Radiative corrections, New Physics and LHC

Victor Novikov

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Sources

- LEPTOP – approach to EWRC worked out by V.A.N., L.B. Okun, A.N. Rozanov and M.I. Vysotsky in the 90s.
- Phys. Lett. B **476** (2000) 107-115
- Phys. Lett. B **572** (2002) 111-116
- ....

Using LEPTOP it was found that the precision data do not exclude an existence of additional generation of quarks and leptons.

- V.A.N., A.N. Rozanov, M.I. Vysotsky arXiv:0904.4570 (hep-ph)

Not excluded yet

Contradictions with New Bible – PDG booklet– claim (2008):

- There is no room for 4th generation of quark and leptons. It is excluded by precision data at the 6σ level.
- Precision data prefer a light higgs

$$m_H = 84^{+32}_{-24}\ \mathrm{GeV} \quad .$$

General introduction

Two strategies to look for a New Physics beyond the SM

- Direct accelerator searches

  LEP and Tevatron search for 4th generation–
  No trace of a New Physics
  L3 $m_E \gtrsim 100.8$ GeV decay to νW;
  CDF, D0 $m_T \sim m_B \gtrsim 256$ Gev (CC decay) ;
  $m_T \gtrsim 220$ GeV, $m_B \gtrsim 190$ Gev (quasi-stable)

- Indirect searches – Precision experiment v.s. Precision

  calculations. Sometimes it works!

A good example in the past

Neptune discovery (Le Verrier, Adams, Galle) (1846)

“Neptune was the first planet found by mathematical prediction rather than by empirical observation” (Wikipedia)

Radiative corrections in the SM

- Interaction in the SM is mediated by gauge bosons exchange.
- Gauge bosons interact in a universal way with any particles, both the standard ones and the new ones.
- If the new particles do not mix with SM particles there are only “oblique” corrections to SM observables

$$\Downarrow$$

Corrections to the propagation of gauge fields only (to self-energy):

$$\left\{ \begin{array}{c} \text{gauge field} \\ \text{propagator} \end{array} \right\} \equiv G(q^2) = \frac{g_0^2}{q^2 - m_0^2 - \Sigma(q^2)}$$

Decoupling of Heavy d.o.f.

Decoupling of Heavy d.o.f. from Low-Energy Physics

- QED – Berestetsky,Krokhin,Klebnikov (1956)
- Vector-like theories– Appelquist–Carazzone Theorem (1975)

"Proof" in QED

Let renormalization procedure respects gauge-invariance:

- Photon is massless and propagator has a pole at $q^2 = 0$

$$G(q^2) = \frac{e_0^2}{q^2(1 - \Pi(q^2))}$$

In equation $G(q^2) = g_0^2/(q^2 - m_0^2 - \Sigma(q^2))$ we take

$$m_0^2 = 0, \quad \Sigma(q^2) = q^2 \Pi(q^2)$$

and assume that $\Pi(q^2)$ is regular near $q^2 = 0$.

- All particles have one and the same electric charge:

$$G(q^2) = \frac{e^2}{q^2}$$

  for small q^2 (large distance). It means that $\Pi(0) \equiv 0$ for any particles! Thus

$$\Pi(q^2) \sim q^2$$

  at $q^2 \sim 0$.

Two step proof of decoupling

The contribution of heavy degrees of freedom into low-energy observables is suppressed by some power if these observables are expressed in terms of renormalized electric charge!

1) First step-dimension argument.

$$[\Pi(q^2)] = (m^2)^0$$

2) Second step-universality of gauge couplings.

$$\Pi(q^2) \sim q^2$$

Thus $\delta\Pi(q^2) \sim q^2/m^2_{\rm heavy}$ for small q^2.

Heavy d.o.f. decouples from low-energy observables!

g-2 in QED

New particles contribute into anomalous magnetic moment of leptons at the level of two loops :

$$a_l = \frac{1}{2}(g_l - 2) = \frac{\alpha}{2\pi} + O(\alpha^2 \frac{m_l^2}{m_{heavy}^2})..$$

Though Berestetsky et al. (1956) argued

$$\delta a_e \sim \alpha^2 \left(\frac{m_e^2}{m_{heavy}^2} \right), \quad \delta a_\mu \sim \alpha^2 \left(\frac{m_\mu^2}{m_{heavy}^2} \right).$$

Enhancement factor $(m_\mu^2/m_e^2) \sim 4 \cdot 10^4$

$(g-2)$ of muon is more suitable for New Physics search.

The electron g-2

What was correct on 60th is not absolutely correct now !!

Theory

4-loop contribution into a_e including μ, τ, hadronic and weak loops

$$a_e^{th} = 1\ 159.652\ 172\ 99(930) \cdot 10^{-6}$$

Experiment

Harvard University experiment (2006) (2008)

$$a_e^{th} = 1\ 159.652\ 180\ 73(28) \cdot 10^{-6}$$

Accuracy 0.24 ppb!!
Need 5-loop calculation to be sensitive to $1TeV$ scale!

The muon g-2

BNL precision experiment E821 on muon anomalous magnetic moment
Theory *vs* Experiment
Long history of mistakes:

1. CERN experiment (1975)
   found missing light-by-light contribution into theoretical calculations of a_μ.
2. BNL experiment (2004)
   found wrong sign in classical Kinoshita calculation (1995) of hadronic contribution into light-by-light calculation
   As a result $7\sigma \rightarrow 3\sigma$ discrepancy.

SM *vs* Exp.

Standard model theory and experiment comparison
(in units 10^{-11})

QED 4-loops and some of 5-loops	116 584 718.1
Hadronic contribution to vacuum polarization	6 903.0 (52.6)
light-by-light	116.0 (39.0)
Weak 2-loops	153.2 (1.8)
Theory	116 591 790.0
Experiment	116 592 080.0
Exp.-Theory 3.2σ	290.0 (90.3)

Current Status of muon (g-2)

Discrepancy with theory

3.2σ if $\alpha(m_\mu)$ is calculated using low-energy e^+e^- data

1.4σ if $\alpha(m_\mu)$ is calculated using data on τ-decay into hadrons

No decoupling in the SM

- An example – the third generation:

$$\begin{pmatrix} t \\ b \end{pmatrix} \quad \text{with} \;\; m_t \gg m_b$$

Thus for low-energy scattering ($E \ll m_t$) we have direct violation of $SU(2) \times U(1)$ symmetry

$$\Downarrow$$

Effective nonrenormalizable theory

$$\Downarrow$$

Power divergencies $\sim \Lambda^2/m_W^2$

Natural cut-off $\Lambda \sim m_t$

Thus EWRC depend on top quark mass as

$$\alpha \left(m_t^2/m_W^2\right) \ , \ \ \alpha^2 \left(m_t^2/m_W^2\right)^2 \quad \text{etc.}$$

$$\Downarrow$$

In this way top quark was found.

(Partly the same is true for c-quark.)

- Degenerate case

$$\begin{pmatrix} U \\ D \end{pmatrix} \quad \text{with} \;\; m_U \to \infty \; ; \;\; m_D \to \infty \; ; \;\; m_U - m_D = \text{finite}$$

In this case we have finite non-zero contribution into observables.

General theory of a heavy d.o.f.

Peskin and Takeuchi (1990, 1992)
Contributions of New Physics can be hidden into universal three variables S, T and U.

$$S = 16\pi \left[\Sigma'_A(0) - \Sigma'_V(0)\right]$$

$$T = \frac{4\pi}{s^2 m_W^2} \left[\Sigma_{11}(0) - \Sigma_{33}(0)\right]$$

$$U = 16\pi \left[\Sigma'_{11}(0) - \Sigma'_{33}(0)\right]$$

This approach equivalent to Effective Field Theory for low-energy d.o.f.
PDG claims that using S, T U analysis one can't find a room for the fourth generation.

Main body of the talk

SM fit by LEPTOP, summer 2008

Observable	Exper. data	LEPTOP fit	Pull
Γ_Z, GeV	2.4952(23)	2.4963(15)	-0.5
σ_h, nb	41.540(37)	41.476(14)	1.8
R_l	20.771(25)	20.743(18)	1.1
$A^l_{\rm FB}$	0.0171(10)	0.0164(2)	0.8
A_τ	0.1439(43)	0.1480(11)	-0.9
R_b	0.2163(7)	0.2158(1)	0.7
R_c	0.172(3)	0.1722(1)	-0.0
$A^b_{\rm FB}$	0.0992(16)	0.1037(7)	-2.8
$A^c_{\rm FB}$	0.0707(35)	0.0741(6)	-1.0
s^2_l ($Q_{\rm FB}$)	0.2324(12)	0.2314(1)	0.8

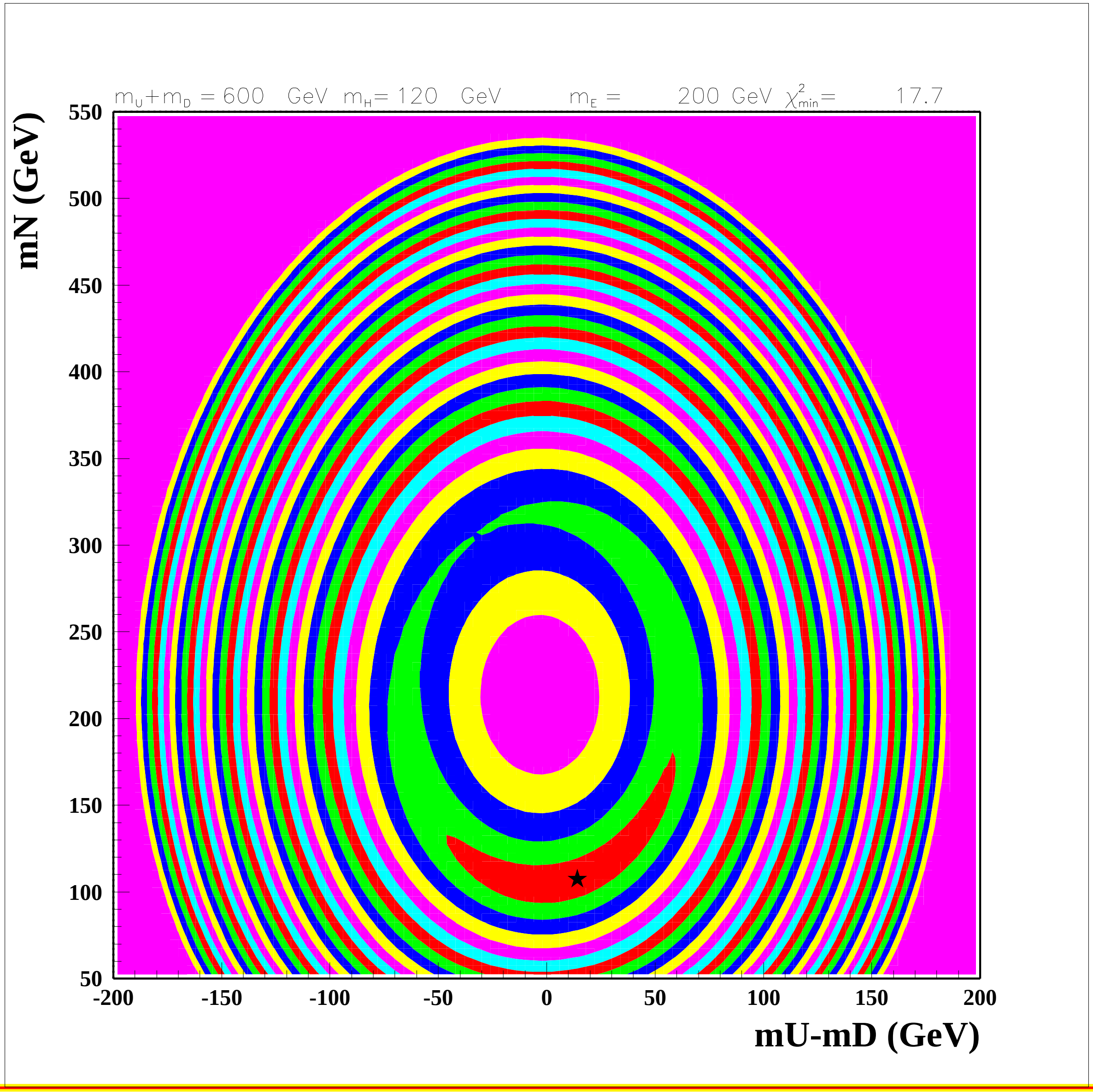
Observable	Exper. data	LEPTOP fit	Pull
A_{LR}	0.1513(21)	0.1479(11)	1.6
A_b	0.923(20)	0.9349(1)	-0.6
A_c	0.670(27)	0.6682(5)	0.1
m_W, GeV	80.398(25)	80.377(17)	0.9
m_t, GeV	172.6(1.4)	172.7(1.4)	-0.1
M_{H}, GeV		84^{+32}_{-24}	
$\hat{\alpha}_s$		0.1184(27)	
$1/\bar{\alpha}$	128.954(48)	128.940(46)	0.3
$\chi^2/n_{\mathrm{d.o.f.}}$		18.1/12	

Fits with the fourth generation

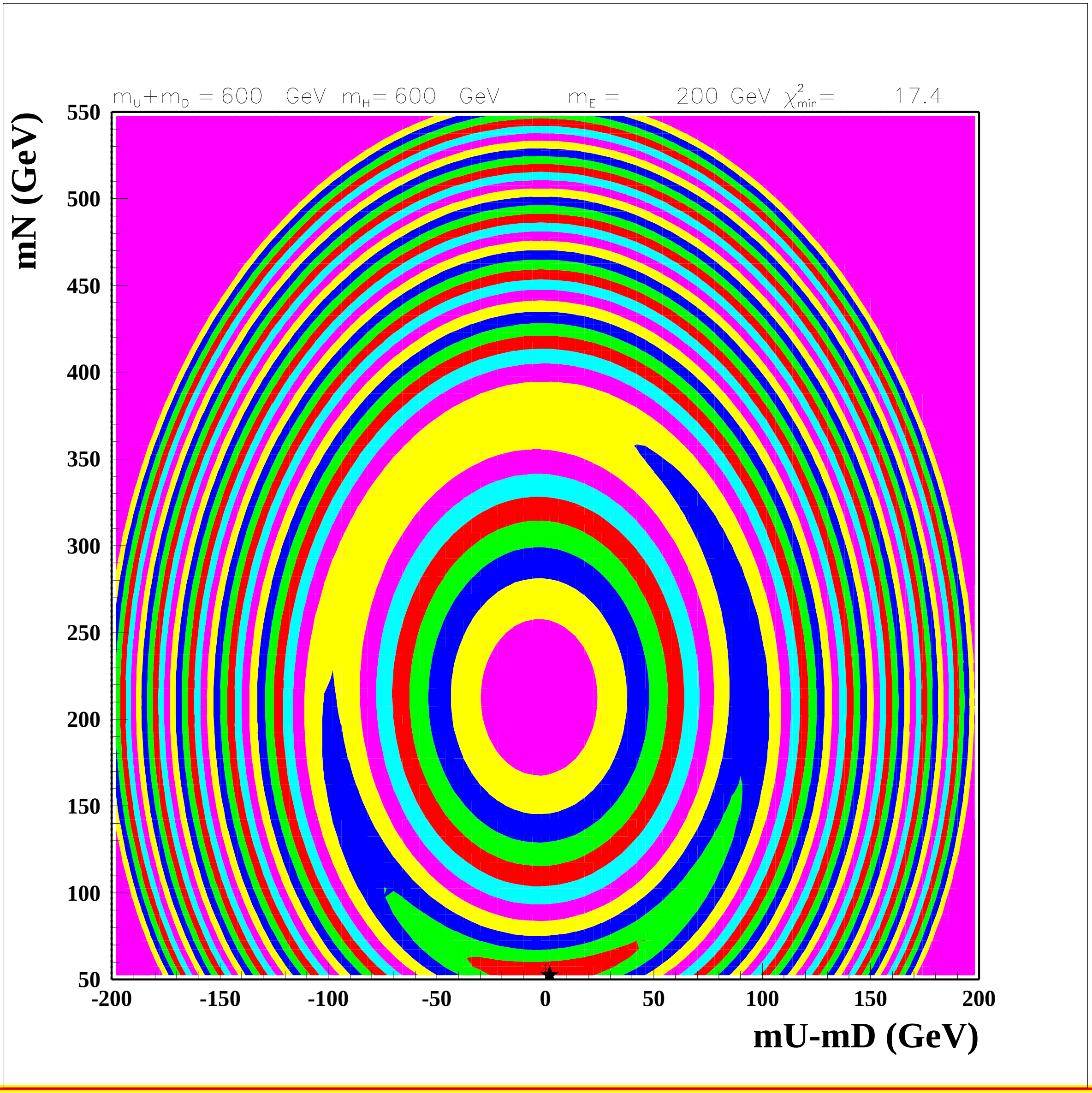
- Let us suppose that mixing is small.
- Separate steep and flat directions in the dependence of χ^2 over new particle masses.
(V.A. Novikov et al. (2002))

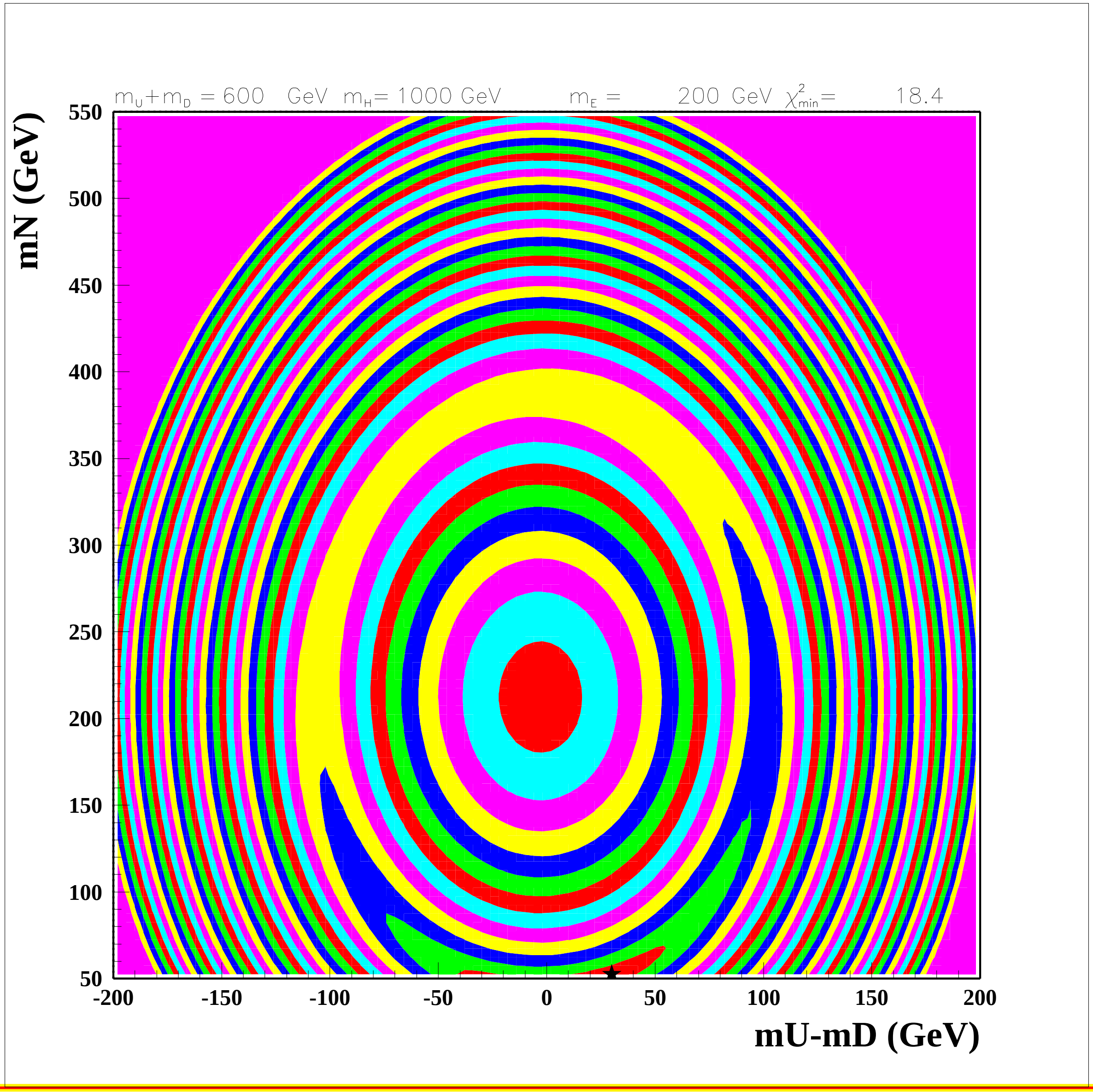
Fix $m_U + m_D = 600$ GeV to avoid Tevatron direct search bounds; fix $m_E = 200$ GeV; vary the difference of neutral lepton mass and the difference of Up- and Down-quark masses.

The results of the fit are presented in Fig. 1 for $m_H = 120$ GeV and in Fig. 2 for $m_H = 600$ GeV and in Fig. 3 for $m_H = 1000$ GeV.



m_U+m_D = 600 GeV m_H= 120 GeV m_E = 200 GeV χ^2_{min}= 17.7
mN (GeV)
550
500
450
400
350
300
250
200
150
100
50
-200
-150
-100
-50
0
50
100
150
200
mU-mD (GeV)


$m_U+m_D = 600$ GeV $m_H= 600$ GeV $m_E =$ 200 GeV $\chi^2_{min}=$ 17.4
mN (GeV)
550
500
450
400
350
300
250
200
150
100
50
-200
-150
-100
-50
0
50
100
150
200
mU-mD (GeV)



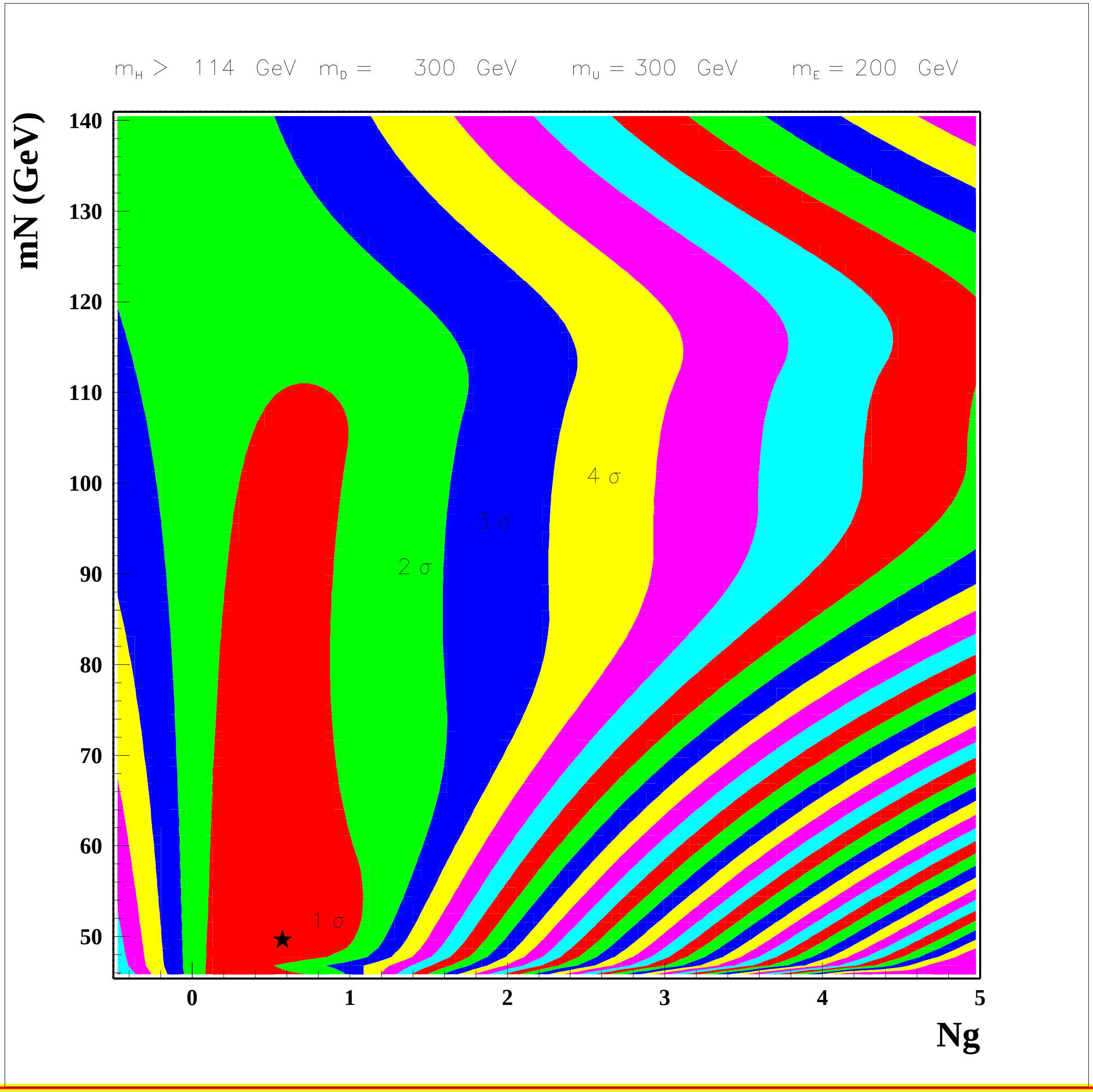
$m_U + m_D = 600$ GeV $m_H = 1000$ GeV $m_E =$ 200 GeV $\chi^2_{min} =$ 18.4
mN (GeV)
550
500
450
400
350
300
250
200
150
100
50
-200
-150
-100
-50
0
50
100
150
200
mU-mD (GeV)

We see that in all cases the quality of the fits is good and not worse than for Standard Model without additional generation.

How many new generations?

- To simplify analysis we assume degeneracy of new particles with identical quantum numbers: $m_{E_1} = m_{E_2} = \ldots$, $m_{N_1} = m_{N_2} = \ldots$, $m_{U_1} = m_{U_2} = \ldots$, $m_{D_1} = m_{D_2} = \ldots$.
- To study this problem we fix $m_E = 200$ GeV, $m_U = m_D = 300$ GeV.
- Take $m_H > 114$ GeV.

The levels of χ^2 are shown in Fig. 4.



m_H > 114 GeV m_D = 300 GeV m_U = 300 GeV m_E = 200 GeV
mN (GeV)
140
130
120
110
100
90
80
70
60
50
0
1
2
3
4
5
Ng
1 σ
2 σ
3 σ
4 σ

The value of χ^2 for Standard Model and for $N_g = 1$ are almost the same, while three and more additional generations are strongly excluded.

$\mathbf{S, T, U}$ **versus** $\mathbf{V_m, V_A, V_R}$

Radiative corrections to electroweak observables were expressed in LEPTOP through three functions V_i:

$$\frac{m_W}{m_Z} = c + \frac{3\bar{\alpha}c}{32\pi s^2(c^2 - s^2)} V_m \quad ,$$

$$g_A = -\frac{1}{2} - \frac{3\bar{\alpha}}{64\pi c^2 s^2} V_A \quad ,$$

$$\frac{g_V}{g_A} = 1 - 4s^2 + \frac{3\bar{\alpha}}{4\pi(c^2 - s^2)} V_R \quad ,$$

$$s^2c^2 \equiv \sin^2\theta_W \cos^2\theta_W = \frac{\pi\bar{\alpha}}{\sqrt{2}G_\mu m_Z^2} \, , \quad \bar{\alpha} \equiv \alpha(m_Z) = (128.87)^{-1} \quad ,$$

$$V_i \equiv V_i^{\mathrm{SM}} + \delta_{NP} V_i \quad .$$

Compare with S, T and U variables.

$$T = \frac{3}{16\pi s^2 c^2} \delta_{NP} V_A + \Delta \equiv T' + \Delta \quad ,$$

$$S = \frac{3}{4\pi} [\delta_{NP} V_A - \delta_{NP} V_R] + 4 s^2 c^2 \Delta \equiv S' + 4 s^2 c^2 \Delta \quad ,$$

$$S + U = \frac{3}{4\pi (c^2 - s^2)} (\delta_{NP} V_m - \delta_{NP} V_R) \equiv S' + U' \quad ,$$

$$\Delta \equiv \frac{1}{\bar{\alpha}} \left[\Pi'_Z(m_Z^2) - \frac{\Pi_Z(m_Z^2)}{m_Z^2} + \frac{\Pi_Z(0)}{m_Z^2} \right] \quad ,$$

$\mathbf{S, T, U}$ **versus** $\mathbf{V_m, V_A, V_R}$

Numbers

Table 2

	$m_H = 120$		$m_H = 600$	
	$m_U = 230$ $m_D = 220$	$m_N = 120$ $m_E = 200$	$m_U = m_D = 225$	$m_N = 50$ $m_E = 200$
T'	-0.001	0.11	-0.006	0.25
T	0.005	0.12	0	0.38
S'	0.15	-0.01	0.15	-0.23
S	0.15	-0.01	0.16	-0.14

Conclusions

- Electroweak data do not contradict the existence of one extra family with specially adjusted masses.
- Three examples corresponding to light and heavy higgs bosons are presented. The properly made analysis based on S, T, U (for $m_H = 120$ GeV) and S', T', U' (for $m_H = 1000$ GeV) confirms the results of the analysis based on V_i.

Global problems with loops

1. Landau pole for Higgs self-coupling, for Yukawa and U(1) coupling

$$\Downarrow$$

Cut-off Λ
for New Physics scale

2. Non-Stable Universe
Heavy Fermions contribution to V_{higgs}^{eff} is negative and makes Universe unstable.

$$V_{higgs}^{eff}(\Phi) \sim \lambda_{eff}(\Phi)\Phi^4$$

$\lambda(\Phi)$ is negative at large Φ.