Fully heavy exotic hadrons

A.V. Nefediev

Long-term fruitful collaboration with my colleagues and co-authors is gratefully acknowledged!

- X. K. Dong, V. Baru, F. K. Guo, C. Hanhart, A. Nefediev,
  "Coupled-Channel Interpretation of the LHCb Double-J/ψ Spectrum and Hints of a New State Near the $J/\psi J/\psi$ Threshold,"
  Phys. Rev. Lett. **126**, 132001 (2021)
- X. K. Dong, V. Baru, F. K. Guo, C. Hanhart, A. Nefediev, B. S. Zou,
  "Is the existence of a $J/\psi J/\psi$ bound state plausible?,"
  Sci. Bull. **66**, 2462 (2021)
- X. K. Dong, F. K. Guo, A. Nefediev, J. T. Castellà,
  "Chromopolarizabilities of fully-heavy baryons,"
  Phys. Rev. D **107**, 034020 (2023)
- A. V. Nefediev,
  "$X(6200)$ as a compact tetraquark in the QCD string model",
  Eur. Phys. J. C **81** 692 (2021)

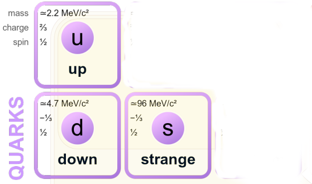
Ordinary hadrons

Quark model: The structure of hadrons

1964 — Quark model by Gell-Mann & Zweig $\Longrightarrow$ $SU(3)$ multiplets

"Ordinary" hadrons*:

- Meson consists of quark and antiquark
- Baryon consists of 3 quarks



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* Compact "exotic" hadrons anticipated

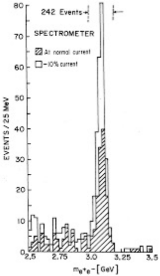
All hadrons understood $\Longrightarrow$ No "exotic" states

Prediction of the fourth quark:

- Glashow & Bjorken (1964)
- Glashow, Iliopoulos & Maiani (1970)

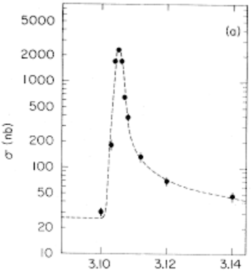
November revolution 1974: Discovery of charm

BNL ($p + Be \to e^+e^- X$)



$m_J = 3.1$ GeV
$\Gamma_J \approx 0$

SLAC ($e^+e^- \to$ hadrons)

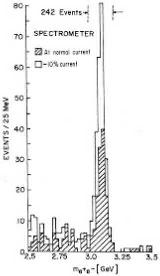


$m_\psi = 3.105 \pm 0.003$ GeV
$\Gamma_\psi \leqslant 1$ MeV

Narrow resonance J/ψ with mass around 3.1 GeV

November revolution 1974: Discovery of charm

BNL ($p + Be \to e^+e^- X$)



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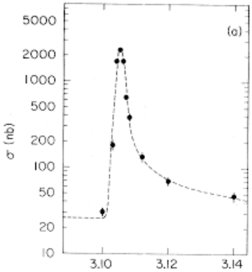


$m_\psi = 3.105 \pm 0.003$ GeV
$\Gamma_\psi \leqslant 1$ MeV

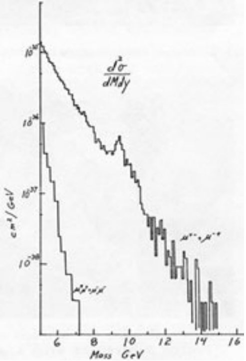
Narrow resonance J/ψ with mass around 3.1 GeV

5 years later $\Longrightarrow$ 10 charmonia states!

Bottomonia

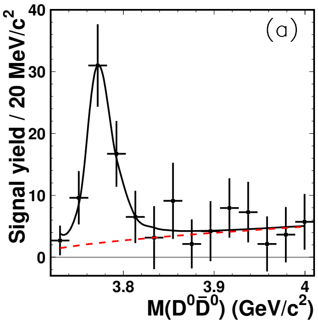
- 1977 — L.Lederman (Fermilab): discovery $\Upsilon(1S)$ with mass 9.54 GeV

$$p + (Cu, Pt) \to \mu^+ + \mu^- + \text{anything}$$



- 1978 — DESY (Germany): discovery of $\Upsilon(2S)$
- 1980 — CESR (USA): discovery of $\Upsilon(3S)$ and $\Upsilon(4S)$

Breit-Wigner parametrisation: Mass, Width, Poles



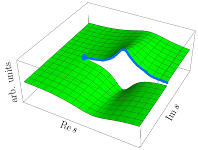
$$\mathcal{A} = \mathcal{A}_{\rm bg} + \mathcal{A}_{\rm BW}$$

$$\mathcal{A}_{\rm BW} \propto \frac{1}{M^2 - {M_0}^2 + iM\Gamma_0}$$

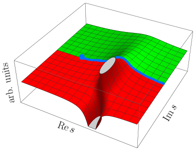
$$M_0 > M_{H_1} + M_{H_2} \quad \Gamma_0 = \Gamma(R \to H_1 H_2)$$

$$s = M^2 = \left(\sqrt{p^2 + M_{H_1}^2} + \sqrt{p^2 + M_{H_2}^2}\right)^2$$

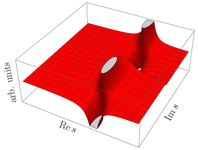
Pole positions: $\begin{cases} M_{\rm pole} \approx M_0 - \frac{i}{2}\Gamma_0 \\ M^*_{\rm pole} \approx M_0 + \frac{i}{2}\Gamma_0 \end{cases}$



first Riemann sheet

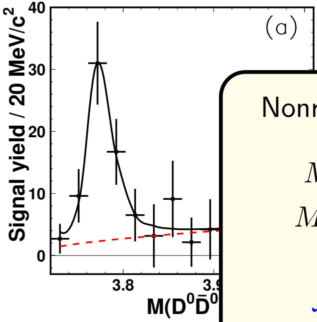


transition from first to second Riemann sheet



second Riemann sheet

Breit-Wigner parametrisation: Mass, Width, Poles



$$\mathcal{A} = \mathcal{A}_{\rm bg} + \mathcal{A}_{\rm BW}$$

Nonrelativistic expansion:

$$M = M_{H_1} + M_{H_2} + E$$
$$M_0 = M_{H_1} + M_{H_2} + E_0$$

$$\mathcal{A}^{\rm nr}_{\rm BW} \propto \frac{1}{E - E_0 + \frac{i}{2}\Gamma_0}$$

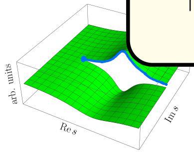
$$|\Psi|^2 \sim \left| e^{-iE_0 t - \frac{1}{2}\Gamma_0 t} \right|^2 \sim e^{-\Gamma_0 t}$$

$= \Gamma(R \to H_1 H_2)$

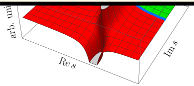
$\sqrt{p^2 + M_{H_2}^2}\Big)^2$

$\approx M_0 - \frac{i}{2}\Gamma_0$

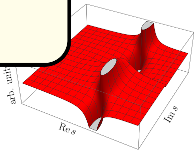
$\approx M_0 + \frac{i}{2}\Gamma_0$



first Riemann sheet

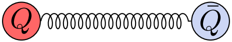


transition from first to second Riemann sheet



second Riemann sheet

Quark model: Adding dynamics



$$\boxed{\hat{H}_0\psi = E_0\psi}$$

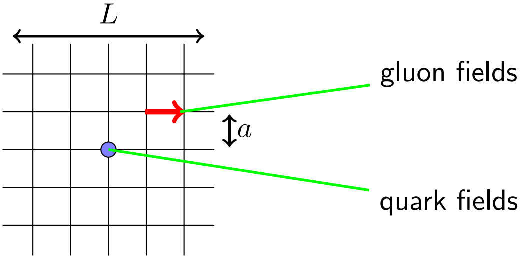
$$\hat{H}_0 = \frac{p^2}{m_Q} + V_0(r) + V_{\mathrm{SD}}(r)$$

$$V_0(r) = \sigma r - \frac{\frac{4}{3}\alpha_s}{r}$$ (Cornell potential)

$$V_{SD}(r) = \underbrace{V_{LS}(r)(\boldsymbol{L}\cdot(\boldsymbol{S}_Q + \boldsymbol{S}_{\bar{Q}}))}_{\text{fine structure}} + \underbrace{V_{SS}(r)(\boldsymbol{S}_Q\cdot\boldsymbol{S}_{\bar{Q}})}_{\text{hyperfine structure}}$$

$$+\underbrace{V_{ST}(r)\Big((\boldsymbol{S}_Q\cdot\boldsymbol{S}_{\bar{Q}}) - 3(\boldsymbol{S}_Q\cdot\boldsymbol{n})(\boldsymbol{S}_{\bar{Q}}\cdot\boldsymbol{n})\Big)}_{\text{spin-tensor force}} \quad \propto \quad \frac{1}{m_Q^2}$$

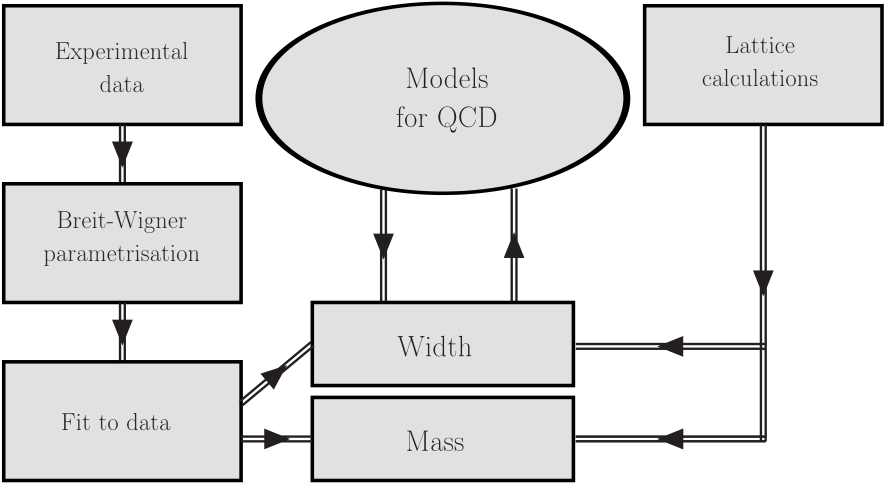
Lattice simulations



$$C_{ij}(t) = \langle 0|O_i(t)O_j(0)|0\rangle = \sum_n \frac{e^{-E_n t}}{2E_n}\langle 0|O_i(0)|n\rangle\langle n|O_j^\dagger(0)|0\rangle$$

- Continuum limit $\Longrightarrow$ $a \to 0$
- Infinite box $\Longrightarrow$ $L \to \infty$
- Unphysical light quark mass $\Longrightarrow$ Chiral extrapolation

Approach to ordinary states



Exotic states with heavy quarks

“Exotic animal is more unusual and rare than normal domesticated pets like cats or dogs“

Revolution of 2003: Enfant terrible $X(3872)$

- $I = 0$, $J^{PC} = 1^{++}$, contains $c\bar{c}$
- Too light compared with Quark Model prediction

$$M^{\rm QM}_{\chi_{c1}(2P)} - M^{\rm exp}_X \sim 100 \text{ MeV}$$

- Strongly attracted to $D\bar{D}^*$ threshold

$$M^{\rm exp}_X - (M_{D^0} + M_{\bar{D}^{*0}}) \;\sim\; 0$$

- Large ($\sim 40\%$) probability of the decay into $D\bar{D}^*$
- Strong isospin violation

$$Br(X \to \pi^+\pi^-\pi^0 J/\psi) \approx Br(X \to \pi^+\pi^- J/\psi)$$

Revolution of 2003: Enfant terrible $X(3872)$

- $I = 0$, $J^{PC} = 1^{++}$, contains $c\bar{c}$

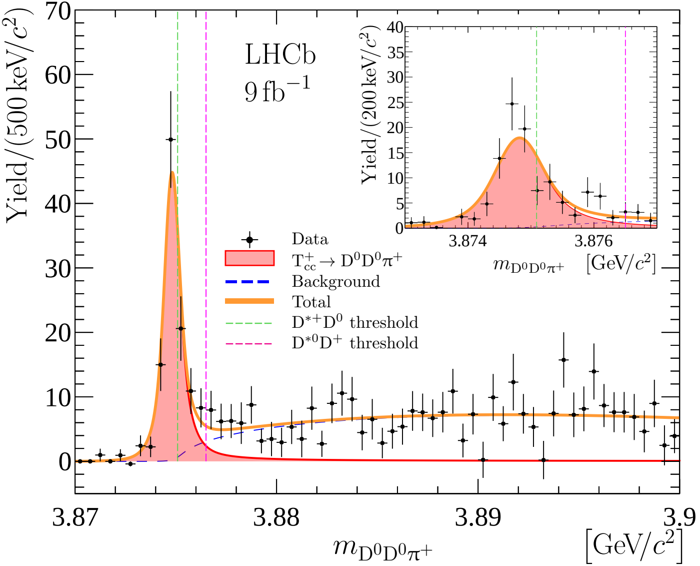
> - $\sim$2500 citations (the most cited paper by Belle)
> - $J^{PC} = 1^{++}$ unambiguously established by LHCb in 2013
> - Nature of $X(3872)$ still under debate
> - New name by PDG — $\chi_{c1}(3872)$

$$M_X - (M_{D^0} + M_{\bar{D}^{*0}}) \sim 0$$

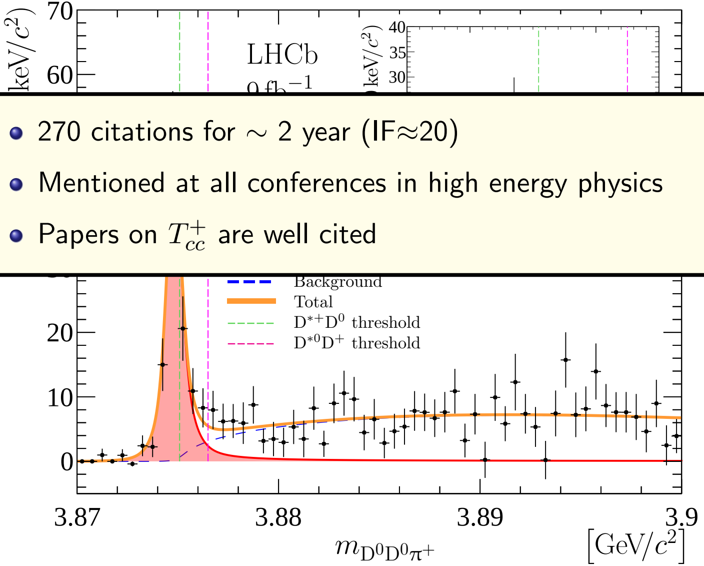
- Large ($\sim 40\%$) probability of the decay into $D\bar{D}^*$
- Strong isospin violation

$$Br(X \to \pi^+\pi^-\pi^0 J/\psi) \approx Br(X \to \pi^+\pi^- J/\psi)$$

$T_{cc}^{+}(cc\bar{u}\bar{d})$ @ LHCb (Nature Phys. 18 (2022) 7, 751)

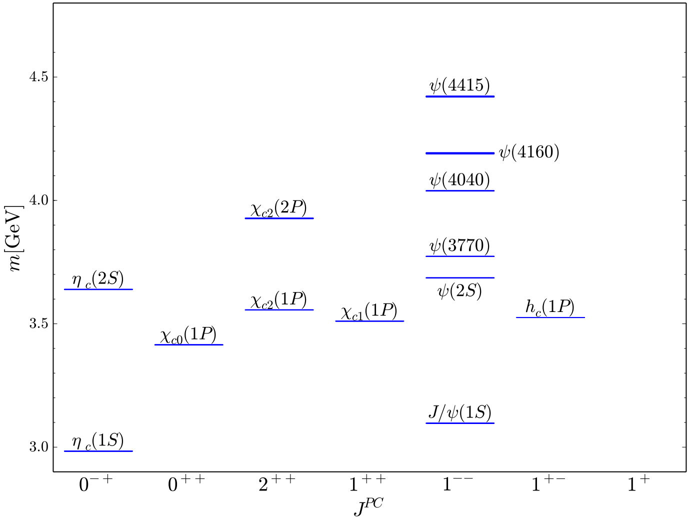


$T_{cc}^{+}(cc\bar{u}\bar{d})$ @ LHCb (Nature Phys. 18 (2022) 7, 751)

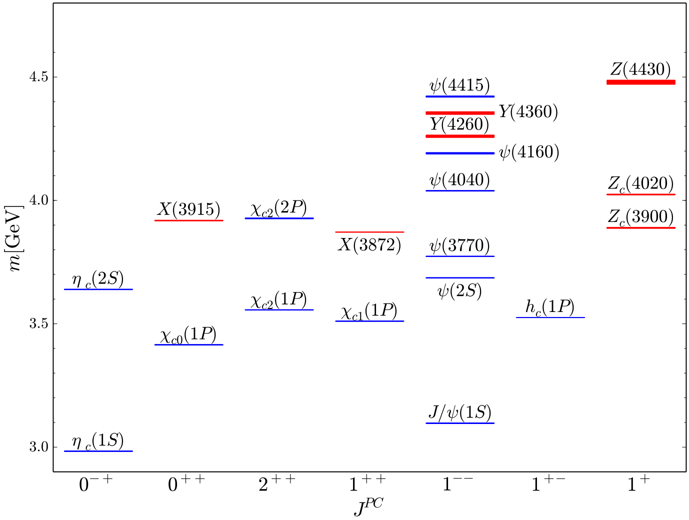


- 270 citations for $\sim$ 2 year (IF$\approx$20)
- Mentioned at all conferences in high energy physics
- Papers on T_{cc}^{+} are well cited

Spectrum of charmonium



Spectrum of charmonium



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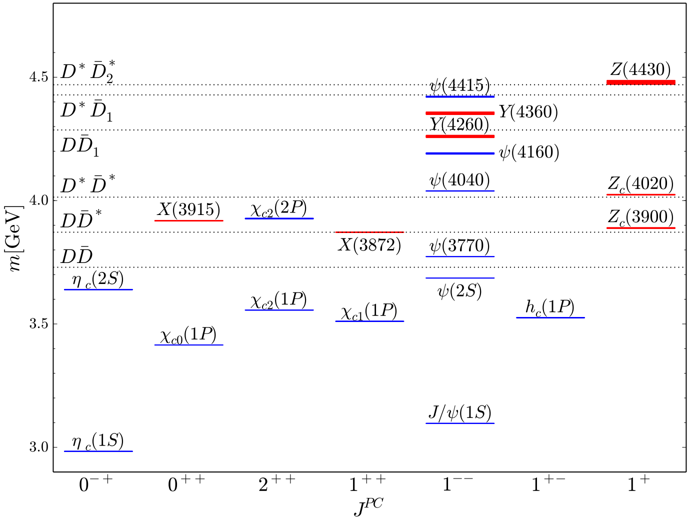

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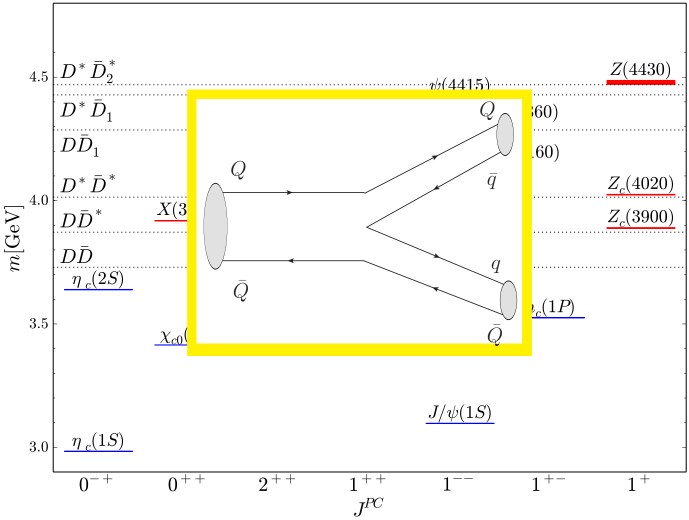
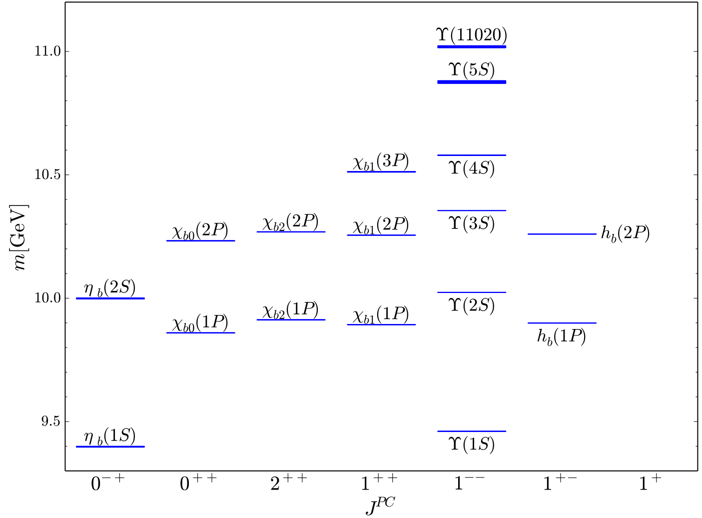
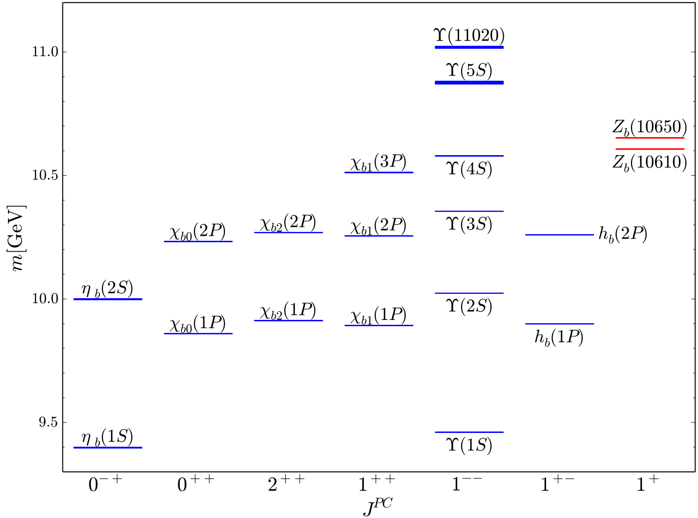


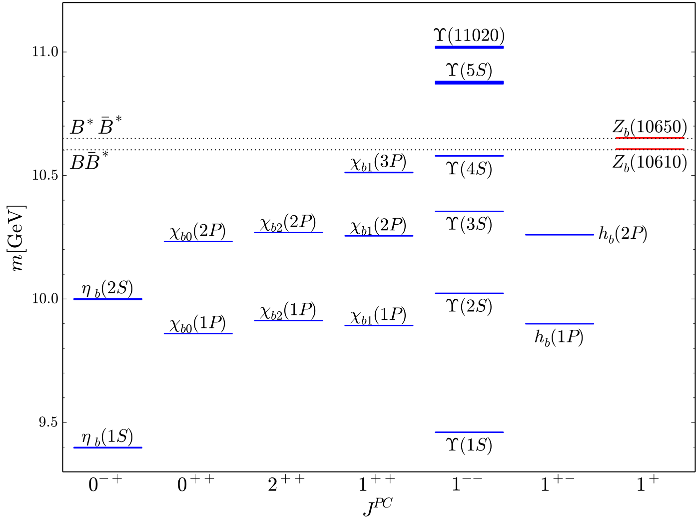
Spectrum of bottomonium



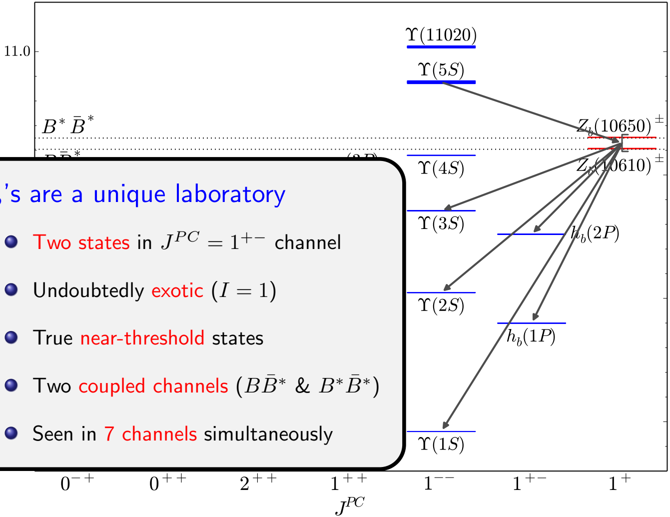
Spectrum of bottomonium



Spectrum of bottomonium



Spectrum of bottomonium

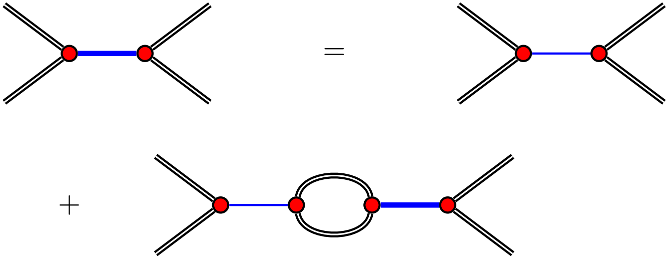


Z_b's are a unique laboratory

- Two states in $J^{PC} = 1^{+-}$ channel
- Undoubtedly exotic $(I = 1)$
- True near-threshold states
- Two coupled channels $(B\bar{B}^*$ & $B^*\bar{B}^*)$
- Seen in 7 channels simultaneously

Effect of hadronic loops

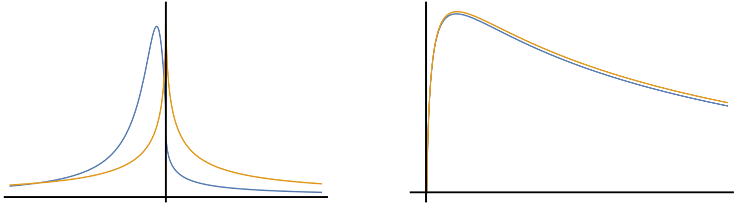
$$|\Psi\rangle = \begin{pmatrix} \sqrt{Z}|\psi_0\rangle \\ \chi(\boldsymbol{k})|H_1 H_2\rangle_{L=0} \end{pmatrix}$$



$$\frac{1}{E-E_0+\frac{i}{2}\Gamma_0} \quad \Longrightarrow \quad \frac{1}{E-E_f+\frac{i}{2}(gk+\Gamma_0)} \qquad k=\sqrt{2\mu E}$$

Examples of line shapes

$$\frac{\Gamma_0}{\left|E - E_f + \frac{i}{2}(gk + \Gamma_0)\right|^2} \qquad \frac{gk}{\left|E - E_f + \frac{i}{2}(gk + \Gamma_0)\right|^2}$$

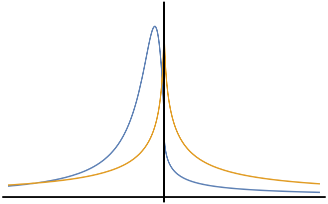


- Bound state ($E_f < 0$) — blue curve
- Virtual state ($E_f > 0$) — yellow curve

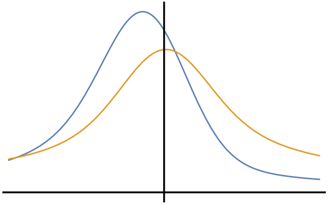
Pole resides on real axis below threshold on RS-I or RS-II

Effect of experimental resolution

$$\frac{\Gamma_0}{\left|E-E_f+\frac{i}{2}(gk+\Gamma_0)\right|^2}$$



$$\int \frac{\Gamma_0 f_{\rm res}(E'-E)dE'}{\left|E'-E_f+\frac{i}{2}(gk+\Gamma_0)\right|^2}$$



- Left plot — before convolution with resolution
- Right plot — after convolution with resolution

Sharp structures turn to broad humps

Interplay of different dynamics

$$|\Psi\rangle = \begin{pmatrix} \sqrt{Z}|\psi_0\rangle \\ \chi(\boldsymbol{k})|H_1H_2\rangle \end{pmatrix} \qquad H = \begin{pmatrix} E_0 & f(k) \\ f(k) & \frac{k^2}{2\mu} + V_{H_1H_2} \end{pmatrix}$$

Scattering amplitude $H_1H_2 \to H_1H_2$ via $V_{H_1H_2}$

$$f_V(E) = \frac{1}{-\gamma_V - ik} \qquad k = \sqrt{2\mu E}$$

Full scattering amplitude $H_1H_2 \to H_1H_2$

$$f(E) = \frac{1}{E - E_f + \frac{i}{2}gk - \frac{(E-E_f)^2}{E - E_C}} \quad \text{with} \quad E_C = E_f - \frac{1}{2}g\gamma_V$$

- $|E_C| \gg |E_f|$

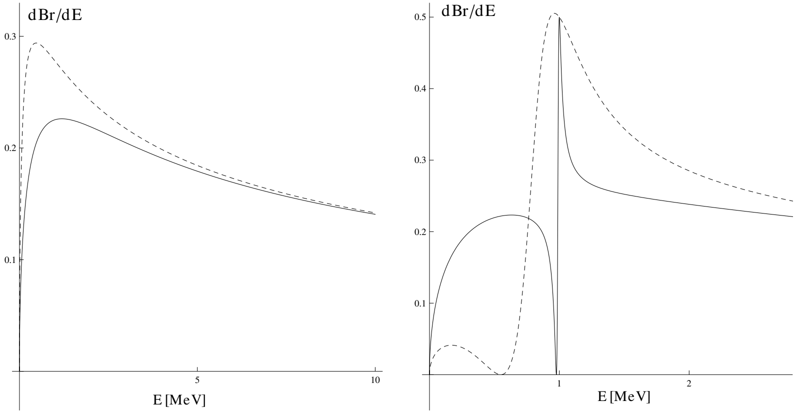
$$f(E) \approx \frac{1}{E - E_f + \frac{i}{2}gk}$$

- $|E_C| \sim |E_f|$

$$f(E) \propto (E - E_C)$$

Interplay of different dynamics

$|E_C| \gg |E_f|$ $\qquad$ $|E_C| \sim |E_f|$



Composite or elementary?

Effective range expansion: $-a^{-1}+\frac{1}{2}rk^2-ik$

$$a=\frac{2(1-Z)}{(2-Z)}\frac{1}{\sqrt{2\mu E_B}}+O\left(\frac{1}{\beta}\right) \qquad r=-\frac{Z}{(1-Z)}\frac{1}{\sqrt{2\mu E_B}}+O\left(\frac{1}{\beta}\right)$$

β $(\gg k)$ — (inverse) range of force

(Weinberg'1960s)

$$\bar{X}_A=1-Z=\frac{1}{\sqrt{1-2r/a}}$$

Elementary (confined) state

- Two near-threshold poles

Composite (molecular) state

- One near-threshold pole

$\Longrightarrow$ pole counting rules (Morgan'1992)

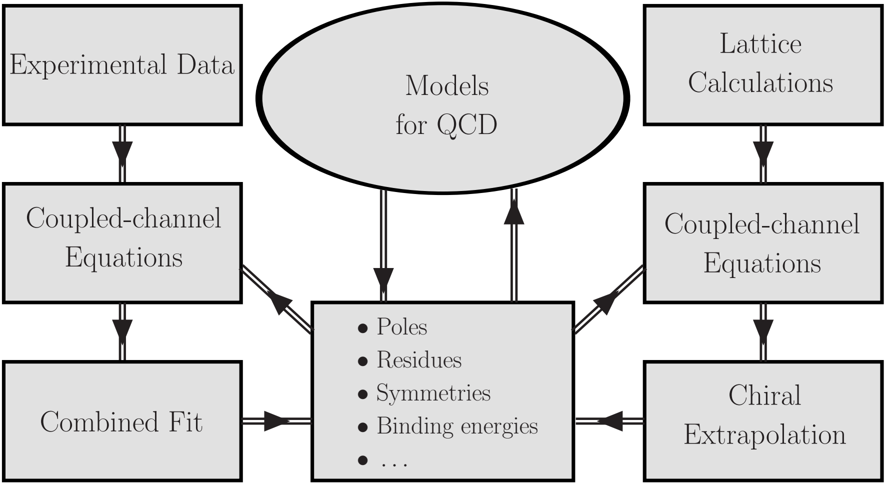
Heavy-quark spin symmetry

- Exotic states contain heavy quarks (HQ)
- In the limit $m_Q \to \infty$ $(m_Q \gg \Lambda_{\rm QCD})$ spin of HQ decouples

  $\Longrightarrow$ Heavy Quark Spin Symmetry (HQSS)
- For realistic m_Q's HQSS is approximate but accurate symmetry of QCD
- HQSS = tool to relate properties of states with different HQ spin orientation

  $\Longrightarrow$ Spin partners

Approach to exotic states



Exotic hadrons with light quarks

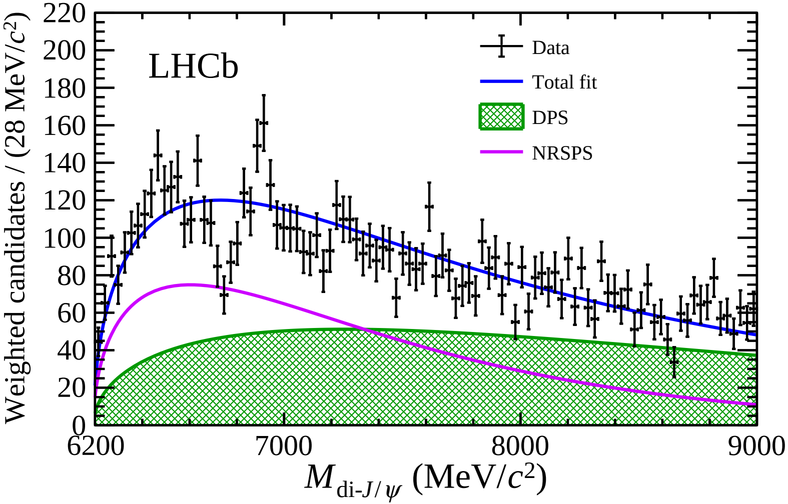
Known exotic hadrons with light quarks:

- Deuteron (bound state of proton & neutron)
- $X(3872)$ (contains large $D\bar{D}^*$ component)
- T_{cc}^+ (DD^* molecule)
- $Z_c(3900)$ and $Z_c(4020)$ ($D^{(*)}\bar{D}^*$ molecules)
- $Z_{cs}(3982)$ ($D^*\bar{D}_s/D\bar{D}_s^*$ molecule)
- $Z_b(10610)$ and $Z_b(10650)$ ($B^{(*)}\bar{B}^*$ molecules)
- ...

How about fully heavy exotic hadrons?

- $M_{\text{bottom}} \gtrsim 18$ GeV
  - $\Longrightarrow$ too heavy for Belle II
  - $\Longrightarrow$ too many b's for LHCb
- $M_{\text{charm}} \gtrsim 6$ GeV $\Longleftarrow$ should be feasible for LHC and KEKB

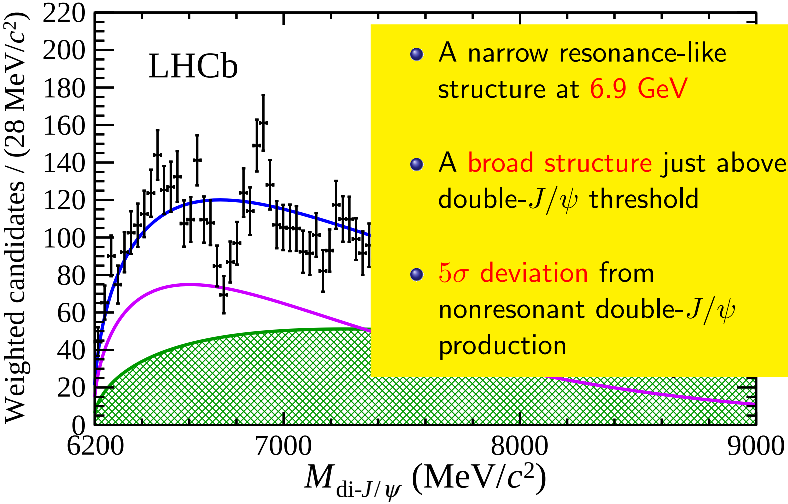
Double-J/ψ production @ LHCb (Sc.Bull. 65 (2020) 1983)



NRSPS=NonResonant Single Parton Scattering

DPS=Double Parton Scattering

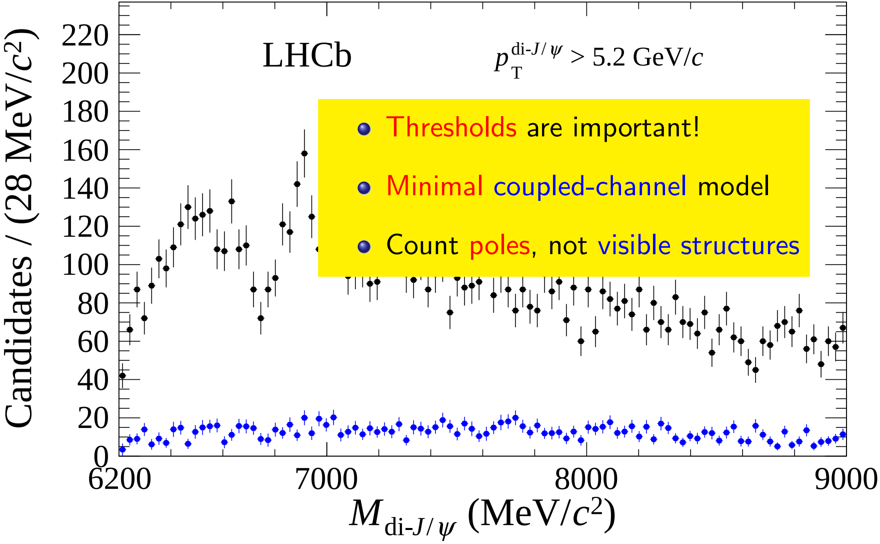
Double-J/ψ production @ LHCb (Sc.Bull. 65 (2020) 1983)



- A narrow resonance-like structure at 6.9 GeV
- A broad structure just above double-J/ψ threshold
- 5σ deviation from nonresonant double-J/ψ production

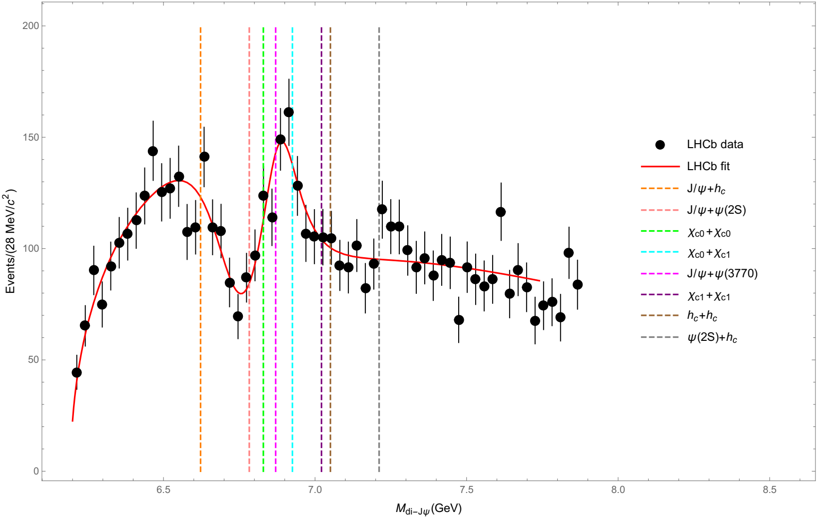
NRSPS=NonResonant Single Parton Scattering
DPS=Double Parton Scattering

Double-J/ψ production: Theoretical data analysis

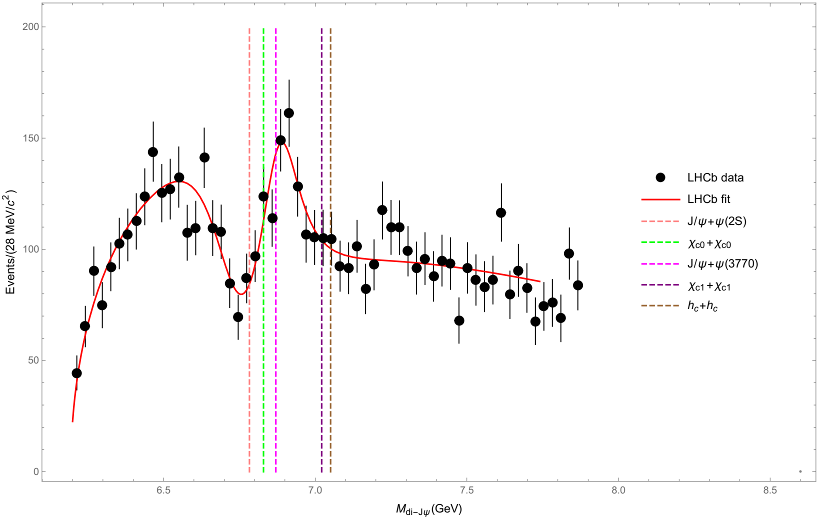


- Thresholds are important!
- Minimal coupled-channel model
- Count poles, not visible structures

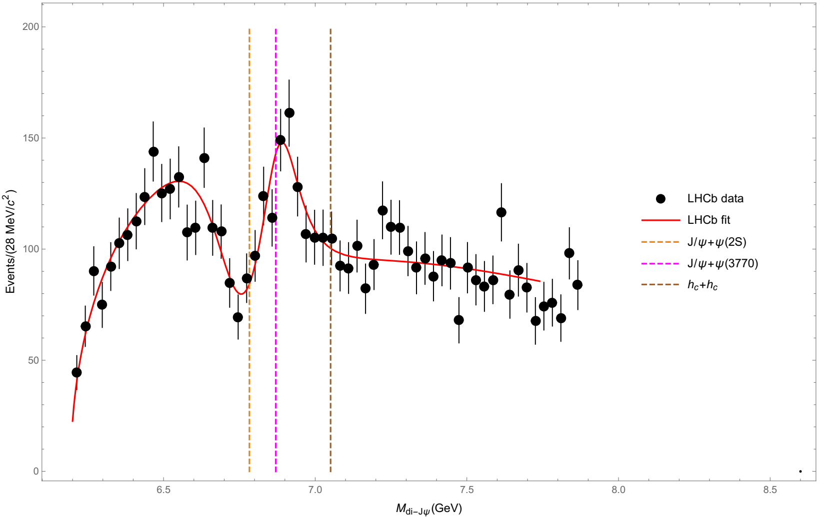
All channels



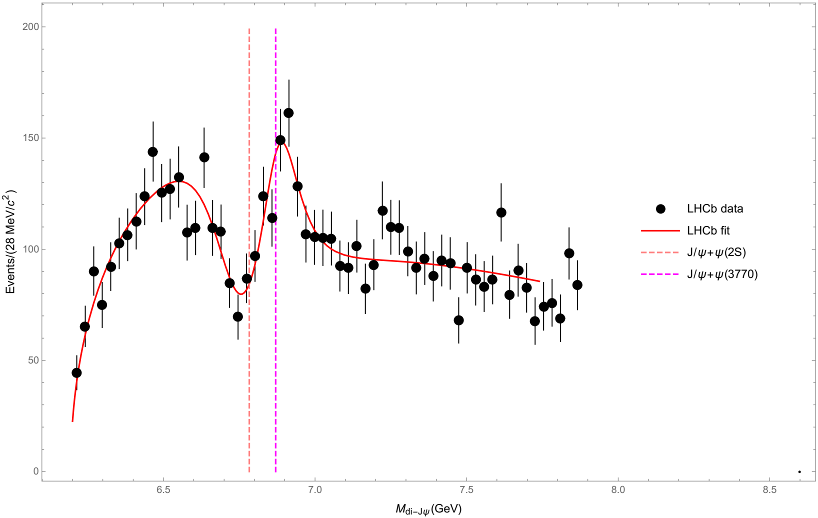
Only S-wave channels (no $J/\psi h_c$, $\psi(2S)h_c$, $\chi_{c0}\chi_{c1}$)



No heavy exchanges (no $\chi_{c0}\chi_{c0}$, $\chi_{c1}\chi_{c1}$)



Only HQSS-allowed channels (no $h_c h_c$)



The models

Two-channel model (7 parameters)
$J/\psi J/\psi$ & $\psi(2S)J/\psi$

Three-channel model (8 parameters)
$J/\psi J/\psi,\ \psi(2S)J/\psi$ & $\psi(3770)J/\psi$

$$V_{2\text{ch}}(E) = \begin{pmatrix} a_1 + b_1 k_1^2 & c \\ c & a_2 + b_2 k_2^2 \end{pmatrix}$$

$$V_{3\text{ch}}(E) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$$

Lippmann-Schwinger equation

$$T(E) = V(E) \cdot [1 - G(E)V(E)]^{-1}$$

Production amplitude in $J/\psi J/\psi$ channel (channel 1):

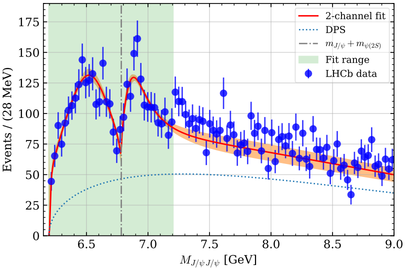
$$\mathcal{M}_1 = \alpha e^{-\beta E^2}\left[b + G_1(E)T_{11}(E) + G_2(E)T_{21}(E) + r_3 G_3(E)T_{31}(E)\right]$$

Slope β fixed to double-parton scattering (DPS): $\beta = 0.0123$ GeV^{-2}

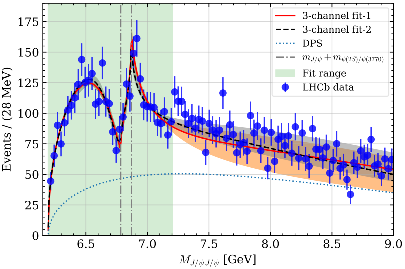
$$r_3 = \begin{cases} 0 & \text{2ch model} \\ 1 & \text{3ch model} \end{cases}$$

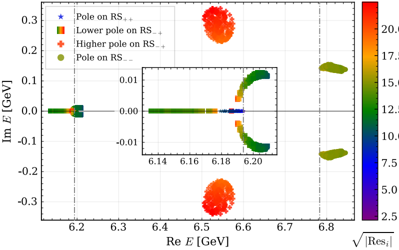
Fits & poles

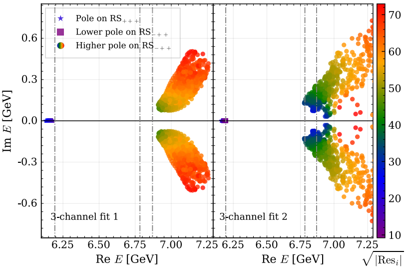
$J/\psi J/\psi$ & $J/\psi\psi(2S)$ (2+5 params)



$J/\psi J/\psi$, $J/\psi\psi(2S)$ & $J/\psi\psi(3770)$ (2+6 params)

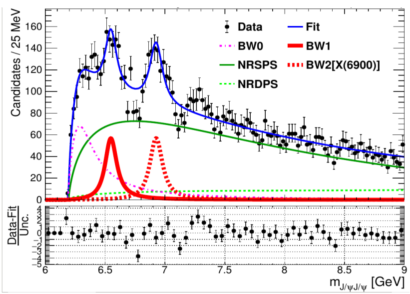


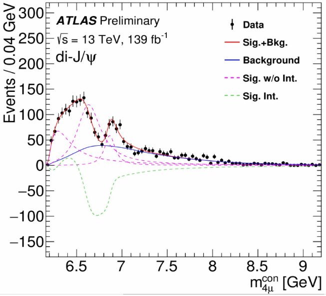




Further tests

- Analyse CMS and ATLAS data in the double-J/ψ channel





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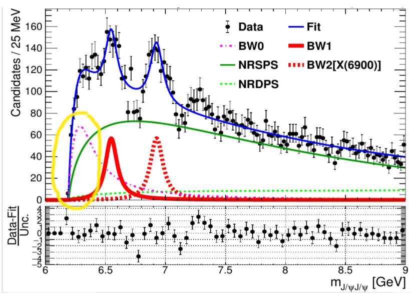
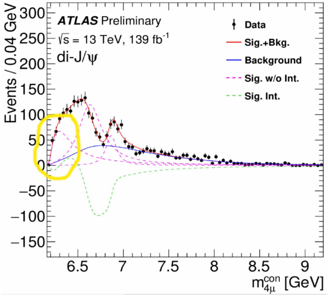
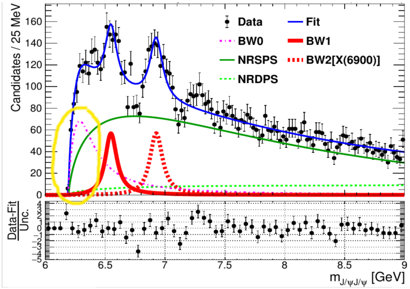
- Analyse CMS and ATLAS data in the double-J/ψ channel

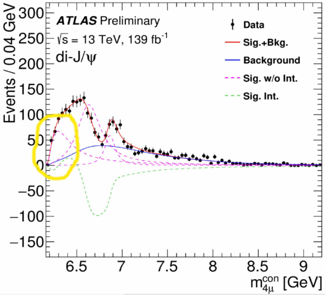




Further tests

- Analyse CMS and ATLAS data in the double-J/ψ channel

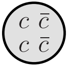




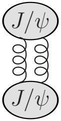
- Analyse data in the complimentary $\psi(2S)J/\psi$ channel
- Combined analysis of all data sets

$X(6200)$ vs $X(6900)$

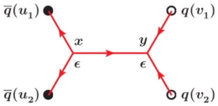
- Poles above the double-J/ψ threshold ($X(6900)$) are badly determined
    - Parameters of $X(6900)$ are uncertain
    - Combined analysis LHCb+CMS+ATLAS ?
- Pole $X(6200)$ near the double-J/ψ threshold is robust
    - Both models predict compositeness of $X(6200)$ $\bar{X}_A = 1 - Z \simeq 1$
    - Yet admixture of compact state (small but finite Z) not excluded
- $X(6200)$ = fully charmed tetraquark
    - Compact Tetraquark



- Hadronic Molecule



Comment of quark clustering tetraquarks



LUCHA, MELIKHOV, and SAZDJIAN PHYS. REV. D **100,** 094017 (2019)

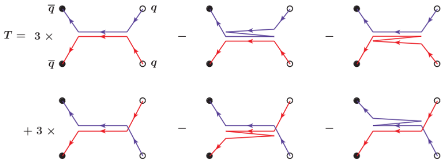


FIG. 5. Decomposition of the tetraquark operator into a combination of products of mesonic operators.

CLUSTER REDUCIBILITY OF MULTIQUARK OPERATORS PHYS. REV. D **100,** 094017 (2019)

Recent review articles on exotic states can be found in Refs. [36,58–63].

V. SUMMARY

The cluster reducibility property of multiquark operators provides a general proof of the nonexistence of completely confined or compact multiquark states. Rather than eliminating the multiquark scheme from the description of multiquark states, taking into account analyses based on lattice and numerical calculations, it streamlines the role played by the various participating operators, according to a qualitative partitioning of configuration space. Existing multiquark states, whether bound states or resonances, would be schematically composed of two layers: an inner core, having a structure governed by a connected string-junction-type interaction, and an outer shell, having a hadronic molecular-type structure. The weight of each layer depends on the masses and flavors of the quarks and on the sectors of quantum numbers that are considered. This unified scheme might provide a better understanding of the structure of multiquark states.

ACKNOWLEDGMENTS

We thank Hans Günter Dosch for enlightening discussions. D. M. acknowledges support from the Austrian Science Fund (FWF), Grant No. P29028. D. M. and H. S. are grateful for support under joint CNRS/RFBR Grant No. PRC Russia/19-52-15022. The figures were drawn with the aid of the package Axodraw2 [64].

$X(6200)$ as compact $\bar{c}\bar{c}cc$ tetraquark

Progress of Theoretical Physics, Vol. 54, No. 2, August 1975

A Possible Model for New Resonances

——*Exotics and Hidden Charm*——

Yoichi IWASAKI

Research Institute for Fundamental Physics
Kyoto University, Kyoto

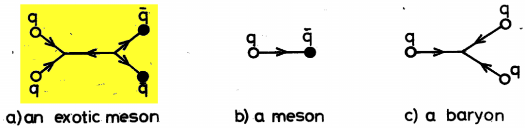


Fig. 1. An exotic state, a meson and a baryon.

Quark model calculations

[1]	[2]	[3]	[4]
6.2 GeV	6.1915 GeV	6.190 GeV	6.196 GeV

[1] Y. Iwasaki, Prog. Theor. Phys. **54** (1975), 492

> We expect at least three exotic mesons with hidden charm, $c\bar{c}(p\bar{p}-n\bar{n})$ [between 3.7∼4.1 GeV], $c\bar{c}\lambda\bar{\lambda}$ [∼4.1 GeV] and $c\bar{c}c\bar{c}$ [∼6.2 GeV], to which we refer as ψ_ρ, ψ_φ and ψ_ψ, respectively. [We may refer to ψ' as ψ_ω.] We assume that the

[2] M. Karliner, S. Nussinov, J. L. Rosner, Phys. Rev. D **95** (2017), 034011

$$M = 2S + 2M_{cc} + B(cc)(\bar{c}\bar{c}) + \Delta M_{\rm HF} = [2(165.1) + 2(3204.1) - 388.3 - 158.5]\ \text{MeV} = 6191.5\ \text{MeV}$$

[3] R. Faustov, V. Galkin, E. Savchenko, Phys. Rev. D **102** (2020), 114030

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\boldsymbol{p}^2}{2\mu_R}\right)\Psi_M(\boldsymbol{p}) = \int \frac{d^3q}{(2\pi)^3} V(\boldsymbol{p},\boldsymbol{q};M)\,\Psi_M(\boldsymbol{q}) \qquad b^2(M) = \frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}$$

[4] A. N., Eur. Phys. J. C **81**, 692 (2021)

QCD string model

Conclusion: Existence of a compact-state pole near the double-J/ψ threshold is supported by model calculations

QCD string model

$$\mathcal{L}(\tau) = -m\sqrt{\dot{x}_1^2} - m\sqrt{\dot{x}_2^2} - \sigma\int_0^1 d\beta\sqrt{(\dot{w}w')^2 - \dot{w}^2 w'^2}$$

$$w_\mu(\tau,\beta) = \beta x_{1\mu} + (1-\beta)x_{2\mu}$$

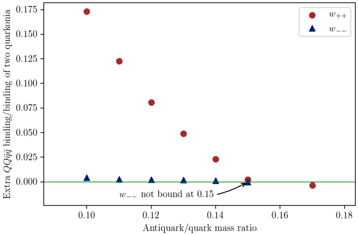
$$H = \frac{p_r^2 + m^2}{\mu} + \mu + \frac{\hat{L}^2/r^2}{\mu + 2\int_0^1(\beta-\frac{1}{2})^2\nu(\beta)d\beta} + \frac{1}{2}\sigma^2 r^2\int_0^1\frac{d\beta}{\nu(\beta)} + \frac{1}{2}\int_0^1\nu(\beta)d\beta$$

- Naturally appearing "constituent" quark mass μ:
  - $\mu \approx m$ for heavy quarks
  - $\mu \sim \sqrt{\sigma}$ for light quarks
- Quark (μ) and string (ν) dynamics entangled via $\hat{L}$
- String inertia taken into account $\Longrightarrow$ Regge slope reduced
- Spin-dependent interaction of Eichten-Feinberg-Gromes-type with $m \to \mu$

Voloshin et al. Phys.Lett.B 778 (2018) 233

- tetraquark $QQ\bar{q}\bar{q}$ with $f = m_q/m_Q$
- Coulombic system
- $N_c \gg 1$
- $\begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix}$ (for $N_c = 3$ u=sextet & w=triplet)
- For $r_{QQ} \to 0$ only w survives and $QQ\bar{q}\bar{q}$ system is weakly bound

Due to the binding between the heavy quarks by the potential (13) the resulting four-quark system is stable under decay to two quarkonium mesons. It should be noted however that this binding is only sub leading in terms of the large N_c counting, as can be seen by comparing the expressions (13) and (15). Thus the discussed 'hierarchy' of the binding energies is only applicable if the ratio f of the masses is small enough at a fixed N_c. In other words there is a critical value of this ratio $f_c(N_c)$ above which the described approximation fails. In order to evaluate the behavior of $f_c(N_c)$ we consider here the effects arising at a finite ratio r_{QQ}/R_q. We find that the main effect arises due to non-vanishing off-diagonal elements in the potential (11):



Hughes et al. Phys.Rev.D 97 (2018), 054505

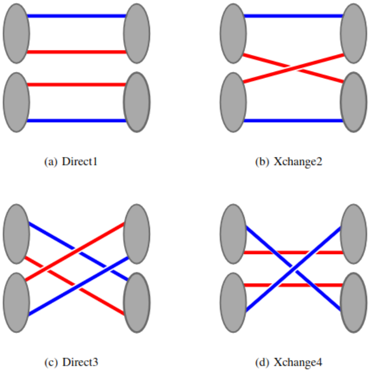


FIG. 1. There are four connected Wick contractions for the two-meson-type correlator when the quarks have the same flavor. The grey region represents a color neutral meson, the blue line a quark and the red line an antiquark. We call these the (a) Direct1 contraction where each meson propagates to itself, (b) Xchange2 where an antiquark is exchanged between the meson pair, (c) Direct3 where each meson propagates to the other, and (d) Xchange4 where a quark is exchanged between the meson pair.

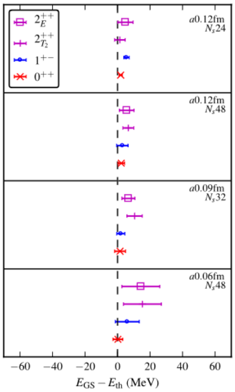


FIG. 9. A summary of the $\bar{b}\bar{b}bb$ ground-state energies with the lowest noninteracting bottomonium-pair threshold subtracted, across the different lattice ensembles listed in Table III, statistical error only. Note, as shown in Table III, fewer configurations were used on the $a = 0.06$ fm ensemble than on the others.

$X(6200)$ as compact $\bar{c}\bar{c}cc$ tetraquark

- Pros
  - Fully charmed compact tetraquark is consistent with QCD
  - Quark models predict fully charmed tetraquark with $M \approx 6.2$ GeV
- Cons
  - Czarnecki,Leng,Voloshin'2018: tetron $QQ\bar{q}\bar{q}$ is unstable for $m_q = m_Q$ *if the system is Coulombic*
  - Hughes,Eichten,Davies'2018: no lattice evidence for $\bar{b}\bar{b}bb$ bound state
- Open questions
  - Are quark model calculations reliable?
  - How stable are fully charmed compact tetraquarks?
  - Is $\bar{c}\bar{c}cc$ system Coulombic enough?
  - Does wisdom from b-sector directly apply to c-sector?

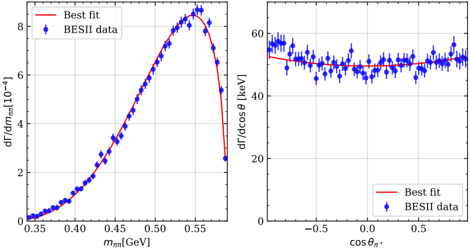
$X(6200)$ as double-J/ψ molecule

Multipole expansion for soft-gluon exchanges ($r_{\bar{Q}Q} \ll \Lambda_{\rm QCD}^{-1}$): $H_{\rm int} \approx -\frac{1}{2}\zeta_a \boldsymbol{r}\cdot\boldsymbol{E}^a$

(Gottfried'1977, Voloshin'1978, Peskin'1979, ... Voloshin&Sibirtsev'2005)

$$\beta_{\psi\psi'} = \frac{1}{9}\langle\psi'|\boldsymbol{r}\frac{1}{\hat{H}_O - M}\boldsymbol{r}|\psi\rangle \Longrightarrow \begin{cases} \mathcal{M}(\psi(2S)\to J/\psi\pi\pi) = \beta_{\psi(2S)J/\psi}\langle\pi\pi|\boldsymbol{E}^a\cdot\boldsymbol{E}^a|0\rangle \\ \boxed{V_{J/\psi J/\psi}(r) \propto \beta^2_{J/\psi J/\psi} \propto \xi^2} \end{cases}$$

$$\left(\xi = \frac{\beta_{J/\psi J/\psi}}{\beta_{\psi(2S)J/\psi}}\right)$$



$$\boxed{|\beta_{\psi(2S)J/\psi}| \approx 1.81\ \text{GeV}^{-3}}$$

(Dong et al.'2021)

$X(6200)$ as double-J/ψ molecule

- Simple estimate

$$\xi \sim I_{J/\psi J/\psi}/I_{\psi(2S)J/\psi} \sim 10 \qquad I_{\psi'\psi} = \langle\psi'|\, e^{-i\Delta \boldsymbol{q}_c\cdot\boldsymbol{r}/2}\,|\psi\rangle$$

(Dong et al.'2021)

- More advanced estimate

$$a_0^{\rm th}(J/\psi\pi \to J/\psi\pi) \approx 0.0036\xi \text{ fm}$$

(Dong et al.'2021)

$$|a_0^{\rm lat}(J/\psi\pi \to J/\psi\pi)| \sim 0.01 \text{ fm} \implies \xi \simeq 3$$

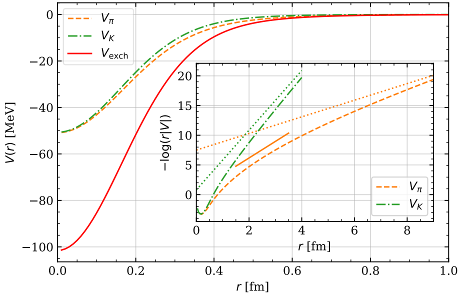
(Yokokawa et al'2006,Liu et al.'2008)

- Quark model calculation

$$\beta_{J/\psi J/\psi} = \frac{0.93}{\alpha_s^4 m_c^3} = 19^{+15}_{-14} \text{ GeV}^{-3} \implies 3 \lesssim \xi \lesssim 19$$

(Brambilla et al.'2016,Dong et al'2023)

$X(6200)$ as double-J/ψ molecule



$$V_{\text{tot}}(r,\Lambda) = V_\pi(r,\Lambda) + V_K(r,\Lambda) = V_{\text{CT}}(r,\Lambda) + V_{\text{exch}}(r,\Lambda)$$

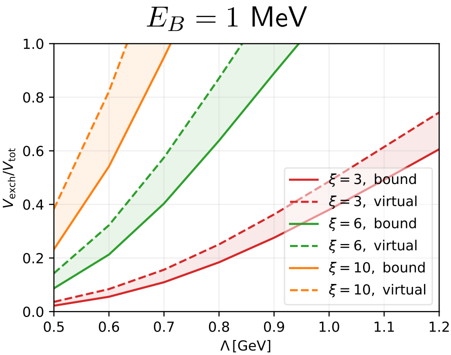
$$V_{\text{exch}}(r,\Lambda) = -\frac{1}{4\pi M_{J/\psi}^2}\int \frac{d^3q}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \int_{4m_\pi^2}^{\infty} d\mu^2 \, \frac{\text{Im}\mathcal{M}_{J/\psi J/\psi}(\mu^2)}{\mu^2+q^2} F\left(\frac{q^2+\mu^2}{\Lambda^2}\right)$$

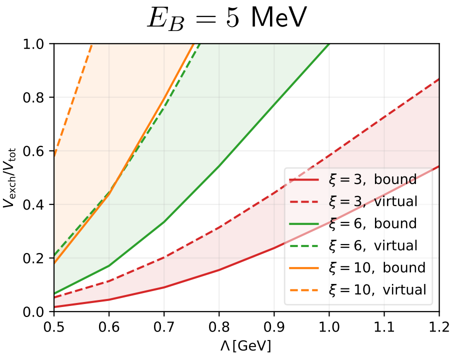
Exchange of charmonia is suppressed as $\Lambda_{\text{QCD}}^2/m_c^2$

$\Longrightarrow$ V_{CT} mainly comes from pion/kaon exchanges

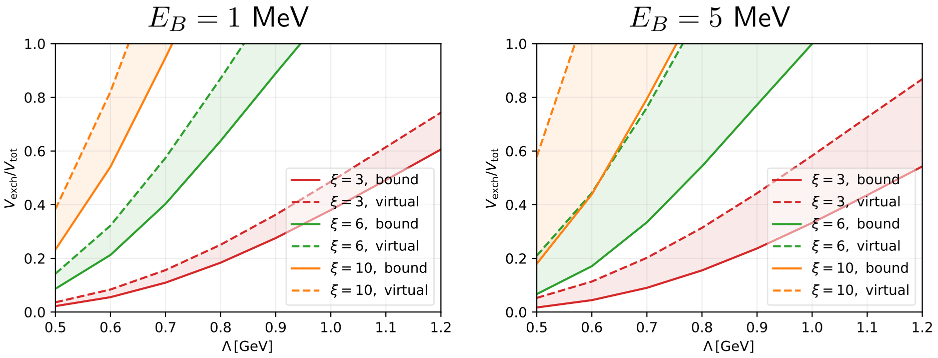
$\Longrightarrow$ For natural $\Lambda \sim 1$ GeV $\boxed{V_{\text{CT}} \sim V_{\text{exch}}}$ $\Longrightarrow$ $V_{\text{exch}}/V_{\text{tot}} \gtrsim \frac{1}{2}$

$X(6200)$ as double-J/ψ molecule





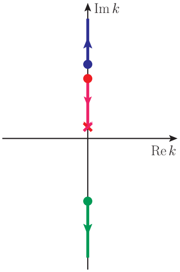
$X(6200)$ as double-J/ψ molecule



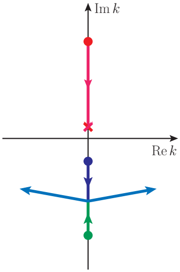
Conclusion: Existence of a molecular pole near the double-J/ψ threshold is consistent with our knowledge on hadron-hadron interactions

Interplay of compact and molecular dynamics

$$f(E) = \frac{E - \overbrace{(E_f - \frac{1}{2} g \gamma_V)}^{E_C}}{(E - E_f)(\gamma_V + ik) + \frac{i}{2} g \gamma_V k}$$



$\gamma_V > 0$
$Z \ll 1$
$E_C < 0$



$\gamma_V < 0$
$Z \sim 1$
$E_C > 0$

Di-heavy-baryon molecules?

- Quarkonium $\psi = \bar{Q}Q$

$$\beta_\psi = \frac{C_\psi}{\alpha_s^4 m_Q^3} \qquad C_\psi \approx 0.93$$

(Brambilla et al.'2016,Dong et al'2023)

- Baryon $\Omega = QQQ$

$$\beta_\Omega = \frac{C_\Omega}{\alpha_s^4 m_Q^3} \qquad C_\Omega \approx 2.4 \approx 2.6 C_\psi$$

(Dong et al'2023)

- Uncertainty from hadron being not Coulombic

$$\frac{\delta\beta}{\beta} \sim \frac{\Lambda_{\rm QCD}^2}{\alpha_s^3 m_Q^2} \sim \begin{cases} 100\% & Q = c \\ 10\% & Q = b \end{cases}$$

Di-heavy-baryon molecules?

- Quarkonium $\psi = \bar{Q}Q$

$$\beta_\psi = \frac{C_\psi}{\cdots} \qquad C_\psi \approx 0.93$$

Conclusion: If $X(6200)$ is a di-J/ψ molecule, then di-Ω_{ccc} and di-Ω_{bbb} molecules are also likely to exist

Supported by recent lattice results:

- $E_B(\mathrm{di}{-}\Omega_{ccc}) \simeq 5..6$ MeV (Lyu et al.'2021)
- $E_B(\mathrm{di}{-}\Omega_{bbb}) \simeq 80..100$ MeV (Mathur et at.'2022)

- Uncertainty from hadron being not Coulombic

$$\frac{\delta\beta}{\beta} \sim \frac{\Lambda_{\rm QCD}^2}{\alpha_s^3 m_Q^2} \sim \begin{cases} 100\% & Q=c \\ 10\% & Q=b \end{cases}$$

Conclusions

- Discovery of $X(3872)$ started new era in hadronic physics of heavy quarks
- LHC studies of double-J/ψ production opened new chapter in this book
- Data collected are analysed
  - using (minimal but realistic) coupled-channel scheme
  - preserving unitarity
  - respecting (approximate but accurate) heavy quark spin symmetry
- Existence of a state at $J/\psi J/\psi$ threshold is predicted from data analysis
- Compact tetraquark assignment for $X(6200)$ is not excluded but calls for additional studies
- Conjecture of molecular nature of $X(6200)$ is consistent with our knowledge of hadron-hadron interactions
- Near-threshold state in double-J/ψ channel may imply existence of double-heavy-baryon molecules

Backup

Values of the parameters found in the fits

Parameters of the two-channel model ($[\bar{a}_i]$=GeV^{-2}, $[\bar{b}_j]$=GeV^{-4}, $[\bar{c}]$=GeV^{-2})

$\bar{a}_1$	$\bar{a}_2$	$\bar{c}$	$\bar{b}_1$	$\bar{b}_2$	α	b
$0.2^{+0.6}_{-0.5}$	-4.2 ± 0.7	$2.94^{+0.36}_{-0.29}$	$-1.8^{+0.4}_{-0.5}$	-7.1 ± 0.4	70^{+8}_{-7}	3.3 ± 0.4

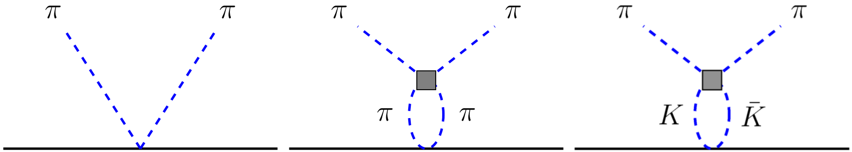
Parameters of the three-channel model ($[\bar{a}_{ij}]$=GeV^{-2})

$\bar{a}_{11}$	$\bar{a}_{12}$	$\bar{a}_{13}$	$\bar{a}_{22}$	$\bar{a}_{23}$	$\bar{a}_{33}$	α	b
$6.0^{+2.2}_{-1.6}$	$10.3^{+3.4}_{-2.8}$	$-0.2^{+1.9}_{-1.3}$	13^{+5}_{-4}	$-2.6^{+2.4}_{-1.3}$	$-2.3^{+1.5}_{-1.1}$	250^{+70}_{-60}	$-0.12^{+0.21}_{-0.22}$
$7.8^{+3.4}_{-2.0}$	16 ± 4	$0.9^{+2.3}_{-2.5}$	26^{+12}_{-6}	-3^{+4}_{-5}	$-2.5^{+2.1}_{-1.0}$	144^{+67}_{-27}	$-0.7^{+0.5}_{-0.4}$

Each parameter with bar needs to be multiplied by $\prod_{i=1}^{4} \sqrt{2m_i}$, where m_i's are the involved charmonium masses

$$m_{J/\psi} = 3.0969 \text{ GeV} \quad m_{\psi(2S)} = 3.6861 \text{ GeV}$$

Final state interaction



From chiral Lagrangian for $\psi \to \psi' \pi\pi$ transition ($s = (p_{\pi_1} + p_{\pi_2})^2$)

$$\mathcal{M}^{\rm tree}_{\pi\pi}(s,\theta) = \mathcal{M}^S(s;m_\pi) + P_2(\cos\theta)\mathcal{M}^D(s;m_\pi)$$

with $\mathcal{M}^{S,D}(s;m_\pi)$ for second-order chiral polynomials (coeffs c_1 and c_2 — free params)

With FSI included

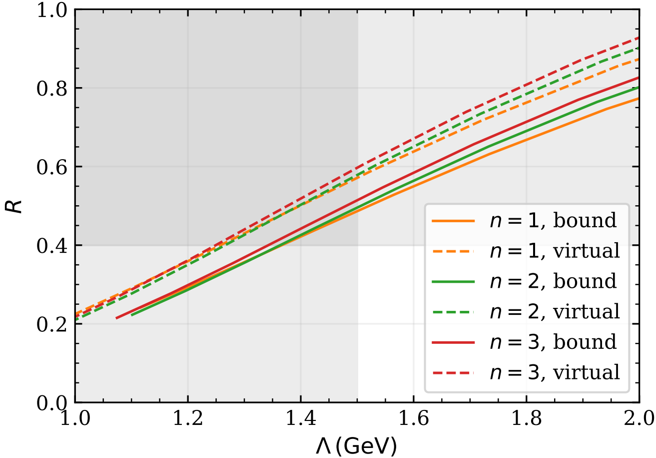
$$\mathcal{M}_{\pi\pi}(s,\theta) = \left[\Omega^S_{11}(s)\mathcal{M}^S(s;m_\pi) + \Omega^D_{11}(s)P_2(\cos\theta)\mathcal{M}^D(s;m_\pi)\right] + \Omega^S_{12}(s)\frac{2}{\sqrt{3}}\mathcal{M}^S(s;m_K)$$

where Omnès functions $(1=\pi,\ 2=K)$

$$\hat{\Omega}^S(s) = \begin{pmatrix} \Omega^S_{11}(s) & \Omega^S_{12}(s) \\ \Omega^S_{21}(s) & \Omega^S_{22}(s) \end{pmatrix} \qquad \Omega^D_{11}(s) = \exp\left(\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dx}{x}\frac{\delta^D_{11}(x)}{x-s}\right)$$

(In)dependence on regularisation scheme

$\xi = 2 \qquad E_B = -1$ MeV $\qquad F(q^2) = \exp[-(q^2/\Lambda^2)^n]$



$$R \equiv \frac{V^S_{\text{exch}}(k'=0, k=0, \Lambda)}{V^S_{\text{tot}}(k'=0, k=0, \Lambda)}$$

Poles trajectories