Estimation of the hadronic contribution to $g_\mu - 2$ using the IHEP total cross section database

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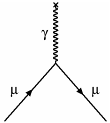
Семинар ОФВЭ ПИЯФ

Introduction

$$\vec{\mu}_\mu = -g_\mu \frac{e}{2m_\mu}\vec{S}$$

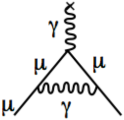
- $a_\mu = (g_\mu - 2)/2$ measured by FNAL Muon g-2 experiment to 0.215 ppm
- $\sim 5\sigma$ theory/experiment tension (with the e^+e^- based HVP estimate)
- ~ 1 ppm precision SM test, sensitive to TeV scale New physics
  - Theory uncertainty mostly due to QCD

Tree level



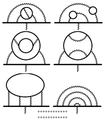
$a_\mu = 0$

Schwinger



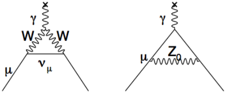
$+\frac{\alpha}{2\pi} = 11614097.3 \times 10^{-10}$

QED 2-5 loops



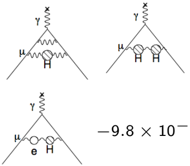
$+44374.6 \times 10^{-10}$

Electroweak



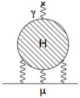
$+15.4 \times 10^{-10}$

Hadron VP NLO



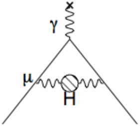
-9.8×10^{-10}

Hadron light-by-light



$+9.2 \times 10^{-10}$

Hadron VP LO

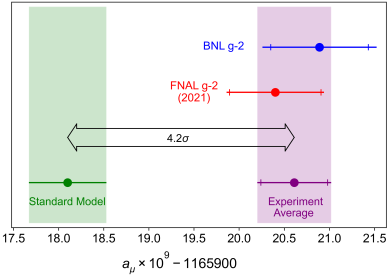


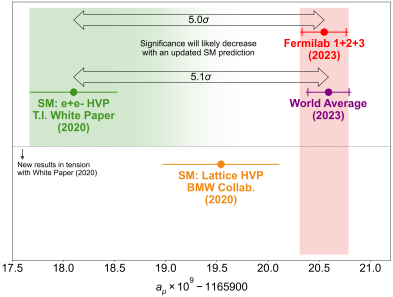
$+695 \times 10^{-10}$ (e^+e^- based)

← **The topic of this talk**

- Controversy in the e^+e^- input
- $\sim 3\sigma$ tension between e^+e^- and lattice estimates

Experiment vs theory

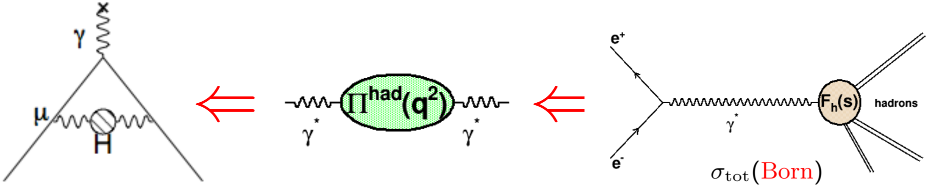




By M. Incagli (Muon g-2 Collaboration), https://indico.cern.ch/event/1312628/

- BNL E821 (2004): 3.7σ experiment/SM tension
- BNL E821 + FNAL g-2 Run-1 (2021, 5% of the full statistics): 4.2σ
- World average including FNAL g-2 Run-1-2-3 (*Muon g-2 Collaboration, arXiv:2308.06230*): **5.1σ tension!**
- SM prediction uncertainty mostly comes from hadron LO VP term:
  - e^+e^- HVP value too low (*the "White Paper": Muon g-2 Theory Initiative, Phys. Rept. 887 (2020) 1*)
  - Lattice HVP calculation gets SM a_μ closer to the experiment (*BMW Collaboration, Nature 593 (2021) 51*)
  - Tension between e^+e^- and lattice HVP
  - New CMD-3 $\pi^+\pi^-$ data $\sim 5\%$ higher than the world average (*CMD-3 Collaboration, arXiv:2309.12910*). $\Rightarrow$ Taken alone, CMD-3 puts SM a_μ estimate within $\sim 2\sigma$ from the experiment
  - More e^+e^- data to come: CMD-3 in other channels, SND, Babar, KLOE ($\pi^+\pi^-$), BESIII ($\pi^+\pi^-$, $\pi^+\pi^-\pi^0$), Belle II ...

$a_\mu(\text{had}, \text{LO})$ via $\sigma(e^+e^- \to \text{hadrons})$



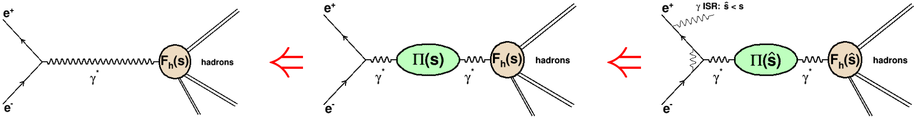
The dispersion relation (Phys. Rev. 105 (1957) 1931):

$$a_\mu(\text{had}, \text{LO}) = 4\alpha_0^2 \int_{m_\pi^2}^{\infty} \frac{ds}{s} K(s) \frac{1}{\pi} \,\mathrm{Im}\,\Pi^{\text{had}}(s) = \frac{\alpha_0^2}{3\pi^2} \int_{m_\pi^2}^{\infty} \frac{ds}{s} K(s) R^{\text{had}}(s)$$

$$R^{\text{had}}(s) = \sigma_{\text{tot}}(e^+e^- \to hadrons, \text{Born}) \Big/ \frac{4\pi\alpha_0^2}{3s}$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m_\mu^2)} \,.$$

$a_\mu(\text{had, LO})$ via $\sigma(e^+e^- \to \text{hadrons})$ *(continued)*



Born | Improved Born Approximation | Experiment

- We need Born cross section for the dispersion integral
- All experiments publish cross sections corrected for ISR + e^+e^- vertex loops
  - An extreme case is the radiative return measurements (BaBar, Belle, KLOE ...)
- Some experiments correct for photon VP (*a caveat: older ones include only leptonic part of the VP*), others leave the VP correction to readers
- Thus, we need first to uniformly rescale all published measurements to Born cross section:
  - Need to know photon $\Pi(s)$ including hadronic VP which is yet unknown as we determine it using a dispersion relation with $\sigma_{\text{tot}}(e^+e^- \to \textit{hadrons}, \textit{Born})$ as the input
  - Do it iteratively: use simple analytical parameterization of the hadronic VP as the first aprroximation, rescale published cross sections to Born, substitute them into the dispersion relation to get the hadronic VP, *etc, etc*

$a_\mu(\text{had, LO})$ via $\sigma(e^+e^- \to \text{hadrons})$ *(continued)*

- $\sigma_{\text{tot}}(e^+e^- \to \textit{hadrons})$ is measured mostly inclusively at $\sqrt{s} > 2$ GeV and for (semi)exclusive final states at $\sqrt{s} < 2$ GeV
- Most final states are measured by multiple experiments
- Parameterize Born cross section in each final state in a model-independent way
- Fit the parameterization taking into account correlated uncertainties within each experiment and between experiments
- Substitute the parameterized cross section into dispersion relations to find final state's contribution to the photon VP and $a_\mu(\text{had, LO})$
- Find total hadronic VP and $a_\mu(\text{had, LO})$ by summing up contributions from individual final states at $0.3 < \sqrt{s} < 11.2$ GeV;
  use ChPT parameterization of $R^{\text{had}}(s)$ at $m_\pi < \sqrt{s} < 0.3$ GeV ($\pi^0\gamma$, $\pi\pi(\gamma)$);
  add contributions from narrow resonances J/Ψ, $\Psi(2S)$, $\Upsilon(1-4S)$;
  insert analytical parameterization of $R^{\text{had}}(s)$ at $\sqrt{s} > 11.2$ GeV into dispersion relations.

So far, one more e^+e^- based HVP estimate (IHEP, Protvino):

- Prerequisites and the workflow:
  - The input: IHEP database of *(particularly)* $e^+e^- \to$ *hadrons* total cross sections
  - Rescale published cross sections to R^{had} (apply/unfold radiative corrections)
  - Parameterize and fit R^{had} in each final state
  - Integrate fitted R^{had} with the $K(s)$ kernel to obtain HVP contribution to a_μ from each final state at $0.3 < \sqrt{s} < 11.2$ GeV, outside this range use analytical parameterizations of R^{had}
- Prerequisites in place since 2003 [V.V. Ezhela et al, hep-ph/0312114]
- The code was used for the PDG minireview "σ and R in e^+e^- collisions" [R.L. Workman et al. (Particle Data Group), Review of Particle Physics, PTEP 2022, 083C01 (2022), also in earlier RPP editions since 2002]
- ⇒ All in place, why not making our HVP estimate?
  - No common code with the Muon g-2 Theory Initiative contributors ⇒ **one more independent cross-check**.

Model-independent parameterization of R^{had}

- Each final state is typically measured by many independent experiments, need to average them.
- Averaging requires to parameterize R^{had} by some continuous function:
  - No prior assumptions about contributions of various amplitudes to the production of the final state.
- A simple choice: parameterize R^{had} by continuous piecewise linear curve. The optimal number and position of the nodes are determined only by the set of experimental measurements $\{s_i, R_i^{\mathrm{had}}\}$, no signal model is assumed.

$\{s_i, R_i^{\mathrm{had}}\}$ points are clustered as follows:
Define the clusterization radius determined by the size of s interval where R^{had} is compatible with a constant within experimental uncertainties. For each s define sliding intervals of "compatibility with a constant": $[s, s + r^+(s)]$, $[s - r^-(s), s]$.
For each pair of measurements $\{i, j\}$ $(s_j > s_i)$ define the proximity metric:

$$w_{ij} = \min\left\{\frac{1}{\sigma_i^2}, \frac{1}{\sigma_j^2}\right\}\left[\frac{s_j - s_i}{\sqrt{a^2 r^+(s_i) r^-(s_j)}}\right]^b,$$

where $\sigma_{i,j}$ are the statistical uncertainties of the measurements and $a, b \sim 1$ are fixed parameters (their variation gives us an estimate of the algorithm's systematics).

$\{i, j\}$ pair with the minimum $w_{ij} = w_{\min}$ is merged into a single point as follows:

continued on the next slide . . .

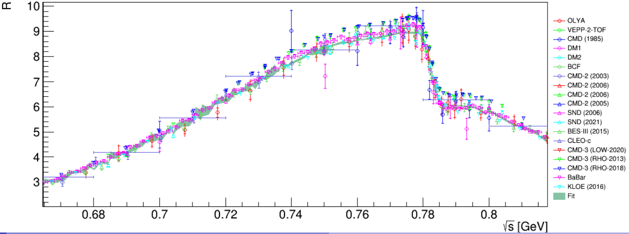
Model-independent parameterization of R^{had} *(continued)*

1. Set w_{min} to a value exceeding any possible w_{ij}.
2. For all $\{i,j\}$ pairs:
    1. Find w_{ij} for the $\{i,j\}$ pair.
    2. If for the $\{i,j\}$ pair $w_{ij} \geq 1/\sigma_i^2$ and $w_{ij} \geq 1/\sigma_j^2$ then move on to the next pair of points.
    3. If for the $\{i,j\}$ pair $w_{ij} < w_{\mathrm{min}}$ then $w_{\mathrm{min}} := w_{ij}$, $\{i,j\}_{\mathrm{min}} := \{i,j\}$.
3. If $\{i,j\}_{\mathrm{min}}$ is not found then **stop the clusterization**.
4. Otherwise merge the pair of points $\{i,j\}_{\mathrm{min}}$ into a single point with $s = w_i s_i + w_j s_j$ and $\sigma^2 = \sigma_i^2 + \sigma_j^2$, where weights $w_{i,j} = \frac{1}{\sigma_{i,j}^2} \Big/ \left(\frac{1}{\sigma_i^2} + \frac{1}{\sigma_j^2} \right)$.
5. Return to **step 2**.

In result, we get a set of $\{s_k\}$ for the nodes of the piecewise linear curve which will approximate the R^{had}. The corresponding $\{R_k\}$ values are then found by a standard χ^2 fit on the set of experimental measurements $\{s_i, R_i\}$ taking into account their binning and statistical and (correlated) systematic uncertainties.

A typical result of the clusterization ($\rho - \omega$ interference region in $\pi^+\pi^-$):

- Multiple experiments with different binning in $\sqrt{s}$, statistical tension between experiments.
- Too detailed parameterization leads to unphysical fluctuations in the fitted R^{had} $\Rightarrow$ smoothing/clusterization needed. See the blue 'Fit' curve



Fitting the $R^{\rm had}$ data

A standard χ^2 minimization:

$$\chi^2 = \sum_{i,j} \left[\frac{1}{\Delta\sqrt{s_i}} \int_{\Delta\sqrt{s_i}} R^{\rm had}_{fit}(s) d\sqrt{s} - R^{\rm had}_i \right] \times {\rm COV}^{-1}_{ij} \times \left[\frac{1}{\Delta\sqrt{s_j}} \int_{\Delta\sqrt{s_j}} R^{\rm had}_{fit}(s) d\sqrt{s} - R^{\rm had}_j \right],$$

where $R^{\rm had}_{fit}(s)$ is the fitted parameterization, $R^{\rm had}_i$ are the measurements in $\Delta\sqrt{s_i}$ bins, and ${\rm COV}_{ij}$ is the full covariance matrix between measurements:

$$\begin{aligned} {\rm COV}_{ij} = \delta_{ij}\sigma^2_{{\rm stat},i} \quad + \quad & \frac{1}{\Delta\sqrt{s_i}} \int_{\Delta\sqrt{s_i}} R^{\rm had}_{fit}(s) d\sqrt{s} \times \frac{1}{\Delta\sqrt{s_j}} \int_{\Delta\sqrt{s_j}} R^{\rm had}_{fit}(s) d\sqrt{s} \times \\ & \times \left\{ \begin{array}{l} \Delta_{{\rm sys},i}\Delta_{{\rm sys},j}, \text{ if } i,j \text{ are from the same experiment} \\ \Delta_{{\rm sys},i}\Delta_{{\rm sys},j} \times ({\rm cross-experiment\ covariation}), \\ \qquad\qquad \text{if } i,j \text{ are from different experiments} \end{array} \right\}, \end{aligned}$$

where $\Delta_{{\rm sys},i}$ are the relative systematic uncertainties as quoted by the experimentalists.

Why $R^{\rm had}_{fit}(s)$ in the systematic term of ${\rm COV}_{ij}$? Naively taking individual measurements $R^{\rm had}_{i,j}$ for the systematic uncertainty leads to a biased ${\rm COV}_{ij}$ and to a biased fit as $R^{\rm had}_{i,j}$ *are already biased themselves* – a manifestation of the well known *Peele's Pertinent Puzzle (PPP)*: "... a phenomenon exhibiting unexpected mean values for experimental data affected by statistical and systematic errors" [R. Frühwirth *et al*, EPJ Web of Conf., Vol. 27 (2012), 00008]

The problem: $\delta\chi^2/\delta R^{\rm had}_{fit}(s)$ is non-linear w.r.t. $R^{\rm had}_{fit}(s)$ ⇒ run the fit iteratively

Fitting the R^{had} data *(continued)*

... $\longrightarrow$ run the fit iteratively:

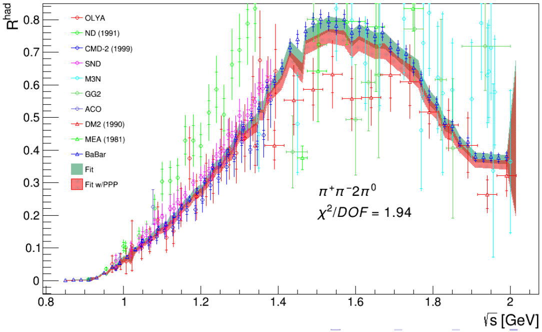
1. Make the fit ignoring the systematic uncertainties to get zeroth approximation for $R^{\mathrm{had}}_{fit}(s)$. Though χ^2/dof is awful, there's no PPP bias in the fit using a diagonal covariance matrix.
2. Rebuild the full covariance matrix using the obtained $R^{\mathrm{had}}_{fit}(s)$.
3. Repeat the fit with the full covariance matrix.
4. Compare just obtained $R^{\mathrm{had}}_{fit}(s)$ with the one from the previous iteration. **Stop** if the convergence condition *(to be refined)* is satisfied, otherwise return to step 2.

In practice, the procedure converges after 2 iterations.

TODO: Estimate the residual bias? Stability w.r.t. the choice of the zeroth approximation for $R^{\mathrm{had}}_{fit}(s)$? Can we start from the non-diagonal covariance matrix using measured R^{had}_i values for its systematic part? ... ?

Significance of the PPP effect:
Most prominent in final states with tension between independent experiments, e.g., in $\pi^+\pi^-2\pi^0$. $\rightarrow$
The PPP bias (red band, always negative!) is comparable to the uncertainty of the unbiased fit (green band).
An integral effect is $\delta a_\mu(\mathrm{had,LO})/a_\mu(\mathrm{had,LO}) \sim -1\%$.

More details and pathological examples in ▸ backup



Fitting the $R^{\rm had}$ data: $a_\mu(\rm had, LO)$ integral

Remarks on fit results:

- Problematic final states:
  - $\pi^+\pi^-$ with $\chi^2/\rm dof = 2.19$.
    $\chi^2/\rm dof$ drops to 1.47 upon exclusion of the latest CMD-3 data being in 5σ tension with other measurements. Precision KLOE and BaBar measurements are also in tension (discussed later).
  - $2\pi^+2\pi^-$, $\chi^2/\rm dof = 2.34$: high precision BaBar measurement in tension with SND and old Orsay data.
  - $\pi^+\pi^-2\pi^0$, $\chi^2/\rm dof = 1.94$: ND (1991) strongly disagrees with the others, still no reason to exclude.
- We don't drop (imprecise) pre-1990 data: different instrumentation, reconstruction and statistical procedures provide a cross-check with newer experiments.
- In channels with $\chi^2/\rm dof > 1.5$ the propagated experimental uncertainty of $R^{\rm had}_{fit}$ is scaled by $\sqrt{\chi^2/\rm dof}$ (cf. Birge factor in PDG prescription).
- $a_\mu(\rm had, LO) = 695.89 \pm 1.93_{e^+e^-\,\rm exp.}$ is in agreement with recent results by other groups (cf. Phys. Rept. 887 (2020) 1) despite an inclusion of CMD-3 2023 data.
  A good channel-by-channel agreement with A. Keshavarzi *et al*, Phys. Rev. D 101 (2020) 1, 014029 (we intentionally chosen identical integration ranges).

Final state	$a_\mu(\rm had, LO)$ $\times 10^{10}$ (exp. err.)	$\sqrt{s}$[GeV]	$\chi^2/\rm dof$
$\pi^+\pi^-(\gamma)$	505.28 (1.37)	[0.30000, 1.937]	2.19
$\pi^+\pi^-\pi^0$	48.48 (0.97)	[0.66000, 1.937]	1.79
$\pi^+\pi^-2\pi^0$	18.78 (0.44)	[0.85000, 1.937]	1.94
$2\pi^+2\pi^-$	15.40 (0.18)	[0.61250, 1.937]	2.34
K^+K^-	23.21 (0.19)	[0.98500, 1.937]	1.99
K_SK_L	13.19 (0.13)	[1.00371, 1.937]	0.95
$K^+K^-\pi^0$	0.20 (0.05)	[1.44000, 1.937]	0.54
$\pi^0\gamma$	4.36 (0.09)	[0.59986, 1.380]	1.70
$2\pi^+2\pi^02\pi^-$	1.73 (0.20)	[1.31250, 1.937]	1.99
$2\pi^+2\pi^-\pi^0$	1.74 (0.14)	[1.01250, 1.937]	1.29
$\pi^+\pi^-3\pi^0$	1.07 (0.11)	[1.12500, 1.937]	0.68
$3\pi^+3\pi^-$	0.24 (0.01)	[1.31250, 1.937]	1.90
$\pi^+\pi^-\pi^0\eta$	0.66 (0.07)	[1.39400, 1.937]	0.82
$\pi^+\pi^-2\pi^0\eta$	0.12 (0.02)	[1.62500, 1.937]	0.85
$2\pi^+2\pi^-3\pi^0$	0.10 (0.01)	[1.57500, 1.937]	0.57
$3\pi^+3\pi^-\pi^0$	0.02 (0.004)	[1.60000, 1.937]	0.65
$K_SK^\pm\pi^\mp$	1.81 (0.10)	[1.24000, 1.937]	0.99
$K_SK_L\eta$	0.12 (0.03)	[1.57500, 1.937]	1.31
$K_SK_L\pi^0$	0.84 (0.11)	[1.42500, 1.937]	1.50
$K^+K^-\pi^+\pi^-$	0.80 (0.03)	[1.40000, 1.937]	1.57
$K^+K^-\pi^0\pi^0$	0.10 (0.01)	[1.50000, 1.937]	1.32
$K_SK_L\pi^+\pi^-$	0.17 (0.03)	[1.42500, 1.937]	-
$K_SK_L\pi^0\pi^0$	0.14 (0.04)	[1.35000, 1.937]	-
$K_SK^\pm\pi^\mp\pi^0$	0.64 (0.04)	[1.51000, 1.937]	1.08
$K_SK_S\pi^+\pi^-$	0.07 (0.01)	[1.63000, 1.937]	1.37
$K^+K^-\pi^+\pi^-\pi^0$	0.13 (0.02)	[1.61250, 1.937]	1.63
$\eta\gamma$	0.69 (0.06)	[0.60000, 1.354]	1.36
$\eta\pi^+\pi^-$	0.57 (0.02)	[1.15000, 1.937]	1.18
$\omega < \pi^0\gamma > \pi^0$	0.89 (0.02)	[0.75000, 1.937]	1.56
$\omega < \pi^+\pi^-\pi^0 > \pi^+\pi^-$	0.098 (0.005)	[1.15000, 1.937]	1.10
$\omega\eta$	0.035 (0.002)	[1.34000, 1.937]	0.85
$\omega\eta\pi^0$	0.055 (0.043)	[1.50000, 1.937]	1.16
$\phi\eta$	0.067 (0.003)	[1.56000, 1.937]	0.98
$p\bar{p}$	0.030 (0.001)	[1.88900, 1.937]	1.24
$n\bar{n}$	0.028 (0.006)	[1.89000, 1.937]	1.24
2hadron(hadrons)	43.51 (0.72)	[1.93700,11.199]	1.35
pQCD	2.065	>11.199	
$\pi^0\gamma, \pi^+\pi^-$ (ChPT)	0.538	[0.140,0.300]	
J/Ψ	6.343	3.096916	
$\Psi(2S)$	1.483	3.686093	
$\Upsilon(1S)$	0.055	9.460300	
$\Upsilon(2S)$	0.018	10.023260	
$\Upsilon(3S)$	0.014	10.355200	
$\Upsilon(4S)$	0.007	10.580000	
Total:	**695.886 (1.928)**		

$R^{\rm had}$ outside the experimental range

- No $\sigma(e^+e^- \to \pi^+\pi^-)$ measurements at $2m_\pi < \sqrt{s} < 0.3$ GeV $\Rightarrow$ use ChPT parameterization of the pion formfactor:

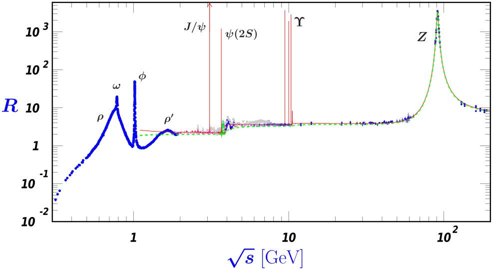
$$F_\pi^{\rm ChPT}(s) = 1 + \frac{< r^2 >_\pi}{6} s + c_1 s^2 + c_2 s^3 + \mathcal{O}(s^4),$$

  where the pion charge radius $< r^2 >_\pi = (11.27 \pm 0.21)\,{\rm GeV}^{-2}$ is extracted from the t-channel scattering and $c_{1,2}$ are from the $\sigma(\pi\pi)$ fit at $0.4 < \sqrt{s} < 0.6$ GeV.
  Though we didn't update the parameters since 2003, the impact would be at $\sim 0.05 \times 10^{-10}$ level
- No $\sigma(e^+e^- \to \pi^0\gamma)$ data at $\sqrt{s} < 0.6$ GeV $\Rightarrow$ parameterize using the $\pi^0 \to \gamma^*\gamma$ transition formfactor [Phys. Rev. D 65 (2002) 073034].
  Much smaller than $\pi\pi$ in the same range.
- Narrow $\Psi(1,2S)$, $\Upsilon(1-4S)$ resonances: the relativistic Breit–Wigner σ parameterization with PDG Γ_{ee}, $\Gamma_{\rm tot}$, M values. A caveat: photon hadronic VP term rapidly varies around a resonance [arXiv:hep-ph/0312114].
- $R^{\rm had}$ at $\sqrt{s} > 11.2$ GeV: measurements do exist up to LEP II energies, still use the 3-loop pQCD expression [K.G. Chetyrkin *et al.*, Phys. Rept 277 (1996) 189]:

$$R^{\rm had}(s) = 3 \sum_{2m_q < \sqrt{s}} Q_q^2 \left(1 - \frac{4m_q^2}{s}\right)^{1/2} \left(1 + \frac{2m_q^2}{s}\right) \left[1 + \frac{\alpha_S(s)}{\pi} + \ldots\right]$$

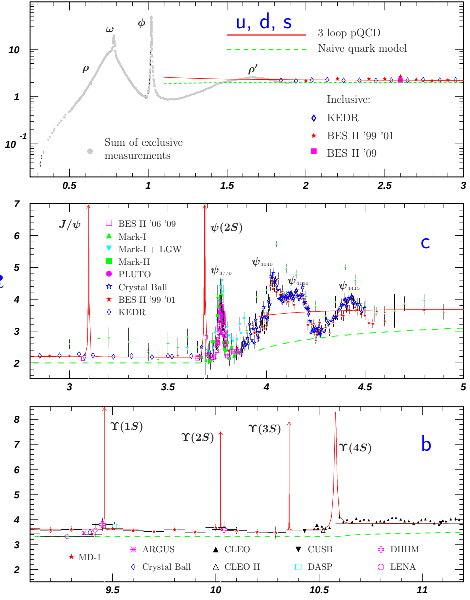
  Switching between data/pQCD in the $11.2 < \sqrt{s} < 40$ GeV range gives a negligible uncertainty on $a_\mu({\rm had, LO})$.

R^{had}: overall picture



R.L. Workman et al., Review of Particle Physics, PTEP 2022, 083C01 (2022) (our contribution)

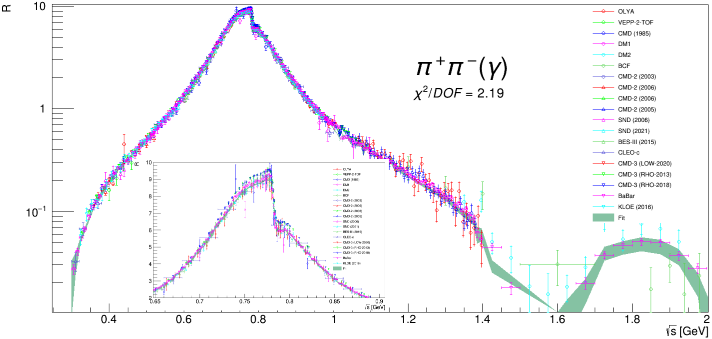
- New CMD-3 and BES III (2023) data not included (the difference would be hardly visible).
- Good agreement between inclusive $e^+e^- \to 2\text{hadron}(\text{hadrons})$ and the sum of exclusive measurements at $\sqrt{s} \sim 2$ GeV. This indicates that we didn't miss (semi)exclusive final states with a non-negligible cross section.
- Good agreement between data and pQCD prediction for R^{had} outside $q\bar{q}$ threshold regions.

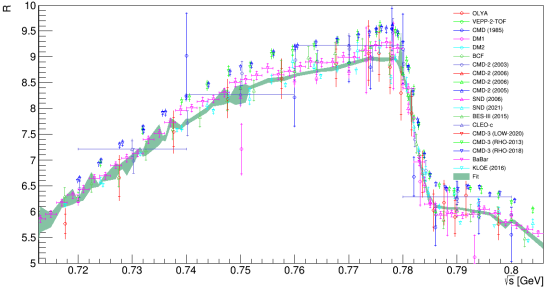


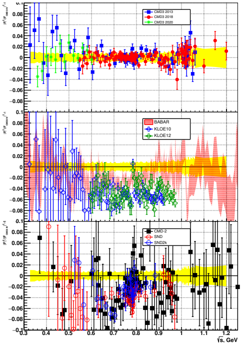
$\pi^+\pi^-$ channel

- $\sim$ 70% contribution to $a_\mu(\mathrm{had, LO})$
- CMD-3 (Novosibirsk) 2013-2020 data $\sim$ 5% higher than others, including CMD-2.
- BaBar/KLOE tension (both using radiative return).
- Fit dominated by KLOE with its $\sim 1\%$ uncertainty.
- Don't drop anything, just rescale the fit uncertainty. $\rightarrow$ $\Delta a_\mu(\mathrm{had, LO}) \times 10^{10} = 505.28 \pm 1.37_{\mathrm{exp}}$

w/o BaBar & KLOE $\Rightarrow$ $511.81 \pm 1.74_{\mathrm{exp}}$, $\chi^2/\mathrm{dof} = 1.43$





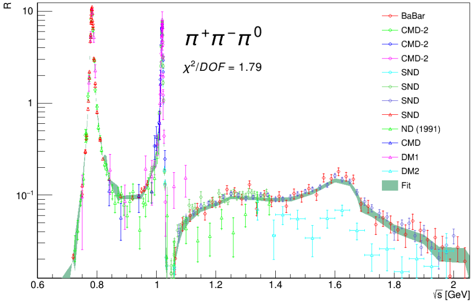


$\Leftarrow$ From CMD-3 Coll., arXiv:2309.12910
Yellow band is the fit to CMD-3 data.

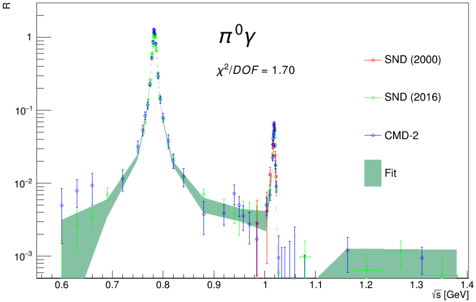
BaBar, KLOE to CMD-3 ratios.

CMD-2, SND, SND2k to CMD-3 ratios.

$\pi^+\pi^-\pi^0$ and $\pi^0\gamma$ channels

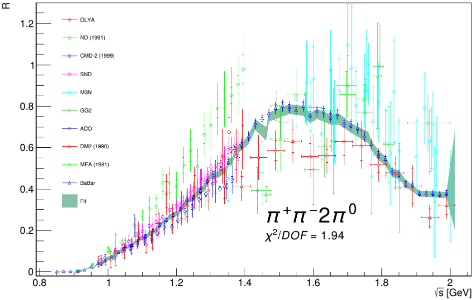


$$\Delta a_\mu(\mathrm{had,LO}) \times 10^{10} = 48.48 \pm 0.96_{\mathrm{exp}}$$

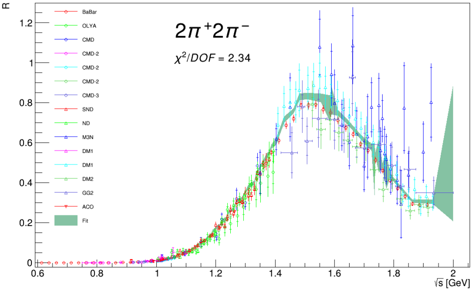


$$\Delta a_\mu(\mathrm{had,LO}) \times 10^{10} = 4.36 \pm 0.09_{\mathrm{exp}}$$

4π channels

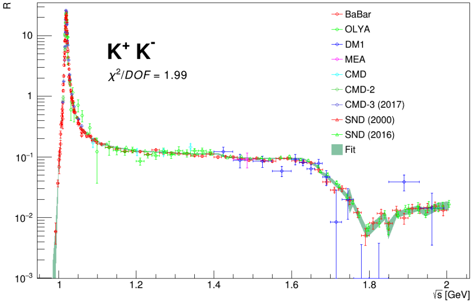


$$\Delta a_\mu(\mathrm{had, LO}) \times 10^{10} = 18.78 \pm 0.44_{\mathrm{exp}}$$

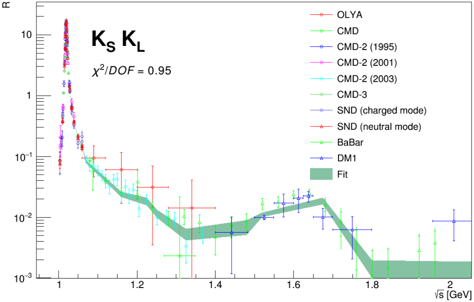


$$\Delta a_\mu(\mathrm{had, LO}) \times 10^{10} = 15.40 \pm 0.18_{\mathrm{exp}}$$

$K\bar{K}$ channels

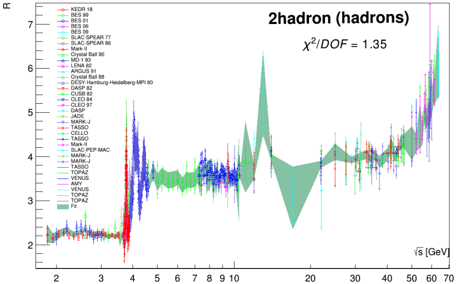


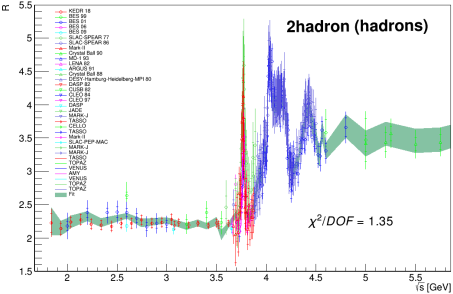
$\Delta a_\mu(\mathrm{had,LO}) \times 10^{10} = 23.21 \pm 0.19_{\mathrm{exp}}$



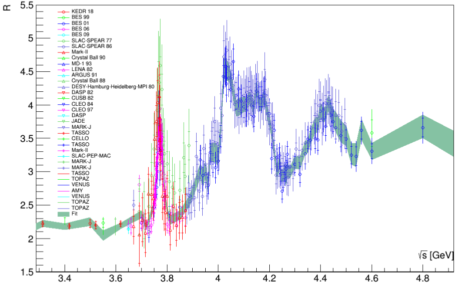
$\Delta a_\mu(\mathrm{had,LO}) \times 10^{10} = 13.19 \pm 0.13_{\mathrm{exp}}$

Inclusive measurements at $\sqrt{s} > 2$ GeV





uds continuum and $c\bar{c}$ threshold region ↑
(J/Ψ and $\Psi(2S)$ not shown)



← Region above $D\bar{D}$ threshold

- Inclusive data above Υ's are well described by pQCD $\Rightarrow$ the data are used (with the correction for γ^*–Z interference) for $a_\mu(\mathrm{had, LO})$ at $1.937 < \sqrt{s} < 11.2$ GeV:
  $\Delta a_\mu(\mathrm{had, LO}) \times 10^{10} = 43.51 \pm 0.72$
- Negligible $a_\mu(\mathrm{had, LO})$ uncertainty due to variation of the integration upper limit within $11.2 \div 40$ GeV range (the correction for γ^*–Z interference is taken into account).

Results

The table shows both propagated experimental uncertainties $(\mathrm{exp.}\, e^+e^-)$ and the systematic uncertainties of our procedure (syst.). The latter is dominated by the cross section parameterization (technically, $E_{\mathrm{c.m.}}$ clustering) uncertainty. Our estimate:

$$a_\mu(\mathrm{had,LO}) = (695.9 \pm 1.9_{\mathrm{exp.}e^+e^-} \pm 1.9_{\mathrm{syst.}}) \times 10^{-10}$$

is consistent with results obtained by the dispersion method by other authors before 2021, though we included 2021-2023 data. The *Muon $g-2$ Theory Initiative* group [Phys. Rept. 887 (2020) 1] quoted an average value of $(693.1 \pm 4.0_{\mathrm{tot}}) \times 10^{-10}$ obtained by merging the recent results [1-6].

We also have a good per final state agreement with [6].

With our $a_\mu(\mathrm{had,LO})$ estimate, the disagreement between the SM a_μ prediction and the experimental a_μ world average remains at 5σ level.

[1] M. Davier *et al.*, EPJ C 77 (12) (2017) 827

[2] A. Keshavarzi *et al.*, PR D 91 (11) (2018) 114025

[3] G. Colangelo *et al.*, JHEP 02 (2019) 006

[4] M. Hoferichter *et al.*, JHEP 08 (2019) 137

[5] M. Davier *et al.*, EPJ C 80 (3) (2020) 241; Erratum *ibid.* 410

[6] A. Keshavarzi *et al.*, PR D 101 (2020) 014029

Final state	$a_\mu(\mathrm{had,LO}) \times 10^{10}$ (exp. err.) (sys. err.)	$\sqrt{s}$ [GeV]	χ^2/dof
$\pi^+\pi^-(\gamma)$	505.279 (1.366) (1.538)	0.300 1.937	2.19
$\pi^+\pi^-\pi^0$	48.481 (0.967) (0.629)	0.660 1.937	1.79
$\pi^+\pi^-2\pi^0$	18.777 (0.431) (0.509)	0.850 1.937	1.94
$2\pi^+2\pi^-$	15.396 (0.181) (0.060)	0.6125 1.937	2.34
K^+K^-	23.211 (0.188) (0.072)	0.985 1.937	1.99
K_SK_L	13.188 (0.130) (0.000)	1.004 1.937	0.95
$\pi^0\gamma$	4.359 (0.093) (0.049)	0.600 1.380	1.70
$K_SK^\pm\pi^\mp$	1.814 (0.100) (0.000)	1.240 1.937	0.99
$2\pi^+2\pi^-\pi^0$	1.744 (0.139) (0.000)	1.0125 1.937	1.29
$2\pi^+2\pi^02\pi^-$	1.728 (0.198) (0.034)	1.3125 1.937	1.99
$2\pi^+2\pi^-3\pi^0$	0.099 (0.013) (0.002)	1.575 1.937	0.57
$3\pi^+3\pi^-$	0.240 (0.014) (0.000)	1.312 1.937	1.90
$3\pi^+3\pi^-\pi^0$	0.020 (0.004) (0.001)	1.600 1.937	0.65
$\eta\gamma$	0.691 (0.051) (0.000)	0.600 1.354	1.36
$\eta\pi^+\pi^-$	0.575 (0.019) (0.000)	1.150 1.937	1.18
$K^+K^-\pi^0$	0.202 (0.050) (0.000)	1.440 1.937	0.54
$K^+K^-\pi^0\pi^0$	0.100 (0.011) (0.000)	1.500 1.937	1.32
$K^+K^-\pi^+\pi^-$	0.799 (0.033) (0.000)	1.400 1.937	1.57
$K^+K^-\pi^+\pi^-\pi^0$	0.129 (0.024) (0.000)	1.612 1.937	1.63
$K_SK_L\eta$	0.119 (0.030) (0.000)	1.575 1.937	1.31
$K_SK_L\pi^0$	0.839 (0.114) (0.000)	1.425 1.937	1.50
$K_SK_L\pi^0\pi^0$	0.137 (0.043) (0.000)	1.350 1.937	-
$K_SK_L\pi^+\pi^-$	0.166 (0.028) (0.000)	1.425 1.937	-
$K_SK^\pm\pi^\mp\pi^0$	0.640 (0.044) (0.000)	1.510 1.937	1.08
$K_SK_S\pi^+\pi^-$	0.066 (0.007) (0.000)	1.630 1.937	1.37
$\omega\eta$	0.035 (0.002) (0.000)	1.340 1.937	0.85
$\omega < \pi^0\gamma > \pi^0$	0.894 (0.021) (0.000)	0.750 1.937	1.56
$\omega < \pi^+\pi^-\pi^0 > \pi^+\pi^-$	0.098 (0.005) (0.000)	1.150 1.937	1.10
$\omega\eta\pi^0$	0.055 (0.043) (0.000)	1.500 1.937	1.16
$\phi\eta$	0.067 (0.003) (0.000)	1.560 1.937	0.98
$\pi^+\pi^-2\pi^0\eta$	0.117 (0.019) (0.000)	1.625 1.937	0.85
$\pi^+\pi^-3\pi^0$	1.067 (0.112) (0.000)	1.125 1.937	0.68
$\pi^+\pi^-\pi^0\eta$	0.663 (0.075) (0.000)	1.394 1.937	0.82
$p\bar{p}$	0.030 (0.001) (0.000)	1.889 1.937	1.24
$n\bar{n}$	0.028 (0.006) (0.000)	1.890 1.937	1.24
2hadron(hadrons)	43.509 (0.722) (0.661)	1.937 11.199	1.35
$\pi^0\gamma, \pi^+\pi^-$ (ChPT)	0.538 (0.003)	0.140 0.300	
pQCD	2.065 (0.000)	>11.199	
J/Ψ	6.343 (0.130)	3.096916	
$\Psi(2S)$	1.483 (0.040)	3.686093	
$\Upsilon(1S)$	0.055 (0.001)	9.460300	
$\Upsilon(2S)$	0.018 (0.001)	10.023260	
$\Upsilon(3S)$	0.014 (0.001)	10.355200	
$\Upsilon(4S)$	0.007 (0.001)	10.580000	
Total	**695.886 (1.928) (1.913)**		

Open issues

Experimental inputs:

- Controversy between experiments:
  - CMD-3 (2023) $\pi^+\pi^-$ cross section is $\sim 5\%$ ($\sim 4\sigma$) higher than the others. Waiting for their final $\pi\pi$ results. Is there an excess in other final states?
  - KLOE vs BaBar tension in $\pi^+\pi^-$.
    More data to arrive: BaBar, Belle ...
- Missing channels?
  - All-neutral final states in inclusive measurements?
  - Unexpected states around π and $\pi\pi$ thresholds?
  - Missing $0^+\gamma$ contributions at $s \sim 1$ GeV?

Our procedure:

- Systematics associated with the unfolding of radiative corrections already applied by experimentalists in their publications.
- Building a non-biased global covariance matrix?
- Cross section parameterization for the fit.
- *Are we missing trivial things known to* $a_\mu(\mathrm{had}, \mathrm{LO})$ *experts?*

Dispersive estimates vs (lattice) QCD and space-like data:

- Lattice QCD gives systematically higher $a_\mu(\mathrm{had}, \mathrm{LO})$ than dispersion method.
- Photon VP from low-t lepton-lepton scattering?

...???

Summary

- Using an up-to-date as of September 2023 compilation of world data on $\sigma_{\rm tot}(e^+e^- \to {\rm hadrons})$ we independently estimated the leading order hadronic contribution to the muon anomalous magnetic moment.
- Our estimate $a_\mu({\rm had, LO}) = (695.9 \pm 1.9_{{\rm exp.}e^+e^-} \pm 1.9_{\rm syst.}) \times 10^{-10}$ is consistent with the value $(693.1 \pm 4.0_{\rm tot}) \times 10^{-10}$ obtained in 2020 by the *Muon $g-2$ Theory Initiative* group by averaging several state-of-the-art dispersion estimates [Phys. Rept. 887 (2020) 1].
- The difference can be attributed, particularly, to inclusion of the new CMD-3 $\pi^+\pi^-$ cross section data lying systematically higher than measured by other experiments.
- The SM prediction of a_μ including our $a_\mu({\rm had, LO})$ estimate remains in $\simeq 5\sigma$ tension with the experimental a_μ value published in 2023 by the *Muon g-2 Collaboration* [arXiv:2308.06230].
- CMD-3 puzzle? New $\sigma(e^+e^- \to hadrons)$ measurements expected: SND, BaBar, KLOE ,BES III, Belle II . . .

Backup

IHEP PPDS CS total cross section database

- Originates from the PPDS CrossSection database maintained at IHEP (Protvino) since 1980s.
- Implemented from scratch for Unix in 2017-2020 (no code from the old BDMS based version).
- Covers total cross section measurements published since 1947. Contains 22146 data records, each comprising cross section measurements for a single reaction published in a single paper (i.e. one paper may be split into several records).
- The data are encoded in a language with a strict grammar (an automatic protection against meaningless content and input mistakes).
- Flexible query language (not SQL).
- Web-based command line interface http://hera.ihep.su:4200/cs/ with basic plotting.
- Coverage of world data is fragmentary since 1990s, still PPDS CS is actively used to maintain our compilations of $e^+e^- \to hadrons$ total cross sections and total (inelastic) cross sections with hadron-hadron beams (cf. the reviews on total cross sections in the Review of Particle Physics before 2023).

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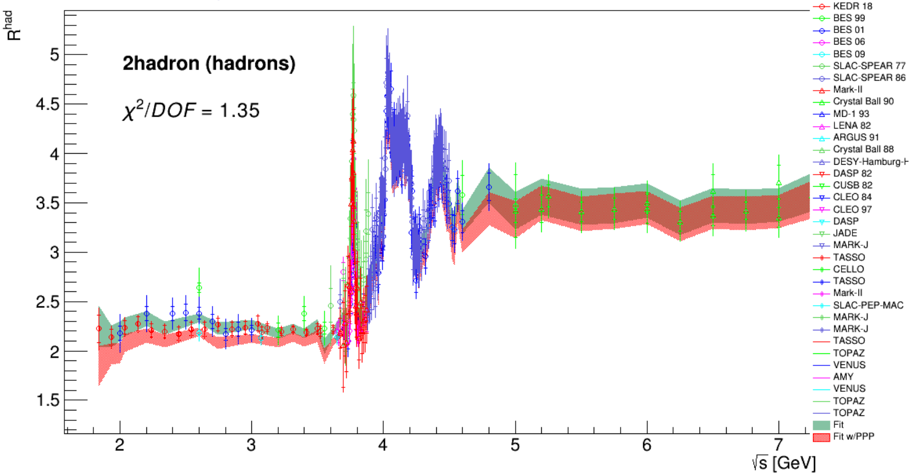
PPP bias: pathological examples



A naive construction of the systematic part of the covariance matrix using inputs (*biased a priori*) from individual experiments leads to PPP bias while fitting correlated data by the least squares method. Generally speaking, the fit can be systematically lower than *any* of the individual measurements, see the example above. [**Yes:** the red curve is the global χ^2 minimum with $\chi^2/\mathrm{dof} = 1.25$]

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