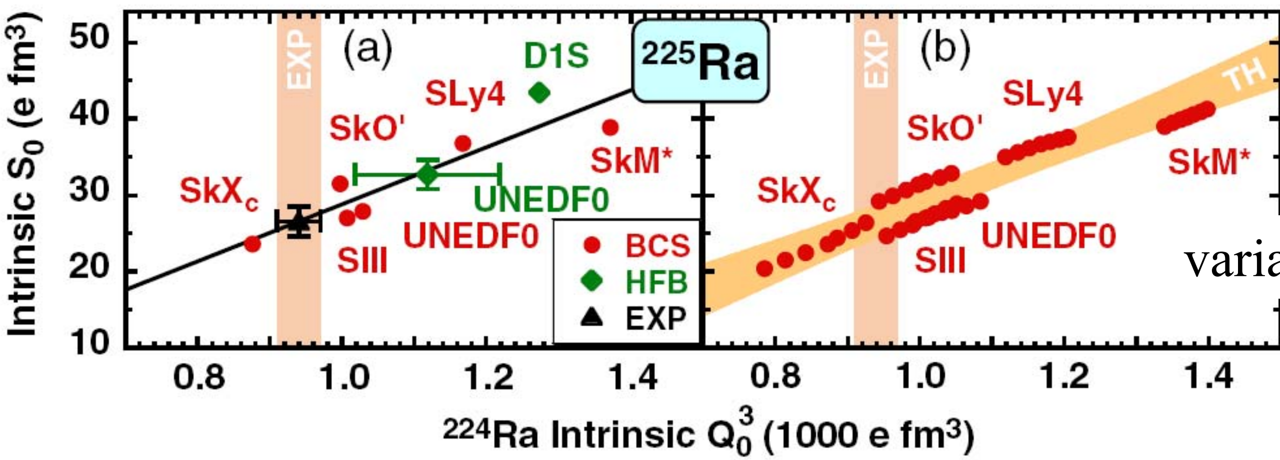
Factor of 2-4!

Correlation of Schiff moment and Q_3

$$\hat{S}_0 = \frac{e}{10}\sqrt{\frac{4\pi}{3}}\sum_i\left(r_i^3 - \frac{5}{3}\overline{r^2_{\rm ch}}\,r_i\right)Y_0^1(\Omega_i) + \ldots \qquad \hat{Q}_0^3 \equiv e\sum_i r_i^3 Y_0^3(i) \qquad \boldsymbol{S_0 \sim Q_3}$$



variable pairing gap



variable pairing gap

J. Dobaczewski, J. Engel, M. Kortelainen, and P. Becker, PRL 121, 232501 (2018)

Intrinsic Schiff moment from measured Q_3

Intrinsic Schiff moments S_0 (e fm^3)

	K	from ^{224}Ra	from ^{226}Ra	from ^{220}Rn
^{221}Rn	$\frac{7}{2}$			19.2(1.9)
^{223}Rn	$\frac{7}{2}$	13.5(2.8)		
^{223}Fr	$\frac{3}{2}$	20.3(1.5)		
^{225}Ra	$\frac{1}{2}$	26.6(1.9)		
^{229}Pa	$\frac{5}{2}$	35.2(2.9)	39.5(2.1)	41.9(2.7)

Q_{30} (expt) = 940(30) e fm^3 for ^{224}Ra,
Q_{30} (expt) = 1080(30) e fm^3 for ^{226}Ra,
Q_{30}(expt) = 810(50) e fm^3 for ^{220}Rn

Summary

1. Октупольная деформация ядра проявляется в целом ряде интересных ядерно-спектроскопических феноменов, требующих для своего описания развития теоретических подходов.
2. Корреляция обратного четно-нечетного эффекта в зарядовых радиусах с октупольной коллективностью требует количественного теоретического описания. Установление границ области обратного четно-нечетного эффекта — одна из актуальных задач ядерной физики.
3. Накопление экспериментальных данных об эффектах октупольности имеет большое значение для формирования надежных ядерных моделей для извлечения информации о T- и P-нечетных компонентах взаимодействия элементарных частиц.