The diffraction cone shrinkage speed up with the collision energy

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Abstract

The multiperipheral ladder structure of the Pomeron leads to the quite natural conclusion that the elastic slope B_{el} is not simple linear function of the colliding particles energy logarithm. The existing experimental data on the diffraction cone shrinkage points to such "complicated" dependence indeed. The diffraction cone shrinkage speed up with the beam energy is directly connected with an extreme rise of total cross-section (Froissart limit).

1 Introduction

At high energies the hadron-hadron scattering are usually described by the Pomeron exchange. A popular parameterization of the elastic scattering amplitude at small momentum transfer takes into account only Reggeon and Pomeron poles exchange . The ab elastic scattering amplitude reads

$$T_{ab}(t) = F_a(t)F_b(t)C_P s^{\alpha_P(t)} + F_R(t)C_R s^{\alpha_R(t)} \qquad (1)$$

where the form factors F_a, F_b, F_R describes the matter distribution in the incoming hadrons a, b. C_P and C_R are the normalization constants. The

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contribution of the secondary Reggeon poles (last term in (1)) becomes negligible at $\sqrt{s} \sim 100 GeV$.

From the microscopic viewpoint the Pomeron is described by the ladder-type diagrams in which the energy (longitudinal momentum fraction) in each next cell is a few times smaller than that in the previous cell [2]. To get the largest cross section we have to consider the chain (sequence) of strong interactions with relatively low partial sub-energies. Such a sequence of interactions provides a large -non-decreasing with energy cross section $\sigma \propto s^{\alpha_P(0)-1}$.

On another hand at each step the interaction radius changes by the value $\delta\rho \sim 1/k_t$ leading to the 'diffusion' in impact parameter plane. At each step the energy of incoming particle diminishes a few times. Thus the number of steps is $n \sim \ln s$ and the final radius is $R^2 = R_0^2 + n \cdot (\delta\rho)^2$.

Therefore the Pomeron trajectory $\alpha_P(t)$ depends on the momentum transferred $t = -q_t^2$ and for a not large $|t|$ it can be written as $\alpha_P(t) = 1 + \epsilon + \alpha'_P t$.

Correspondingly the elastic ab-cross section takes the form

$$\frac{d\sigma_{ab}}{dt} = \frac{\sigma_0^2}{16\pi} F_a^2(t) F_b^2(t) \left(\frac{s}{s_0}\right)^{2\epsilon + 2\alpha'_P t} . \tag{2}$$

The power growth of the "single Pomeron exchange" cross section generated by the ladder diagram reflects the growth of the parton multiplicity, N. Since at each (ladder) step the longitudinal momentum decreases by a few times the mean number of steps $< n > \sim c * \ln s$. At each splitting (step) the multiplicity of parton increases by a factor two. Thus the final parton multiplicity $N \sim 2^{c \ln s} = s^{c \ln 2}$.

The slope of Pomeron trajectory α'_P accounts for the growth of interaction radius caused by a long chain of intermediate (relatively low energy) interactions which length increases with $\ln s$. In the case of Gaussian form factors $F_a^2 F_b^2 = \exp(B_0 t)$ we get the slope of elastic cross section $d\sigma/dt = |T(t)|^2/16\pi s^2 \propto \exp(B_{el} t)$

$$B_{el} = B_0 + 2\alpha_P^{\prime eff} \ln(s/s_0). \tag{3}$$

[2]This multiperipheral ladder structure of the Reggeon was considered first in [1].

While the first term B_0 in (3) depends on the sort of incoming hadrons a and b,the second term $2\alpha_P'^{eff}\ln(s/s_0)$ is universal. In the case of one Pomeron exchange it should be the same at any energy and for any type of incoming hadrons. This universality was confirmed at the fixed target experiments[2]($\sqrt{s} = 24$ GeV) where the value of $\alpha'_P = 0.14$ GeV^{-2} was measured. [3]

Donnachie-Landshoff [3](equation 7) have deduced from the shape of $d\sigma_{el}/dt$ at measured $\sqrt{s} = 52.8$ GeV [4] much larger $\alpha'_P = 0.25$ GeV^{-2}.

2 More complicated slope $B_{el}(s)$ behavior

The growth of $\alpha_P'^{eff}$ should be expected indeed. When the optical (parton) density, i.e. the opacity $\Omega(\rho, s)$, becomes too large we have to account for the multiple interactions which are described by the multi-Pomeron diagrams. Like in the case of nuclear-nuclear AA-collisions, where few nucleon-nucleon pairs may interact simultaneously and screen each other, the corresponding absorptive corrections stop the growth of elastic amplitude near the black disk limit, when in impact parameter representation the imaginary part of the elastic scatterng amplitude $ImT(\rho) \to 1$. [4]. Note that while at the center of the disk (at small ρ) the amplitude saturates at $ImT = 1$, it still continues to increase with energy at the periphery (at large ρ) leading to the growth of the mean interaction radius and thus to the growth of t-slope B_{el}. Another way to see the variation of B_{el} with energy is to consider just two first diagrams - the one Pomeron exchange and the two Pomeron cut. In comparison with the one Pomeron exchange the two Pomeron contribution falls down with $-t$ slowly, since using two (few) Pomerons we may distribute the whole transferred momentum more homogeneously between the compo-

[3]At a not too large fix target energies it was important to account for the secondary Reggeon contribution in fit [2].

[4]Recall that these multi-Pomeron diagrams are generated just by the s-channel two particle unitarity. Within the eikonal model, the amplitude given by the sum of multi-Pomeron contributions reads $T(\rho) = i(1 - \exp(-\Omega(\rho, s)/2))$ where the value of Ω is described by the one Pomeron (ladder) exchange.

nents (partons) of the initial hadron. However the two Pomeron contribution describes the absorptive correction and has the sign opposite to that of the one Pomeron exchange. Therefore the t dependence of the whole amplitude becomes steeper and the slope B_{el} increases for the case of $\alpha_P(0) > 1$, when at larger energies the relative size of the two Pomeron cut increases.

Therefore the effective shrinkage of the diffractive cone is described by the value of $\alpha_P^{'eff}$ which accounts for both- the growth of the radius of individual Pomeron (α'_P of the 'bare' Pomeron trajectory) and the decrease of optical density in the center of the disk (in comparison with the periphery) because of absorptive effects which provide the radius growth with energy . So $\alpha_P^{'eff} > \alpha'_P$.

3 The increase $\alpha_P^{'eff}$ with collisions energy

Fig. 1 shows measured values of the elastic t-slope $B_{el}(s)$ (NA8-Gatchina-Cern [2],ISR [4], UA4 [5], CDF [6]) including new TOTEM result [7].

One can see clearly that the value of $\alpha_P^{'eff}$ does grow with energy. Fitting the $\ln s$ dependence of B_{el} by the second order polynomial

$$B_{el} = B_0 + b_1 \ln(s/s_0) + b_2 \ln^2(s/s_0) \tag{4}$$

we get

$b_1 = (-.22 \pm .17)GeV^{-2}$ and $b_2 = (.037 \pm .006)GeV^{-2}$

with good $\chi^2/NoF = 7.5/5$ while fit with the linear function is unacceptable $\chi^2/NoF = 37.8/6$. In all fits we use $s_0 = 1$ GeV^2. Recall that the coefficient b_2 (and the analogous coefficient c_2 in the expression for the total cross section in sect.4) does not depend on the value of s_0. Changing s_0 we only re-define the coefficients B_0 and b_1 . Moreover at a given beam energy the value of $2\alpha_P^{'eff} = dB_{el}/d(ln(s/s_0))$ is also independent on the scale s_0. Note that in the case of $s_0 = 1$ GeV^2 the value of b_1 is consistent with zero. The exclusion of this parameter and the fit with the function

$$B_{el} = B_0 + b_2 \ln^2(s/s_0) \tag{5}$$

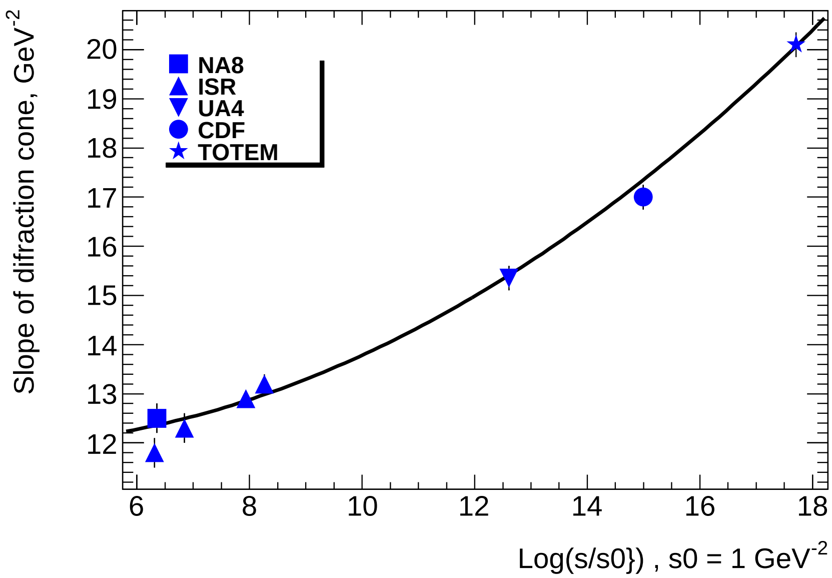

Figure 1: The existing measurement of the diffraction cone slope B_el.

gives

$$b_2 = (0.02860 \pm 0.00050) GeV^{-2} \tag{6}$$

and does not change statistical significance : $\chi^2/NoF = 3.3/4$ against of $\chi^2/NoF = 3.9/5$.

The energy dependence of $2\alpha_P^{\prime eff} = dB_{el}/d(ln(s/s_0))$ is shown in Fig. 2

4 Froissart limit for the diffraction cone shrinkage

Let us compare the behavior of the slope B_{el} and the total pp cross section in the asymptotic black disk (Froissart) limit, when $\sigma_{tot} = 2\pi R^2$ and $B_{el} = R^2/4$ (here R is the black disk radius).

The recent fit $\sigma_{tot} = \sigma_0 + c_1 \ln(s/s_0) + c_2 \ln^2(s/s_{)}$ gives $c_2 = (0.2817 \pm 0.0064)$ mb (see Table 1 of [8])while from $b_2 = 0.037$ GeV^2 we get $c_2(B_{el}) = 0.375$ mb and from $b_2 = (0.0286 \pm 0.0005) GeV^2$, obtained in two parameters

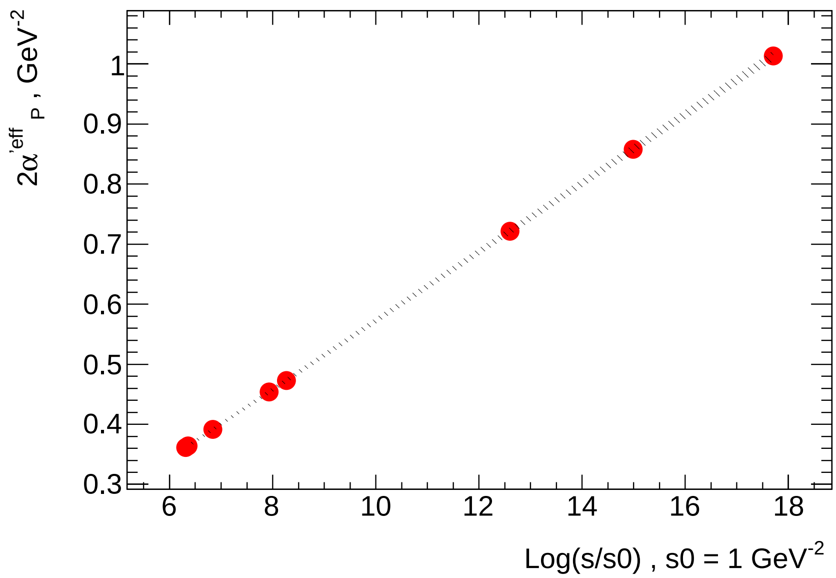

Figure 2: The energy dependence of $2\alpha_P^{'eff}$.

fit, we get a very close value – $c_2(B_{el}) = (0.294 \pm 0.005)$ mb. This demonstrates the present uncertainty in the coefficent c_2 extracted from the elastic slope behaviour. Of course, even at the LHC we are rather far from the complete black disk limit. The proton is still relatively transparent and the cross section σ_{tot} is less than its geometric value $2\pi R^2$.

However it is interesting that both the elastic t-slope and the total cross section have the same $\ln^2 s$ high energy behavior. Starting from the elastic slope we get from the coefficient b_2 the value of c_2 close to that obtained from the total cross sections.

Nontrivial fact is that the value of $2\alpha_P^{'eff} = (0.26 \pm 0.17)GeV^{-2}$ for 3-parameters fit or $2\alpha_P^{'eff} = (0.364 \pm 0.003)GeV^{-2}$ for two parameters fit at $\sqrt{s} = 24$ GeV are similar to $2\alpha'_P = (0.28 \pm 0.03)GeV^{-2}$ found in the Regge Poles analysis of " low energy" elastic scattering [2].

Unfortunately, our conclusion about the non-linear $\ln(s)$ behaviour of the slope B_{el} is based (besides the Regge *Theory*) on the only ONE *measurement* - TOTEM [7]. It looks that in the energy region $\sqrt{s} = 2 - 7$ TeV the role of multi-Pomeron contributions strongly increases. The multi-Pomeron effects should reveal itself not only in elastic scattering but in the multiparticle production as well (see the discussion in [9]).

Recall that the recent Donnachei-Landshoff fit[10] includes *two* Pomeron poles. The pole with high intercept $\epsilon = 0.362$ and the pole with $\epsilon = 0.093$. Each of these 'effective' poles should produce its own secondaries and it would be important to observe the two different power of s in the behaviour of the inclusive cross sections, $d\sigma/d^3p$, and in two particle correlations, including the Bose-Einstein correlations where these two poles will act as two different sources of secondary mesons. Since the slope of the trajectory with a higher intercept is smaller than that for the pole with $\epsilon = 0.093$, we expect that the emission size corresponding to the pole with $\epsilon = 0.362$ should be smaller as well.

Only the LHC can investigate this energy region performing the energy scan in the manner previously realized with the S$p\bar{p}$S collider.

References

[1] D. Amati, S. Fubini, A. Stanghellini, Nuovo Cim. **26**, 896, (1962).

[2] J.P.Burq et al ,Nuclear Physics **B217** (1983) 285-335.

[3] A. Donnachie, P.V. Landshoff, Nucl.Phys. **B231** (1984) 189.

[4] N.Amos et al, Nuclear Physics **B262** (1985) 689-714.

[5] C. Augier, et al.,Phys. Lett. **B316** (1993) 448.

[6] Phys Rev, **D50** (1994) 5518.

[7] TOTEM Coll , Eur.phys. Lett. **96** (2011) 21002.

[8] Martin M. Block, Francis Halzen, e-Print: arXiv:1109.2041 [hep-ph]

[9] M.G. Ryskin, A.D. Martin, V.A. Khoze, A.G. Shuvaev, J.Phys.G **36** (2009) 093001; M.G. Ryskin, A.D. Martin, V.A. Khoze, J.Phys.G **38** (2011) 085006

[10] A. Donnachie, P.V. Landshoff, arXiv:1112.2485 [hep-ph]