DIS as a probe of entanglement

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- D. E. Kharzeev and E.L.: *"Deep inelastic scattering as a probe of entanglement,"* Phys. Rev. D **95** (2017), 114008;
- D. E. Kharzeev and E.L.: *"Deep inelastic scattering as a probe of entanglement: confronting experimental data,"* [arXiv:2102.09773 [hep-ph]]
- E. Gotsman and E. L.: *"High energy QCD: multiplicity distribution and entanglement entropy,"* Phys. Rev. D **102** (2020) 074008

Motivations and Disclaimers:

- How does the pure state with $S = 0$ in the r.f. evolves to the set of 'quasi free' partons in the IMF with $S \neq 0$?
- What is the rigorous definition of 'quasi free' parton distribution?
- Why do partons have kind of Bolzman distribution while the number of collisions turns out to be small?
- How does parton distribution relate to parton density functions?
- What do we need to use instead of patrons deep in the saturation region?

Our answers lie in the entanglement of quantum states

Do not expect:

- **A thorough knowledge of EE**
  - **C. Holzhey, F. Larsen and F. Wilczek,** *"Geometric and Renormalized Entropy in Conformal Field ",* **Theory Nucl. Phys. B 424 (1994) 443, [hep-th/9403108];**
  - **P. Calabrese and J. L. Cardy,** *"Entanglement Entropy and Quantum Field Theory: A Non-Technical Introduction",* **Int. J. Quant. Inf. 4 (2006) 429, [quant-ph/0505193];**
  - **M.Martinelli,** *"Photons, Bits and Entropy: From Planck to Shannon at the Roots of the Information Age"*, **Entropy,19, 347 (2017);**

- **A rigorous answer to every questions.**

- **A list of prediction for DIS deep in the saturation region.**

This talk is an attempt to give the answers to all above questions, based on simple calculations and the observed similarities between CFT and the parton cascade if we discuss it in terms of entropy.

Ideas and results (1) :

- In toy (1+1) dimensional model as well as in there full QCD cascade we computed von Neumann entropy $S(x)$;

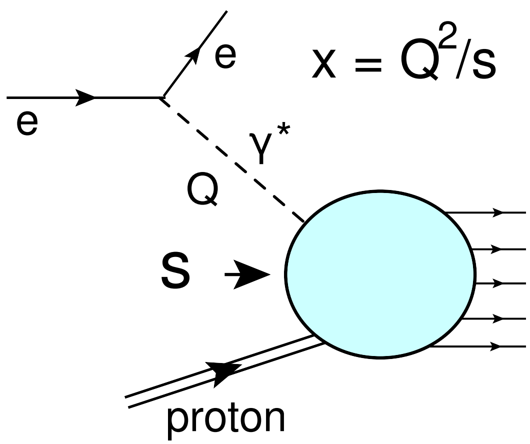
- We found that $S(x) = \ln\left(xG\left(x, Q^2\right)\right)$

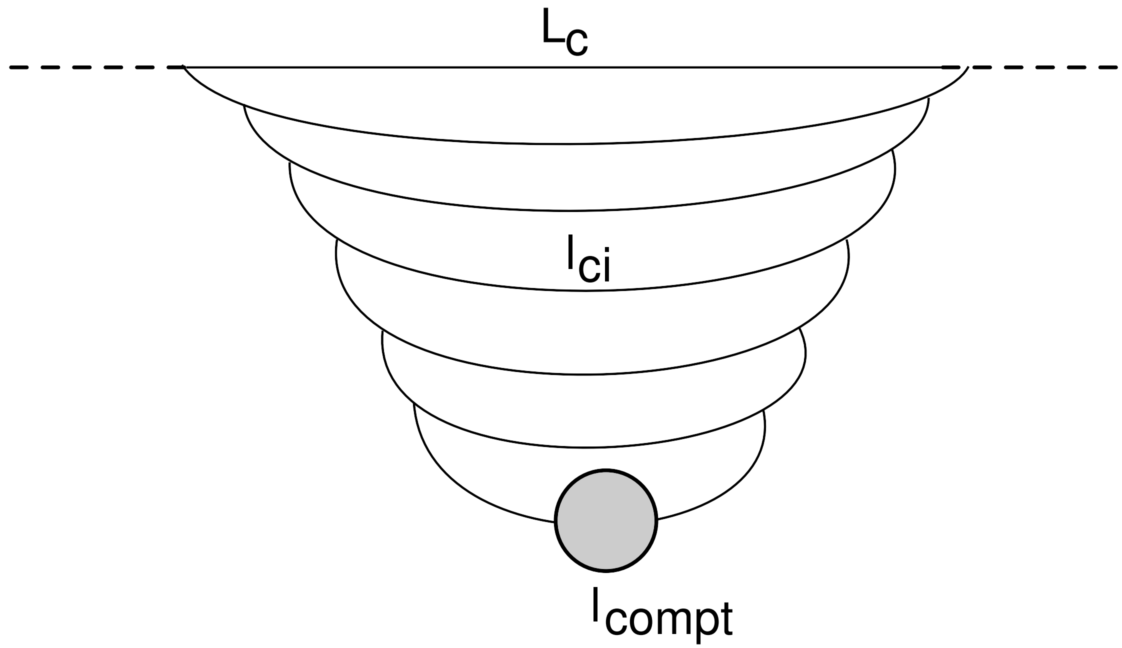
  where $xG(x, Q^2)$ is the multiplicity of partons(gluons);

- This equation implies that all microstates of the system are equally probable and S is maximal;

- This equipartitioning of microscopic states that maximizes the von Neumann entropy corresponds to the parton saturation;

- S **diverges logarithmically at** $x \to 0$; $S(x) \; = \; \Delta \ln\left(\frac{1}{x}\right) \; = \; \Delta \ln\left(\frac{L}{\epsilon}\right)$ **with** $L = 1/(mx)$ **and** $\epsilon = 1/m \leftarrow$ **proton's Compton wave length,** Δ **is the BFKL intercept** $\Delta = 2.8\bar{\alpha}_S$**;**





Ideas and results (2) :

- Reminds the expression for EE in (1+1) CFT: $S(x) \;=\; \frac{c}{3}\ln\frac{L}{\epsilon}$
- We argue that this agreement is not coincidental, and propose that the parton distributions, and the entropy associated with them, arise from the entanglement between the spatial domain probed by DIS and the rest of the target;
- The maximal value of the entanglement entropy attained at small x implies that the corresponding partonic state is *maximally entangled*;
- Unlike the parton distribution, the EE is an appropriate observable even at strong coupling when the description in terms of quasi-free partons fails.

QM of parton entanglement, as I understood it

A is the region that we measure in DIS, The physical sates are in $H_A(n_A)$.

B is a complementary region, unobserved state $\in H_B(n_B)$

the entire space: $A \cap B$. the composite system in $H_A \otimes H_B$

$$|\Psi_{AB}\rangle = \sum_{ij} c_{ij} |\phi_i^A\rangle \otimes |\phi_j^B\rangle; \quad \text{matrix } C \text{ has } n_A \times n_B \text{ dimension}$$

If $y \in A$ and $z \in B$ the density matrix:

$\rho\,(y,z,y',z') = \Psi_{AB}\,(y,z)\,\Psi^*_{AB}\,(y,z') \leftarrow$ pure state with S=0.

$$\rho_A\,(y,y') = \int dz\;\rho\,(y,z,y',z) \equiv \mathrm{tr}_B \rho_{AB}$$

Schmidt decomposition theorem (Schmidt (1907)):

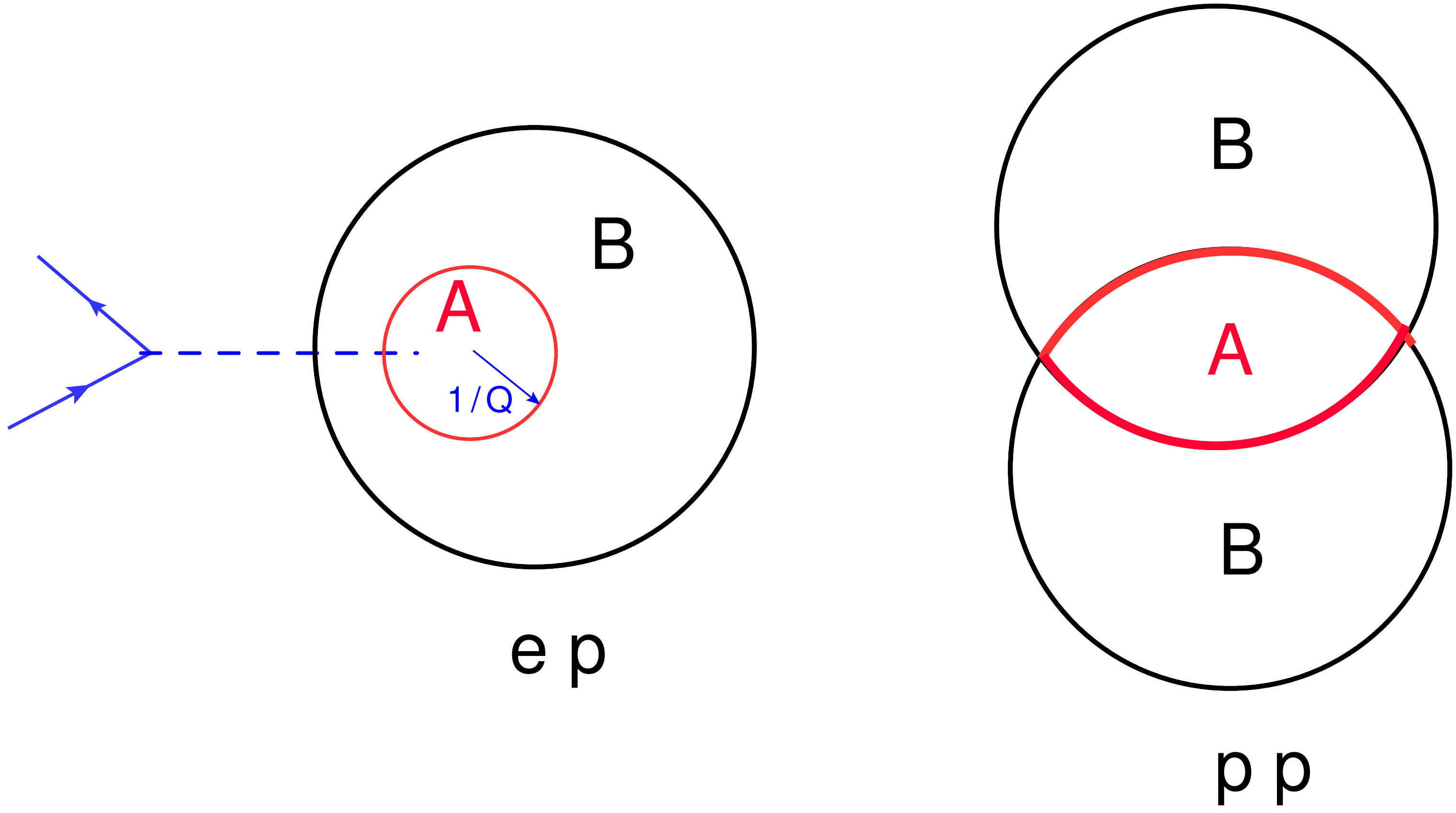
$$|\Psi_{AB}\rangle = \sum_n \alpha_n |\Psi^A_n\rangle |\Psi^B_n\rangle$$

where $\alpha_n = \sqrt{CC^\dagger}$.

• • • $\rho_A = |\Psi_{AB}\rangle\langle\Psi_{AB}| = \mathrm{tr_B}\,\rho_{\mathrm{AB}} = \sum_{\mathrm{n}} \alpha^2_{\mathrm{n}}\,|\Psi^{\mathrm{A}}_{\mathrm{n}}\rangle\langle\Psi^{\mathrm{A}}_{\mathrm{n}}|$ • • •

where $\alpha^2_n \equiv p_n \leftarrow$ the probability of a state with n partons.

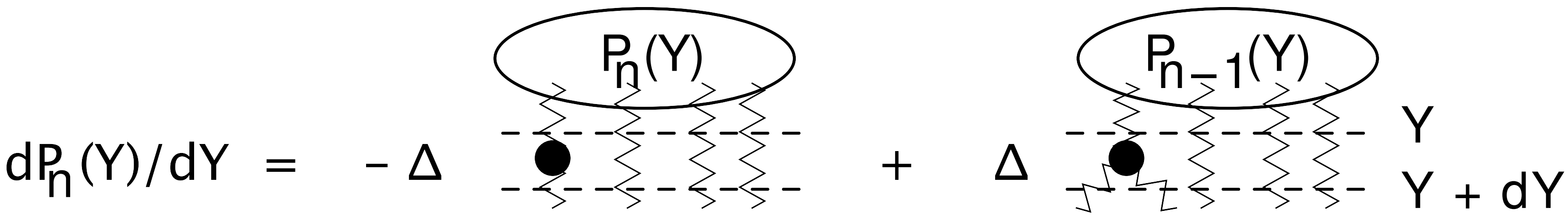
• • • $S_{\text{von Neumann}} = -\sum_n p_n \ln p_n \equiv S_{\mathrm{E}}$ • • •


B
A
1/Q
e p
B
A
B
p p

$1 + 1$ toy model of non-linear QCD evolution:

BFKL Pomeron:

- $$\frac{d\sigma(Y)}{dY} = \Delta\,\sigma(Y) \quad \textbf{where} \quad \Delta = 2.8\,\bar{\alpha}_S$$



- $$\frac{dP_n(Y)}{dY} = \underbrace{-\Delta\,n\,P_n(Y)}_{\text{depletion of the probability}} + \underbrace{(n-1)\,\Delta P_{n-1}(Y)}_{\text{growth due to splitting}}$$

Generating function:

$Z(Y,u) = \sum_n P_n(Y)\,u^n$, with $Z(Y=0,u) = u$; $Z(Y,u=1) = 1$

Equation:

- $$\frac{\partial Z(Y,u)}{\partial Y} \;=\; -\Delta\, u\,(1 - u)\,\frac{\partial Z(Y,u)}{\partial u} \;\xrightarrow{Z(u(Y))}\; \frac{\partial\, Z}{\partial\, Y} \;=\; -\,\Delta\left(Z \;-\; Z^2\right)$$

For scattering amplitude

$$N\,(Y;\gamma) = 1 - Z\,(Y, 1 - \gamma) \;\longrightarrow\; dN\,(Y)\,/dY \;=\; \Delta\left(N - N^2\right)$$

Solution:

- $$Z\,(Y,\,u) \;=\; \frac{u\;e^{-\,\Delta\,Y}}{1\,+\,u\,\left(e^{-\,\Delta\,Y}\,-1\right)} \;=\; u\,e^{-\,\Delta\,Y}\sum_{n=1}^{\infty} u^n\left(1\;-\;e^{-\,\Delta\,Y}\right)^n$$
- $$P_n\,(Y) \;=\; e^{-\,\Delta\,Y}\left(1\;-\;e^{-\,\Delta\,Y}\right)^{n-1}$$

Gluon sructure function:

$$xG\,(x) \;=\; \langle n \rangle \;=\; \sum_n n P_n\,(Y) \;=\; u\frac{dZ\,(Y,u)}{du}\bigg|_{u=1} \;=\; e^{\Delta\,Y} \;=\; \left(\frac{1}{x}\right)^{\Delta}$$

- $$\boxed{P_n\,(N) \;=\; \frac{1}{N}\left(1\;-\;\frac{1}{N}\right)^{n-1}}$$ **with** $N \equiv < n >$

Entropy: $S_{\text{von Neumann}} = -\sum_n p_n \ln(p_n) = -\sum_n P_n(Y) \ln(P_n(Y))$

$$S = -\sum_n \frac{1}{N}\left(1 - \frac{1}{N}\right)^{n-1}\left(-\ln(N-1) + n\,\ln\left(1-\frac{1}{N}\right)\right)$$

$$= \ln(N-1) + N\,\ln\left(\frac{1}{1+\frac{1}{N+1}}\right) \xrightarrow{N\gg 1} \ln N = \Delta \ln\left(\frac{1}{x}\right)$$

$$S_{\text{v. N.}} \longrightarrow \begin{cases} \ln\left(xG(x)\right) & \text{if } \Delta Y \gg 1 \\ -\ln\left[\frac{xG(x)-xG(x=x_0)}{xG(x=x_0)}\right]\left[\frac{xG(x)-xG(x=x_0)}{xG(x=x_0)}\right] & \text{if } \Delta Y \ll 1 \end{cases}$$

$$S_{\text{von Neumann}} = -\sum_n p_n \ln(p_n) = -\underbrace{\sum_{n=0}^{N} ln\left(\frac{1}{N}\right)\left(\frac{1}{N}\right)}_{maximal\ entanglement} = \ln N$$

$$N = xG(x, Q^2)$$

Multiplicity distributions: $(\tilde{N} = \tilde{n} - 1)$

$$P_n(Y) = e^{-\Delta Y}\left(1 - e^{-\Delta Y}\right)^{n-1} = \frac{1}{\bar{n}}\left(\frac{\bar{n}-1}{\bar{n}}\right)^{n-1} = \frac{1}{\bar{N}}\left(\frac{\bar{N}}{\bar{N}+1}\right)^{n}$$

Negative binomial distribution:

$$\frac{\sigma_n}{\sigma_{in}} = P^{\rm NBD}(r, \bar{n}, n) = \left(\frac{r}{r+\langle n\rangle}\right)^{r}\frac{\Gamma(n+r)}{n!\,\Gamma(r)}\left(\frac{\langle n\rangle}{r+\langle n\rangle}\right)^{n}$$

- $$\frac{\sigma_n}{\sigma_{in}} = \frac{\bar{n}-1}{\bar{n}}P^{\rm NBD}(1, \bar{n}-1, n)$$

with r $= 1$(number of failures) and $p = \bar{N}/\left(\bar{N}+1\right) = 1 - 1/\bar{n}$(probability of success)

Cumulants: $C_q = < n^q > / < n >^q = \left(u\frac{d}{du}\right)^q Z(Y,u)\Big|_{u=1}$

$$C_2 = 2 - 1/\bar{n}; \quad C_3 = \frac{6(\bar{n}-1)\bar{n}+1}{\bar{n}^2};$$

$$C_4 = \frac{(12\bar{n}(\bar{n}-1)+1)(2\bar{n}-1)}{\bar{n}^3}; \quad C_5 = \frac{(\bar{n}-1)(120\bar{n}^2(\bar{n}-1)+30\bar{n})+1}{\bar{n}^4}$$

Predictions: $C_2 \simeq 1.83$, $C_3 \simeq 5.0$, $C_4 \simeq 18.2$ and $C_5 \simeq 83$.

Experiment: $C_2^{\rm exp} = 2.0\pm0.05$, $C_3^{\rm exp} = 5.9\pm0.6$, $C_4^{\rm exp} = 21\pm2$, and $C_5^{\rm exp} = 90\pm19$

QCD cascades:

$$\frac{\partial P_n\left(Y,\{\vec{r},\vec{b}_i\}\right)}{\partial Y} = -\sum_{i=1}^{n} \omega_G(r_i)\, P_n\left(Y,\{\vec{r}_i,\vec{b}_i\}\right) + \bar{\alpha}_S \sum_{i=1}^{n-1} \frac{(\vec{r}_i+\vec{r}_n)^2}{(2\pi)\, r_i^2\, r_n^2}\, P_{n-1}\left(Y,\{\vec{r}_i,\vec{b}_i\}\right)$$

$$\omega_G(r) = \bar{\alpha}_S \int \frac{d^2 r'}{2\pi} \frac{r^2}{r'^2\,(\vec{r}-\vec{r}')^2}$$

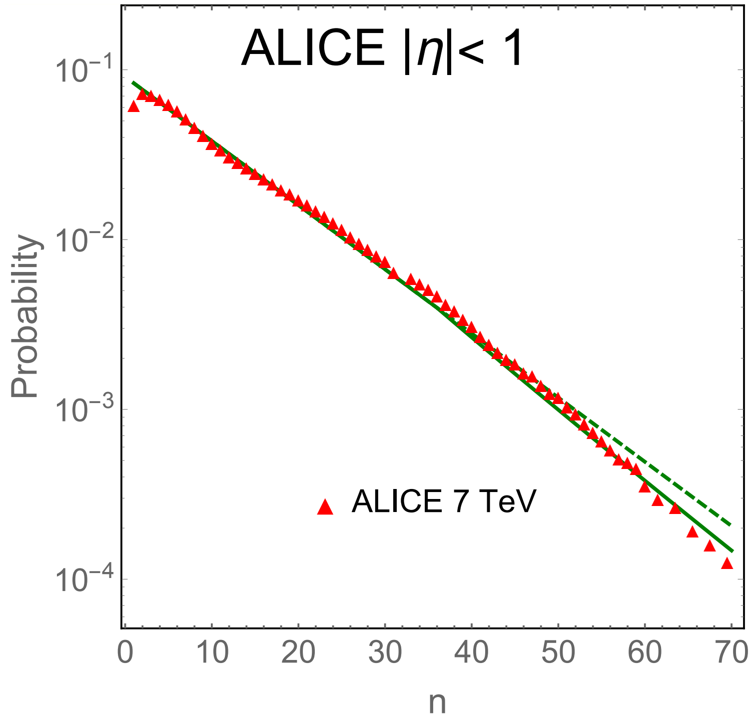
Kharzeev &E.L(2017), Gotsman &E.L(2020):

$$\longrightarrow P_n(N) = \frac{1}{N}\left(1 - \frac{1}{N}\right)^{n-1}$$

- **For DIS** $N = \Sigma_{\rm sea}(x, Q^2)$;

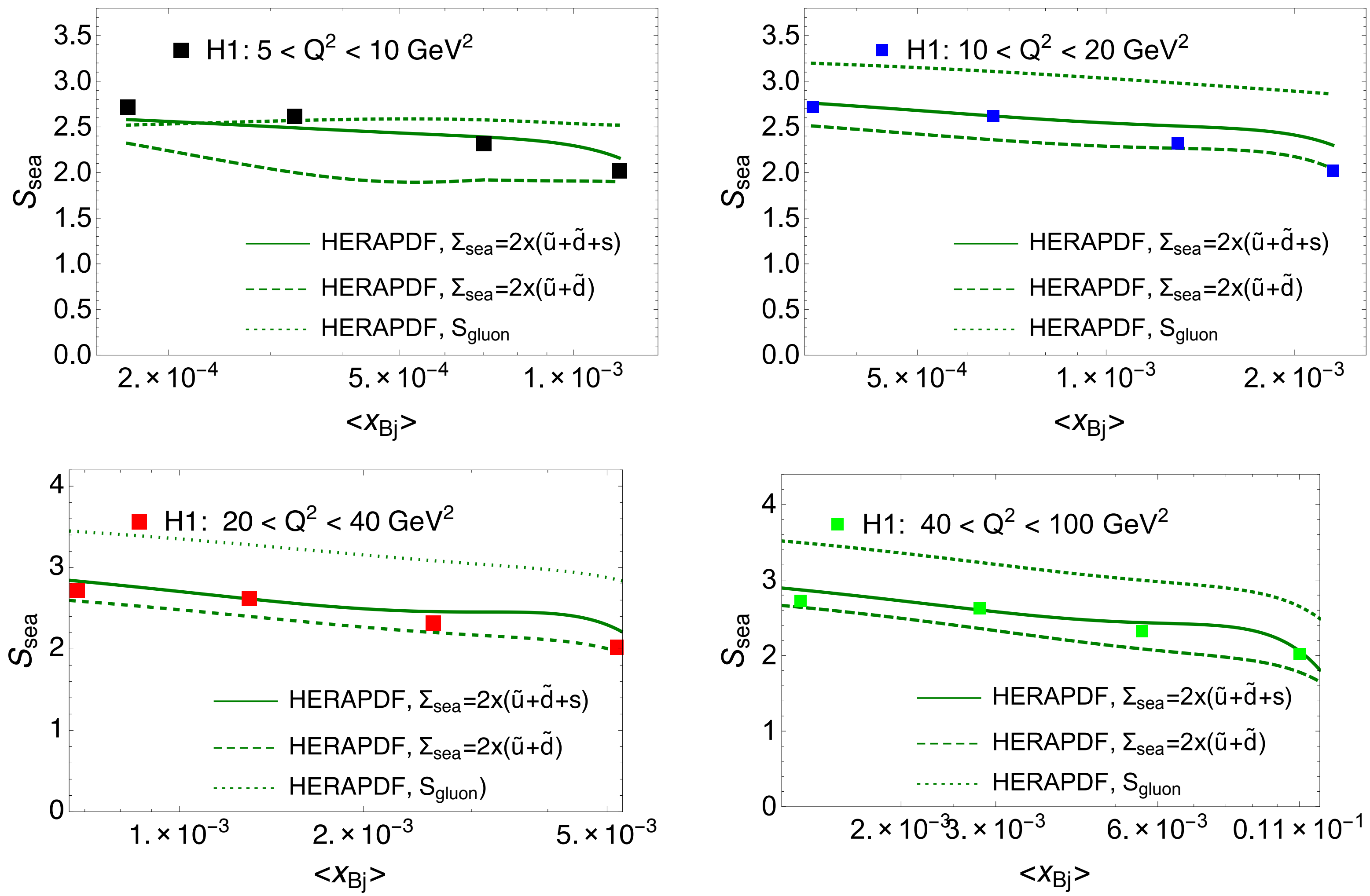
$$\Sigma_{\rm sea}(x,Q^2) = 2x\left(\bar{u}(x,Q^2) + \bar{d}(x,Q^2) + \bar{s}(x,Q^2)\right)$$

- **For pp -scattering** $N = Q_s^2(s)$;

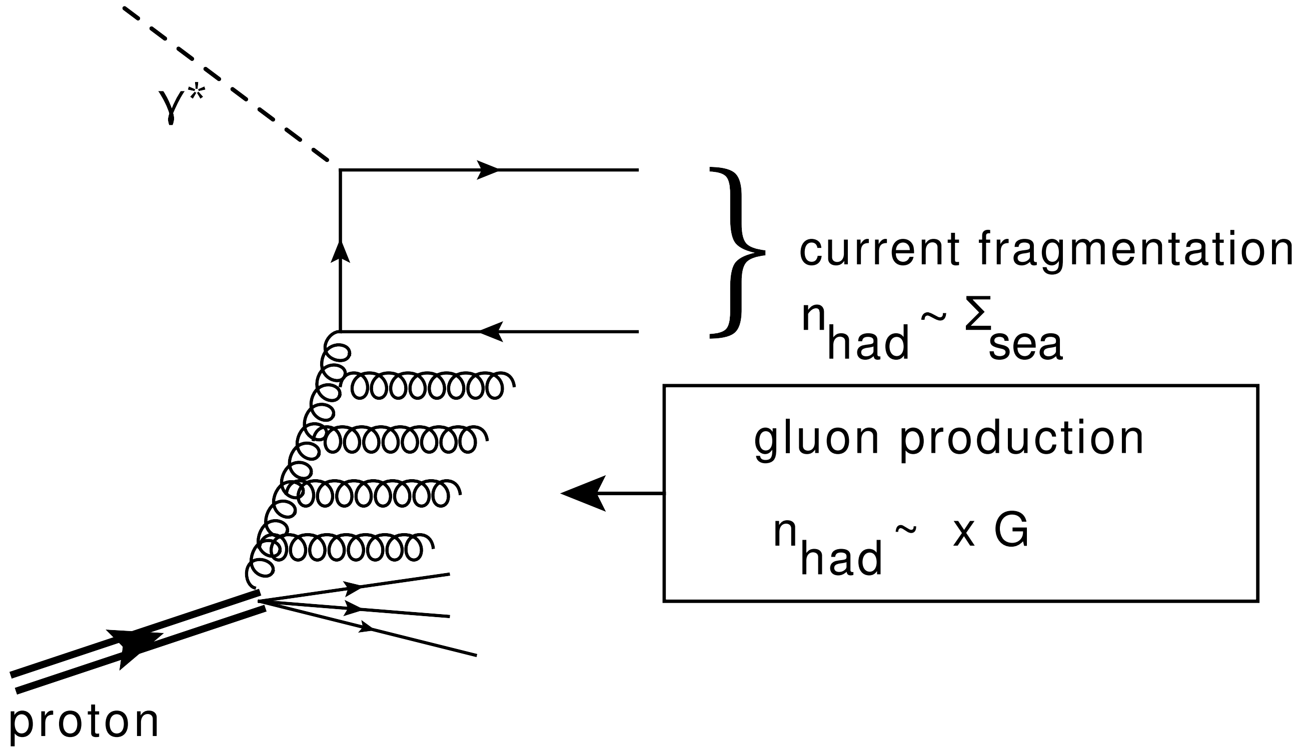


Dedicated experiment : H1(2020)

"Measurement of charged particle multiplicity distributions in DIS at HERA and its implication to entanglement entropy of partons", **DESY 20-176 arXiv:2011.01812 [hep-exp]:** $\mathbf{S} \neq \ln\left(xG(x,Q^2)\right)$



$$\Sigma_{\rm sea}(x,Q^2) \;=\; 2x\left(\bar{u}(x,Q^2) \,+\, \bar{d}(x,Q^2) + \bar{s}(x,Q^2)\right)$$



• • • $S_E = S_{\rm parton} = S_{\rm hadrons}$ • • •

Dokshitzer,Khoze, Troian & Mueller (1988))

Beyond of perturbative QCD:

1+1 CFT C. Holzhey, F. Larsen and F. Wilczek(1994), P. Calabrese and J. L. Cardy (2006) **:**

$$S_E = \frac{c}{3} \ln \left(\frac{L}{\epsilon} \right)$$

where

- **L is the length of the probed region;**
- **ϵ is the regularization scale (the resolution of the measurement);**
- **c is the central charge of CFT that counts the number of d.o.f ;**

In DIS: L $= \frac{1}{mx}$ and $\epsilon = \frac{1}{m}$ **S** $= \Delta \ln \left(\frac{L}{\epsilon} \right)$

Our conjecture:

At small x the field theory describing parton evolution approaches a fixed point corresponding to a CFT with the central charge

$$c \;=\; 3\,\Delta$$

- c **increases with evolution and hence** $\Delta \,\leq\, \frac{1}{3}$ (Zamolodchikov (1986))
- **In pQCD and in high energy phenomenology** $\Delta \,\approx\, 0.25$;
-
$$xG\left(x, Q^2\right) \;\leq\; \frac{\text{Const}}{x^{1/3}}$$

Back to results

- Identifying the entropy of partonic system as the entanglement entropy explains the apparent loss of quantum coherence in the parton model;

- Parton distributions have a well-defined meaning only for weakly coupled partons at large momentum transfer Q^2 – but the entanglement entropy is a universal concept that applies to states at any value of the coupling constant;

- Unlike the parton distributions, the entanglement entropy is subject to strict bounds – for example, if the small x regime is described by a CFT, the growth of parton distributions should be bounded by $xG(x, Q^2) \leq \text{Const}\ x^{-1/3}$;

- $S_{\text{hadrons}} \geq S_{\text{partons}}$ $\leftarrow$ the II law of thermodynamics.

  $S_{\text{hadrons}} = S_{\text{partons}}$ $\leftarrow$ local hadron parton duality;