

# Обзор методов анализа данных в физике нейтрино по материалам PHYSTAT- $\nu$ 2019

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Семинар ОФВЭ ПИЯФ 19.02.2019

# Введение

PHYSTAT - серия “рабочих совещаний” по статистическим методам в физике высоких энергий.

[https://espace.cern.ch/phystat/\\_layouts/15/start.aspx#/](https://espace.cern.ch/phystat/_layouts/15/start.aspx#/)

## Introduction:

PHYSTAT is a workshop series dealing with statistical methods in particle physics. It was founded in 2000 by Louis Lyons. A wide range of topic is covered: frequentist and bayesian inference, parameter estimation, hypothesis testing, Goodness of Fit testing, confidence interval estimation, unfolding, multivariate analysis techniques, systematic uncertainties, data combination and more. The workshop is a unique meeting point of Physicists and Statisticians, where the latest advances in statistical techniques and procedures are exchanged. The workshop consists of a mixture of invited and contributed talks plus panel discussions. The proceedings of the workshop provide a comprehensive reference on statistical issues and state of the art methods used in particle physics and neighbouring fields.

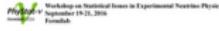
Since beginning of 2018 Olaf Behnke (DESY) is acting as chair person of PHYSTAT.

## Next Events:

- PHYSTAT-LHC 2019 (DESY) Oct. 2019

Please send your suggestion for possible future meetings to [olaf.behnke@desy.de](mailto:olaf.behnke@desy.de).

## Links to past events:

- **PHYSTAT-nu 2019 (CERN)** Jan 22-25   
 Workshop on Statistical Issues in Experimental Neutrino Physics  
January 22-25, 2019  
CERN, Geneva
- **PHYSTAT-nu 2016 (FNAL)** 
- **PHYSTAT-nu 2016 (Kavli, Japan)** 
- **PHYSTAT 2011 (CERN)** Proceedings: "Statistical issues related to discovery claims in search experiments, concentrating on those of the LHC. + Unfolding workshop"
- **PHYSTAT 2007 (CERN)** Link to proceedings: "Statistical issues for LHC physics."

- **PHYSTAT 2005 (Oxford)** Proceedings  "Statistical Problems in Particle Physics, Astrophysics and Cosmology"
- **PHYSTAT 2003 (SLAC)**  "Statistical Problems in Particle Physics, Astrophysics and Cosmology"
- **PHYSTAT 2002 (Durham)** "Advanced statistical analysis techniques as used in measurements and searches in Particle Physics, including Astroparticle Physics"
- **Workshop on Confidence Limits 2000 (FNAL)**
- **1st Workshop on Confidence Limits 2000 (CERN)**

## PHYSTAT Committee (2019):

- Olaf Behnke (DESY) [Chair] [olaf.behnke@desy.de](mailto:olaf.behnke@desy.de)
- Louis Lyons (Imperial) [louis.lyons@physics.ox.ac.uk](mailto:louis.lyons@physics.ox.ac.uk)
- Robert Cousins (UCLA) [cousins@physics.ucla.edu](mailto:cousins@physics.ucla.edu)
- Glen Cowan (RHUL) [g.cowan@rhul.ac.uk](mailto:g.cowan@rhul.ac.uk)
- Kyle Cranmer (NYU) [kyle.cranmer@cern.ch](mailto:kyle.cranmer@cern.ch)
- Thomas R. Junk (FNAL) [trj@fnal.gov](mailto:trj@fnal.gov)
- Mikael Kuusela (CMU) [mikael.kuusela@cern.ch](mailto:mikael.kuusela@cern.ch)
- Nicholas Wardle (Imperial) [nw709@ic.ac.uk](mailto:nw709@ic.ac.uk)

# Введение

<https://indico.cern.ch/event/735431/>

PHYSTAT- $\nu$  2019 посвящено методам анализа данных в физике нейтрино.



## PHYSTAT- $\nu$ 2019

22-25 January 2019  
CERN  
Europe/Zurich timezone

Search...

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The workshops deals with the statistical issues in the broad range of modern neutrino physics, in particular in view of the increased precision of current and future experiments. Topics to be included are

- Measuring model parameters, choosing between models
- Setting limits, discovery
- Determination of systematic uncertainties
- Unfolding
- Machine learning for event reconstruction and classification

The workshop will be a mixture of invited and contributed talks from physicists and statisticians.

# Введение

~ 100 neutrino physicists

~ 50 LHC physicists

6 statisticians.



# Почему $5\sigma$ -discovery?

Louis Lyons (Imperial College (GB)). Introductory statistics course, part I.

## Why $5\sigma$ for Discovery?

Statisticians ridicule our belief in extreme tails (esp. for systematics)

Our reasons:

- 1) Past history (Many  $3\sigma$  and  $4\sigma$  effects have gone away)
- 2) LEE
- 3) Worries about underestimated systematics
- 4) Subconscious Bayes calculation

$$\frac{p(H_1|x)}{p(H_0|x)} = \frac{p(x|H_1)}{p(x|H_0)} * \frac{\pi(H_1)}{\pi(H_0)}$$

Posterior      Likelihood      Priors  
prob               ratio

“Extraordinary claims require extraordinary evidence”

N.B. Points 2), 3) and 4) are experiment-dependent

Alternative suggestion:

L.L. “Discovering the significance of  $5\sigma$ ”      <http://arxiv.org/abs/1310.1284>

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# Почему $5-\sigma$ -discovery?

Louis Lyons (Imperial College (GB)). Introductory statistics course, part I.

## How many $\sigma$ 's for discovery?

SEARCH	SURPRISE	IMPACT	LEE	SYSTEMATICS	No. $\sigma$
Higgs search	Medium	Very high	M	Medium	5
Single top	No	Low	No	No	3
SUSY	Yes	Very high	Very large	Yes	7
$B_s$ oscillations	Medium/Low	Medium	$\Delta m$	No	4
Neutrino osc	Medium	High	$\sin^2 2\theta, \Delta m^2$	No	4
$B_s \rightarrow \mu \mu$	No	Low/Medium	No	Medium	3
Pentaquark	Yes	High/V. high	M, decay mode	Medium	7
$(g-2)_\mu$ anom	Yes	High	No	Yes	4
H spin $\neq 0$	Yes	High	No	Medium	5
4 <sup>th</sup> gen q, l, v	Yes	High	M, mode	No	6
Dark energy	Yes	Very high	Strength	Yes	5
Grav Waves	No	High	Enormous	Yes	8

Suggestions to provoke discussion, rather than 'delivered on Mt. Sinai'



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Bob Cousins: "2 independent expts each with  $3.5\sigma$  better than one expt with  $5\sigma$ "

# Полезные асимптотические формулы

Glen Cowan (Royal Holloway, University of London). Introductory Statistics Course - Part II.

Measure two Poisson distributed values:

$n \sim \text{Poisson}(s+b)$  (primary or “search” measurement)

$m \sim \text{Poisson}(\tau b)$  (control measurement,  $\tau$  known)

## Asymptotic significance

Use profile likelihood ratio for  $q_0$ , and then from this get discovery significance using asymptotic approximation (Wilks’ theorem):

$$\begin{aligned} Z &= \sqrt{q_0} \\ &= \left[ -2 \left( n \ln \left[ \frac{n+m}{(1+\tau)n} \right] + m \ln \left[ \frac{\tau(n+m)}{(1+\tau)m} \right] \right) \right]^{1/2} \end{aligned}$$

for  $n > \hat{b}$  and  $Z = 0$  otherwise.

Essentially same as in:

Robert D. Cousins, James T. Linnemann and Jordan Tucker, NIM A 595 (2008) 480–501; arXiv:physics/0702156.

Tipei Li and Yuqian Ma, Astrophysical Journal 272 (1983) 317–324.

Имеется и более простая формула для известной величины  $b$ .

# Фактор Байеса

James Berger (Duke University). Bayesian techniques.

PHYSTATnu 1/22,2019

## Two Types of Bayesian Problems

**I. Estimation (confidence limit) problems:** In principle, they are straightforward.

- There are optimal prior distributions for most problems (e.g. *priors* - although their derivation can be difficult).
- Implementation of Bayes is usually easy, through MCMC.

**II. Hypothesis testing or model uncertainty problems:** Not so easy.

- Sometimes one can use the optimal estimation priors, but often not.
- In the latter case, the answers can be quite sensitive to the choice of prior,
  - so that one often seeks a *robust* conclusion over the choice.
- Computations can be much more difficult.

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Bayes factor (*evidence in physics*) of  $H_0$  to  $H_1$ : ratio of likelihood under  $H_0$  to average likelihood under  $H_1$  (or "odds" of  $H_0$  to  $H_1$ )

$$B_{01}(N) = \frac{\text{Poisson}(N \mid 0 + b)}{\int_0^{\infty} \text{Poisson}(N \mid s + b) \pi(s) ds} = \frac{b^N e^{-b}}{\int_0^{\infty} (s + b)^N e^{-(s+b)} \pi(s) ds}.$$

**Subjective approach:** Choose  $\pi(s)$  subjectively (e.g., using the standard physics model predictions of the mass of the Higgs).

**Objective approach:** Choose  $\pi(s)$  to be the 'intrinsic prior' (not discussed here)  $\pi^I(s) = b(s+b)^{-2}$ . (Note that this prior is proper and has median  $b$ .)

**Bayes factor:** is then given by

$$B_{01} = \frac{b^N e^{-b}}{\int_0^{\infty} (s + b)^N e^{-(s+b)} b(s+b)^{-2} ds} = \frac{b^{(N-1)} e^{-b}}{\Gamma(N-1, b)},$$

where  $\Gamma$  is the incomplete gamma function.

Референсные априорные обычно не интегрируемы (improper)!

Это не проблема для формулы Байеса (произвольная константа сокращается).

# Статус физики нейтрино

Alain Blondel (Universite de Geneve (CH)).

The Nobel Prize in Physics 2015  
Takaaki Kajita, Arthur B. McDonald

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## The Nobel Prize in Physics 2015

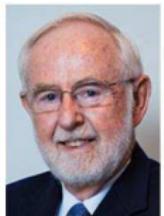


Photo © Takaaki Kajita  
**Takaaki Kajita**  
Prize share: 1/2

Photo: K. MacFarlane,  
Queen's University  
/SNOLAB  
**Arthur B. McDonald**  
Prize share: 1/2

The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita and Arthur B. McDonald "for the discovery of neutrino oscillations, which shows that neutrinos have mass"

A massless particle cannot be seen to transform  $\tau_{\text{lab}} = \tau_{\text{particle}} * E/m \rightarrow \infty$

# Статус физики нейтрино

Alain Blondel (Universite de Geneve (CH)).

## neutrino definitions

the **electron neutrino** is present in association with an **electron** (e.g. beta decay)

the **muon neutrino** is present in association with a **muon** (pion decay)

the **tau neutrino** is present in association with a **tau** ( $W \rightarrow \tau v$  decay)

these **flavor-neutrinos** are not (as we know now) quantum states of well defined **mass** (neutrino mixing)

the **mass-neutrino** with the highest **electron neutrino content** is called  $\nu_1$

the **mass-neutrino** with the next-to-highest **electron neutrino content** is  $\nu_2$

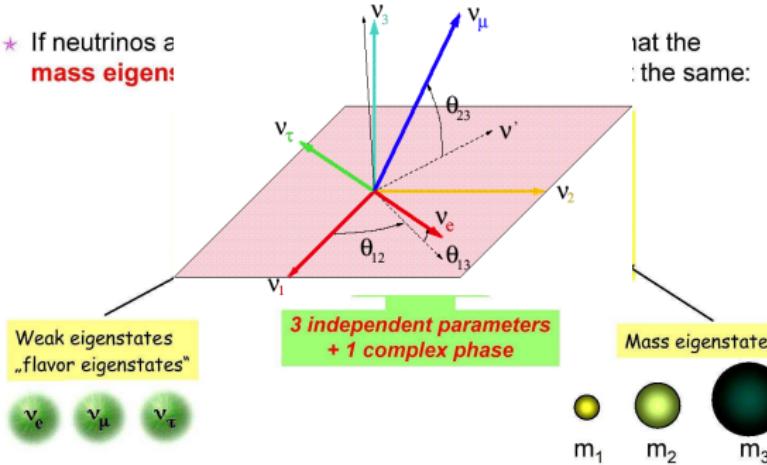
the **mass-neutrino** with the smallest **electron neutrino content** is called  $\nu_3$

# Статус физики нейтрино

Alain Blondel (Universite de Geneve (CH)).

## Lepton Sector Mixing

- \* If neutrinos  $\neq$  mass eigenstates



Pontecorvo 1957

# Статус физики нейтрино

Alain Blondel (Universite de Geneve (CH)).

## General framework and status:

1. We know that there are **three** families of active, light neutrinos ([LEP](#))
2. **Solar** neutrino oscillations are **established** ([Homestake+Gallium+Kam+SK+SNO](#))
3. **Atmospheric** neutrino ( $\nu_\mu \rightarrow \nu$ ) oscillations are **established** ([IMB+Kam+SK+ K2K](#))
4. At that frequency, ( $\nu_\mu \rightarrow \nu_e$ ) oscillations, small (5%) have been observed (T2K, NOvA) and  $\nu_e$  disappearance has been measured (Daya Bay, Reno, Double Chooz)

This allows a consistent picture with 3-family oscillations preferred:

LMA:  $\theta_{12} \sim 30^\circ$   $\Delta m_{12}^2 \sim 8 \cdot 10^{-5} \text{ eV}^2$ ,  $\theta_{23} \sim 45^\circ$   $\Delta m_{23}^2 \sim \pm 2.5 \cdot 10^{-3} \text{ eV}^2$ ,  $\theta_{13} < \sim 9^\circ$   
with several unknown parameters though 2018 revealed hints of CPV and NH.  
=> an **exciting** experimental program for at least 20 years \*)  
including **leptonic CP & T violations.**

5. There are unexplained phenomena interpreted as possible higher frequency oscillation (LSND miniBooNe, reactors) but they are inconsistent with excellent disappearance experiments (MINOS, MINOS+, ICECUBE and DayaBay) so sterile neutrino explanation is ruled out, but further investigation will be performed with time-sensitive experiments (SBN,

\*)to set the scale: **CP violation in quarks** was discovered in 1964  
and there is still an important program (K0pi0, B-factories, Neutron EDM, BTeV, LHCb..)  
to go on for 10 years...i.e. a total of ~50 yrs.

and we have not discovered leptonic CP yet!

# Статус физики нейтрино

Particle Data Group, 2018.

## Neutrino Properties

See the note on "Neutrino properties listings" in the Particle Listings.

Mass  $m < 2$  eV (tritium decay)

Mean life/mass,  $\tau/m > 300$  s/eV, CL = 90% (reactor)

Mean life/mass,  $\tau/m > 7 \times 10^9$  s/eV (solar)

Mean life/mass,  $\tau/m > 15.4$  s/eV, CL = 90% (accelerator)

Magnetic moment  $\mu < 0.29 \times 10^{-10} \mu_B$ , CL = 90% (reactor)

## Number of Neutrino Types

Number  $N = 2.984 \pm 0.008$  (Standard Model fits to LEP-SLC data)

Number  $N = 2.92 \pm 0.05$  ( $S = 1.2$ ) (Direct measurement of invisible  $Z$  width)

## Neutrino Mixing

The following values are obtained through data analyses based on the 3-neutrino mixing scheme described in the review "Neutrino Mass, Mixing, and Oscillations" by K. Nakamura and S.T. Petcov in this Review.

$$\sin^2(\theta_{12}) = 0.307 \pm 0.013$$

$$\Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2$$

$$\sin^2(\theta_{23}) = 0.421^{+0.033}_{-0.023} \quad (S = 1.3) \quad (\text{Inverted order, quad. I})$$

$$\sin^2(\theta_{23}) = 0.592^{+0.025}_{-0.030} \quad (S = 1.1) \quad (\text{Inverted order, quad. II})$$

$$\sin^2(\theta_{23}) = 0.417^{+0.028}_{-0.028} \quad (S = 1.2) \quad (\text{Normal order, quad. I})$$

$$\sin^2(\theta_{23}) = 0.597^{+0.028}_{-0.030} \quad (S = 1.2) \quad (\text{Normal order, quad. II})$$

$$\Delta m_{32}^2 = (-2.56 \pm 0.04) \times 10^{-3} \text{ eV}^2 \quad (\text{Inverted order})$$

$$\Delta m_{32}^2 = (2.51 \pm 0.05) \times 10^{-3} \text{ eV}^2 \quad (S = 1.1) \quad (\text{Normal order})$$

$$\sin^2(\theta_{13}) = (2.12 \pm 0.08) \times 10^{-2}$$

## Three-Neutrino Oscillations

$$U_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \times \begin{pmatrix} \cos \theta_{13} & 0 & \sin \theta_{13} e^{i\delta_{CP}} \\ 0 & 1 & 0 \\ -\sin \theta_{13} e^{-i\delta_{CP}} & 0 & \cos \theta_{13} \end{pmatrix} \times \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Previous oscillations correspond to the "1-2" and "2-3" parameters
- The third type of oscillation would correspond to "1-3"
- Zero or very small  $\theta_{13}$  combined with very different  $\Delta m^2$  values for the two existing oscillations implies strong decoupling between them (Two-neutrino oscillations)
- Finite  $\theta_{13}$  means two of the three  $\Delta m^2$  values are very similar and oscillation effects become complicated  
⇒ Three-neutrino mixing

Yoshi Uchida (Imperial College London)

## Oscillations in the Three Large Mixing-Angle Paradigm (and beyond)

Since 2012

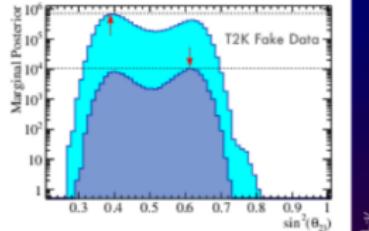
- Ten years ago we thought we might still be chasing a very small third mixing angle
- Now we know that all three mixing angles are “large”:
  - $\theta_{12} \simeq 34$  degrees,  $\theta_{23} \simeq 50$  degrees,  $\theta_{13} \simeq 8.6$  degrees
- Clear signs of flavour change when seen: basic “rate analyses” were enough to be convincing in discovery mode
- Next steps could be more subtle
  - Determination of parameter values within the PMNS model
  - Mass Hierarchy / Ordering
  - CP-violation and  $\delta_{\text{CP}}$
  - Deviations from the Unitary  $3 \times 3$  PMNS matrix
  - Oscillations into additional states
  - Non-standard interactions, Lorentz violation,....
- Consequences for statistical methods...

## Three Large Mixing-Angle Paradigm

From PhyStat- $\nu$  2016 IPMU

### Marginalization vs Projection

- \* T2K found that treatment of systematics can have noticeable effects on analyses, especially when treating oscillation parameters, which are very non-gaussian
- \* T2K has moved to marginalization for all oscillation analyses, using both MCMC and likelihood averaging over toys



$$f_{\text{marg}}(x) = \int dy f(x, y)$$
$$f_{\text{proj}}(x) = \max_y f(x, y)$$

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- Fake MC data generated at  $\sin^2 \theta_{23} = 0.4$
- **Marginalising** and **Profiling** give different preferred points
- Differences in meaning between Marginalising and Profiling

Asher Katoh, T2K(LBL- $\nu$ )

Yoshi Uchida (Imperial College London)

## Presentation of Results

### Elimination of nuisance parameters

- “Did you marginalise or did you profile?”: Asked frequently at 2016 meetings
- **For each set of values in the parameters of interest:**
  - Profiling: Fix nuisance parameters to their best-fit points (with external constraints)
  - Marginalising: Integrate over nuisance parameters (according to their priors)
- Nuisance parameters not so problematic if they are for detector effects etc.
- But they also include physics parameters that are not well-measured yet
  - Such as  $\theta_{23}$  when presenting  $\theta_{13}$  and  $\delta_{CP}$ ; not at all Gaussian
  - Using the prior  $\pi(\theta_{23})$  to integrate over
- Differences of coverage may also need to be studied
- “Never profile if you are a Bayesian”
- Many experimental systematics are also non-trivial, for example, neutrino production and interaction uncertainties
- **Q: Can we be agree on when we marginalise and when we profile?**

Yoshi Uchida (Imperial College London)

## Frequentist and Bayesian Statistics

- From 2016 meetings: general open-mindedness towards use of Frequentist and Bayesian statistics
- Bayesian methods used quite enthusiastically for model selection, event classification and study of discrete outcomes (e.g., mass hierarchy)
- Frequentist methods may be preferred for discovery searches
- Tend to agree for parameter estimation, which is also less dependent on prior choice than model selection
- “Why throw away half your toolbox?”
- INSPIRE HEP numbers for papers with “Neutrino” in the title:
  - 1548 mention “Frequentist” and variants
  - 2816 mention “Bayesian” and variants
  - 560 mention both
  - 35044 mention neither!

## Use of Bayesian Statistics

### Mass Hierarchy

#### Strong Bayesian evidence for the normal neutrino hierarchy

Fergus Simpson,<sup>a</sup> Raul Jimenez,<sup>a,b</sup> Carlos Pena-Garay<sup>c,d</sup> and  
Licia Verde<sup>a,b</sup>

**Abstract.** The configuration of the three neutrino masses can take two forms, known as the normal and inverted hierarchies. We compute the Bayesian evidence associated with these two hierarchies. Previous studies found a mild preference for the normal hierarchy, and this was due to the fact that the normal hierarchy is slightly more likely than the inverted hierarchy under prior ignorance. Here we identify the presence of a normal symmetry, which is imposed by data from neutrino oscillations. By combining constraints on the squared-mass splittings [1], with the limit on the sum of neutrino masses of  $\Sigma m_\nu < 0.12\text{eV}$  [2], and using a minimally informative prior on the masses, we infer odds of 42:1 in favour of the normal hierarchy, which is classified as ‘strong’ in the Jeffreys’ scale. We explore how these odds may evolve in light of higher precision cosmological data, and discuss the implications of this finding with regards to the nature of neutrinos. Finally the individual masses are inferred to be  $m_1 = 3.89^{+0.12}_{-0.13}\text{meV}$ ;  $m_2 = 8.8^{+0.12}_{-0.13}\text{meV}$ ;  $m_3 = 50.4^{+1.0}_{-1.0}\text{meV}$  (95% credible interval).

#### Objective Bayesian analysis of neutrino masses and hierarchy

Alan F. Heavens,<sup>a</sup> Elena Sellentin<sup>a</sup>

**Abstract.** Given the precision of current neutrino data, priors still impact markedly the inference on neutrino masses and their hierarchy. To avoid our understanding of neutrinos being driven by prior assumptions, we construct a prior that is mathematically minimally informative. Using the considered uninformative prior, we find that the normal hierarchy is favoured but with inconclusive posterior odds of 5:1:1. Better data is hence needed before the neutrino masses and their hierarchy can be well constrained. We find that the next decade of cosmological data should provide conclusive evidence if the normal hierarchy with negligible neutrino mass is correct, and if the uncertainty in the sum of neutrino masses drops below 0.025 eV. On the other hand, if neutrinos obey the inverted hierarchy, achieving strong relevance will be difficult. In this case, an uninformative prior was constructed and found priors of the Objective Bayesian approach. The prior is called ‘reference prior’ and is minimally informative in the specific sense that the information gain after collection of data is unchanged. The prior is computed for the combination of neutrino oscillation data and cosmological data and still applies if the data improve.

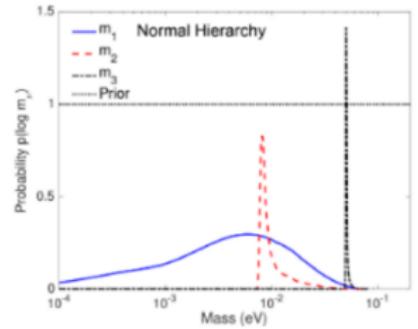
- “minimally-informative prior on the masses”
- “infer **odds of 42:1 in favour of the normal hierarchy**”
- “which is classified as ‘strong’ on the Jeffreys’ scale”
- “we construct a prior that is minimally-informative”
- “we find that the **normal hierarchy is favoured but with inconclusive posterior odds of 5:1:1**”
- “our uninformative prior was constructed from principles of the Objective Bayesian approach”

## Use of Bayesian Statistics

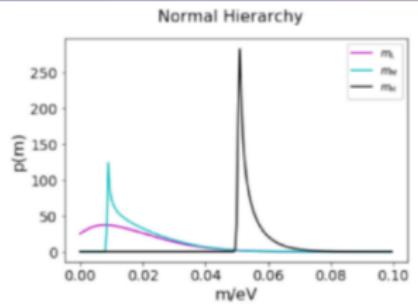
### Mass Hierarchy

- Conflicting conclusions for model selection
- Quite similar results for parameter fitting
- Simpson uses logarithms of masses

Simpson et al.:



Heavens et al.:



# Последние оценки иерархии масс

Stefano Gariazzo (IFIC-CSIC/University of Valencia)

## Can current data tell us the neutrino mass ordering?

- 1 [Hannestad, Schwetz, 2016]: extremely weak (2:1, 3:2) preference for NO (cosmology + [Bergstrom et al., 2015] neutrino oscillation fit)  
Bayesian approach;
- 2 [Gerbino et al., 2016]: extremely weak (up to 3:2) preference for NO (cosmology only), Bayesian approach;
- 3 [Simpson et al., 2017]: strong preference for NO (cosmological limits on  $\sum m_\nu$  + constraints on  $\Delta m_{21}^2$  and  $|\Delta m_{31}^2|$ )  
Bayesian approach;
- 4 [Schwetz et al., 2017], "Comment on ..." [Simpson et al., 2017]: effect of prior?
- 5 [Capozzi et al., 2017]:  $2\sigma$  preference for NO (cosmology + [Capozzi et al., 2016, updated 2017] neutrino oscillation fit)  
frequentist approach;
- 6 [Caldwell et al., 2017] very mild indication for NO (cosmology + neutrinoless double-beta decay + [Esteban et al., 2016] readapted oscillation results)  
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- 7 [Wang, Xia, 2017]: Bayes factor NO vs IO is not informative (cosmology only).

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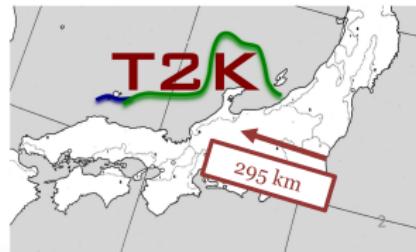
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# Эксперименты с “длинной базой”

Alexander Himmel (Fermilab)

## What is a Long-Baseline Neutrino Experiment?

- A beam of neutrinos (primarily  $\nu_\mu$ ) is produced by an accelerator.
- The neutrinos are allowed to travel and oscillate over a long distance.
- Detectors at the near and far end observe the neutrinos before and after oscillations.
- While energy and distance vary, the ratio of L/E is chosen to maximize “atmospheric” oscillations.



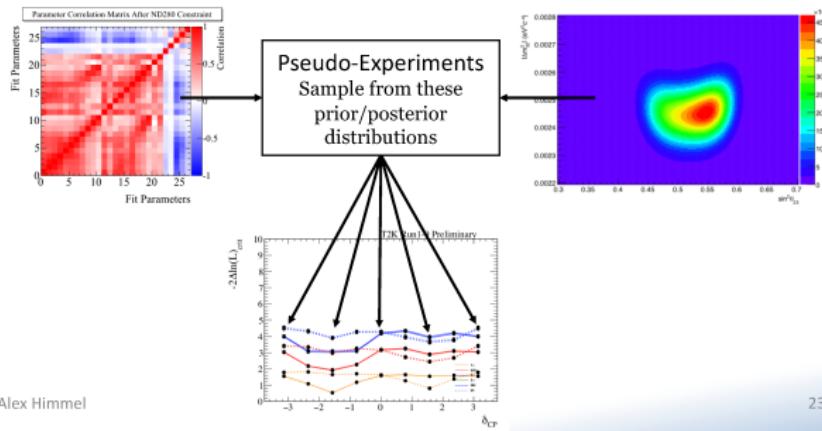
Alex Himmel

# Эксперименты с “длинной базой”

Alexander Himmel (Fermilab)

## Feldman-Cousins Pseudo-experiments: T2K

- For  $\theta_{13}$  and systematics:
  - Draw from prior distributions (PDG or output of ND fit)
- For  $\sin^2\theta_{23}$  and  $\Delta m^2$ :
  - Generate an Asimov dataset at best fit values, and construct a likelihood for this simulated dataset.
  - Convert the likelihood to a PDF, and draw values for the experiments from that distribution.



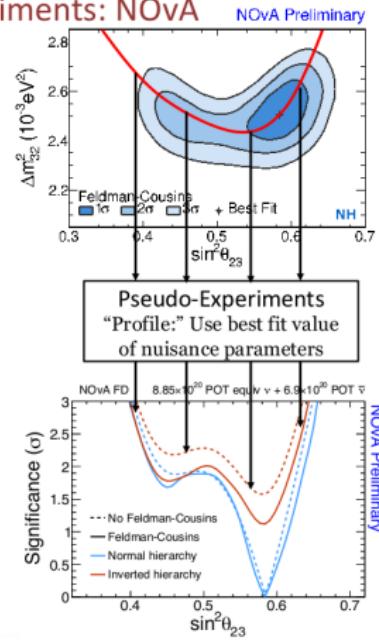
Процедура сильно отличается от стандартной.

# Эксперименты с “длинной базой”

Alexander Himmel (Fermilab)

## Feldman-Cousins Pseudo-experiments: NOvA

- Fit the data and extract parameters with all possible values of  $\theta$ .
- When generating experiments, always use the best fit to the nuisance parameters from the fit to data for each  $\theta$ .
  - Minimizes over-coverage of all methods we examined while still never under-covering.
- Tested coverage with a method from Berger and Boos for handling  $p$ -values with unknown nuisance parameters.
  - R. L. Berger and D. D. Boos, J. Amer. Statist. Assoc., 89, 1012 (1994)
  - Tested coverage at a variety of choices of oscillation nuisance parameters within  $3\sigma$ 
    - Reducing quoted significance by a very small amount
    - In all cases, the other choices of nuisance parameters produced stronger rejection than the quoted rejection at the nominal profiled values.
    - This is as expected if everything is working correctly since the profiled point should give the widest CIs or lowest significance.



Alex Himmel

Underlines are mine.

В частном случае подчеркнутое может быть верно. В общем случае подчеркнутое неверно!

## Выбор $p$ -величины

- Максимизация  $p$ -величины подбором мешающих параметров:
  - 1) Дает консервативную оценку, но насколько консерватизм оправдан?
  - 2) Нетривиальные максимумы существуют не для всех тестовых статистик и выборок.
  - 3) Существование таких тестовых статистик и выборок не доказано для произвольной задачи.
  - 4) В некоторых случаях выбранные мешающие параметры несовместимы с измерениями.
  - 5) Установление ограничений при подборе приводит к зависимости от (произвольных) ограничений.
- Максимизация  $p$ -величины подбором мешающих параметров на доверительном множестве: [R. L. Berger, D. D. Boos, J.Am.Stat.Ass., 89 (1994) 1012] “Valid  $p$ -value”:  $P(\rho \leq \alpha) \leq \alpha$  для любого  $\alpha \in [0, 1]$ .  
Пусть  $C_\beta$  есть доверительное множество соотв.  $1 - \beta$  для  $\theta$ , если  $H_0$  — истинна.  $\rho_\beta = \sup_{\theta \in C_\beta} \{\rho(\theta)\} + \beta$ ,  $\rho_\beta$  заменяется на  $\min\{\rho_\beta, 1\}$ , и этот минимум — “valid  $p$ -value”.  
Устраниет (формально) проблему 4), но только её.

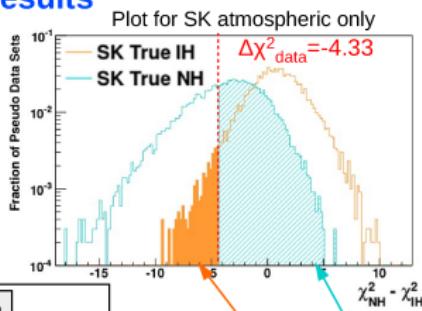
# Атмосферные нейтрино

Christophe Bronner (University of Tokyo)

## Mass hierarchy significance Super-K results

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- Used CLs to report significance: not truly frequentist, but conservative
- Computed p-values and CLs for lower/upper edges of the 90% CL intervals for  $\sin^2(\theta_{23})$  and  $\delta$
- Quoted a range of CLs-based significance in the paper



$$CL_s = \frac{p_0(IH)}{1 - p_0(NH)}$$

PRD 97, 072001 (2018)

P-values and CLs for IH exclusion			
P-values	Lower	Best fit	Upper
SK only	0.012	0.027	0.020
SK+T2K model	0.004	0.023	0.024
CLs	Lower	Best fit	Upper
SK only	0.181	0.070	0.033
SK+T2K model	0.081	0.075	0.056

Формула в дискретном случае не верна!

# Проблема моделирования реакторных антинейтрино

Emilio Ciuffoli (IMP, CAS)

## Reactor Neutrino Spectrum

Theoretical model for reactor neutrino fluxes:

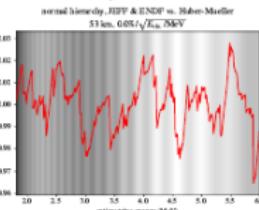
- *Ab initio* approach: calculate spectrum branch-by-branch (however, problematic for the large number of isotopes,  $\simeq 10^3$ , and branching ratio,  $\simeq 10^4$ )
- Conversion method: measure the beta spectrum directly and then convert to  $\bar{\nu}_e$

In the last years with the measurement of the “5 MeV bump” it became clear that the current models for reactor neutrinos are not very reliable.

Moreover, only available measurements of reactor neutrino spectrum with low energy resolution ( $\simeq 6\%/\sqrt{E}$ ; for MH  $3\%/\sqrt{E}$  needed).

A fine structure (“sawtooth-like features”) is most likely present (showed also in “ab initio” calculations) and currently undetected; it could be a problem for the MH determination

See also D. Forero, R. Hawkins, P. Huber, arXiv:1710.07378, and Danielson *et al.*, arXiv:1808:03276



Danielson *et al.*, arXiv:1808:03276

Uncertainty in the Reactor Neutrino Spectrum and MHD

## My advocacy for >10 years (Section 16):

Have in place tools to allow computation of results using a variety of recipes, for problems up to intermediate complexity:

- Bayesian with analysis of sensitivity to prior
- Profile likelihood ratio (Minuit MINOS)
- Frequentist construction with approximate treatment of nuisance parameters
- Other “favorites” such as LEP’s  $CL_s$  (an HEP invention)

The community can (and should) then demand that a result shown with one’s preferred method also be shown with the other methods, *and sampling properties studied.*

When the methods all agree, we are in asymptotic nirvana.

When the methods disagree, we are reminded that the results are answers to different questions, and we learn something! E.g.:

- Bayesian methods can have poor frequentist properties
- Frequentist methods can badly violate likelihood principle

Ещё около 20 докладов, в том числе по unfolding и deep neural networks.

## Заключение

На совещании PHYSTAT- $\nu$  2019 обсуждены

- ▶ основные методы статистического анализа данных,
- ▶ некоторые другие методы анализа,
- ▶ статус нейтринных экспериментов и перспективы на будущее.

Спасибо за внимание!