Infrared Contribution to the Small-x Asymptotics of Non-Singlet Structure Functions

S.I.Manayenkov and M.G.Ryskin

Petersburg Nuclear Physics Institute
Russian Academy of Science,
Gatchina, St.Petersburg district, 188350,
Russia

Abstract

A simple model for non-perturbative effect contribution to the non-singlet structure functions f_{1NS} and g_{1NS} has been considered. The double logarithmic integral equation taking into account both perturbative and non-perturbative contributions has been proposed and its analytical solution has been found. It is shown that the non-perturbative contributions reduce the value of f_{1NS} and increase the value of g_{1NS} significantly but they change the logarithmic derivatives of these structure functions less than by 20%.

1 Introduction

The small-x behaviour of the deep inelastic structure functions plays an important role both for a description of parton densities in hadrons and for checking of sum rules. As it is well known [1], [2], [3], [4] one cannot use the conventional DGLAP evolution equation [5], [6], [7] to study the non-singlet structure functions f_{NS} at $x \ll 1$. Instead of the usual ordering $|k^2_{i+1,T}| \ll |k^2_{i,T}|$ of transversal momenta k_T in the ladder diagrams for the DGLAP case the leading logarithmic contribution for $x \ll 1$ comes sometimes from regions with inverse ordering ($|k^2_{i,T}| \ll |k^2_{i+1,T}|$). Even more, not excluded that in the middle of the evolution chain values of $k_{i,T}$ become again close to the infrared cat-off parameter μ as at the beginning of the evolution.

It is hard to believe that the double-logarithmic approximation is valid at k_T close to the infrared region where $\alpha_s(k^2_T) \sim 1$. It is reasonable for the time being to consider the perturbative QCD evolution for rather large transverse momenta (say, $|k^2_T| > t_0 = 1$ (GeV/c)^2) and for low k_T to use the effective Lagrangian of QCD

$$L = i\bar{q}\gamma^\mu\partial_\mu q - M\bar{q}\exp\{i\gamma_5\tau^a\pi^a/f_\pi\}q \tag{1}$$

which takes into account the lightest degrees of freedom (light constituent quarks with a mass $M \approx 350$ MeV and the Goldstone pseudoscalar mesons π^+, π^0, π^-). In (1) q and π^a denote the quark and pion field operators, respectively. The constant f_π in (1) is equal to 93 MeV, the Pauli matrices τ^a act on isospin indices of q. As it has been shown in [8] starting from the simplest $SU(6)$ wave function of the nucleon at low $Q^2 \sim M^2$ the leading logarithmic evolution described with the aid of Lagrangian (1) gives at $Q^2 \geq 1$ (GeV/c)^2 a reasonable explanation of both "spin crisis" ($\Delta\Sigma = \Delta u + \Delta d + \Delta s \sim 0.3$) and the violation of the Gottfried sum rule

$$\int_0^1 \left[F_2^p(x,Q^2) - F_2^n(x,Q^2)\right]\frac{dx}{x} \approx 0.24 < \frac{1}{3} \tag{2}$$

observed by NMC [9]. These examples give us a hope that effective Lagrangian (1) provides reasonable predictions of small k_T contributions to non-singlet structure functions also. It has been argued in [10] that the application of the Lagrangian (1) to strong interaction phenomena is reasonable at momenta ≤ 1 GeV/c. This Lagrangian has been considered also in [11] where spontaneous chiral symmetry breaking in the instanton QCD vacuum has been discussed.

2 Integral Equation Summing up Double-Logarithmic Contributions to Structure Functions

For the positive signature (f_{1NS}) the double-logarithmic (DL) contributions are described by ladder Feynman diagrams shown in Fig. 1. As the spin-dependent structure function g_{1NS} corresponds to the negative signature [12], in this case the double-logarithmic contributions come also from the non-ladder Feynman graphs [13]. However the non-ladder corrections for the non-singlet quark distribution are suppressed by the colour factor $1/2N_c^2 \sim 5\%$ [13] and we shall neglect them here. To get a DL contribution the unequality

$$|k^2_{j,T}| \gg |k^2_{j,||}| = s\alpha_j\beta_j \tag{3}$$

is to be valid for any j where $k_{j,||}$ denotes the longitudinal quark momentum. In (3) α_j, β_j are the Sudakov's parameters [14] in the decomposition of k_j into transversal ($k_{j,T}$) and longitudinal components $k_j = -\alpha_j \tilde{q} + \beta_j p + k_{j,T}$ with $\tilde{q} = q - pq^2/2(pq)$. When a transferred momentum in the ladder Feynman graph $|k_j - k_{j+1}|$ is larger than 1 GeV/c we consider the gluon exchange in the framework of the perturbative QCD otherwise we take into account the pion exchange with the effective coupling equal to M/f_π. In our simple model we introduce the function

$$\theta(-(k_j - k_{j+1})^2 - t_0) = 1 - \theta(t_0 - t_j)\theta(t_0 - t_{j+1}) \tag{4}$$

which "switch on" the perturbative contributions. In (4) $t_0 = 1$ (GeV/c)^2 and $t_j = -k_{j,T}^2 > 0$. We have omitted in (4) $k_{j,||}$ and $k_{j+1,||}$ owing to relation (3). It has been taken into account that the DL contributions are originated from two regions $|k_{j,T}| \ll |k_{j+1,T}|$ and $|k_{j+1,T}| \ll |k_{j,T}|$ and relation (4) is valid for these regions only. When both $|k_{j,T}^2|$ and $|k_{j+1,T}^2|$ are smaller than t_0 we consider the pion exchange which is switched on by the function $\theta(t_0 - t_j)\theta(t_0 - t_{j+1})$.

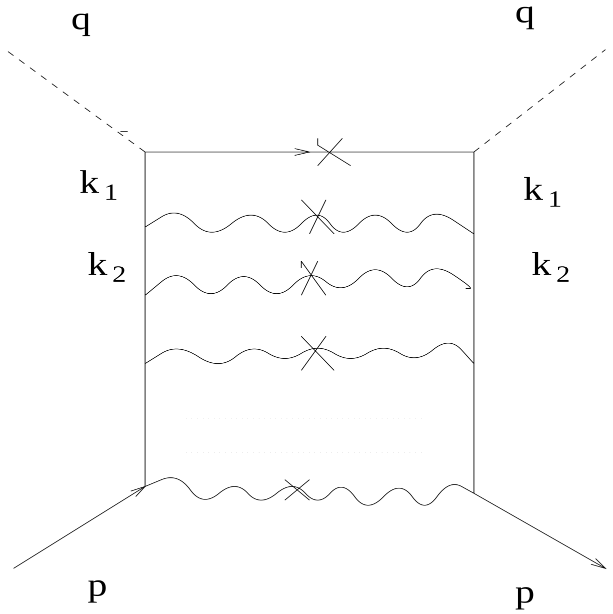

Figure 1: Ladder Feynman graph. Dashed lines correspond to virtual photons and the straight ones correspond to quarks. Wavy lines are assign to gluons (or pions when $|(k_{j,T} - k_{j+1,T})^2| < t_0$). Crosses on the lines mark on-shell particles.

The generalization of the equation proposed in [1] which includes the pion exchange at low k_T looks like

$$A^I(t,\beta) = A_0^I(t,\beta) + \int_\beta^1 \frac{d\beta'}{\beta'} \int_{\mu^2}^{t_m} G^I(t_m) A^I(t',\beta) \frac{dt'}{t'} \tag{5}$$

where

$$G(t_m) = \frac{C_F}{2\pi}\alpha_s(t_m)\Big[1 - \theta(t_0 - t)\theta(t_0 - t')\Big] + w\theta(t_0 - t)\theta(t_0 - t') \,, \tag{6}$$

with $t_m = t\beta'/\beta$ [3], $w = (aM^2)/(16\pi^2 f_\pi^2)$, $C_F = (N^2-1)/(2N)$ for the colour group $SU(N)$, I denotes a value of the isospin in the t-channel and the inhomogeneous term in (5) is

$$A_0^I(t,\beta) = \frac{C_F}{2\pi}\alpha_s(t/\beta)\theta(t - t_0) + w\theta(t_0 - t) \,. \tag{7}$$

A value of the coefficient a depends both on I and a type of the structure function under consideration. For f_{1NS} $(I = 1)$ $a = -1$ and for spin-dependent non-singlet structure function

g_{1NS} $a = 1$. The relation of $A^I(t, \beta)$ with the non-singlet structure functions looks like

$$F_{NS}(x, Q^2) = \delta(1 - x) + \int_{\mu^2}^{S_m} A^I(t, \tilde{\beta}) \frac{dt}{t} \tag{8}$$

with $\tilde{\beta} = x(1 + t/Q^2)$ and $S_m = Q^2/(4x)$. Here and below F_{NS} is a general notation for f_{1NS} or g_{1NS} and $Q^2 = -q^2 > 0$.

It is convenient to express the analytical solution of equation (5) through $\Phi^I(y, \rho)$ where

$$A^I(t, \beta) = \int_{d-i\infty}^{d+i\infty} d\rho e^{\rho z} \Phi^I(y, \rho) . \tag{9}$$

In (9) the parameter d should be chosen so that the integration contour is to the right from all singularities of $\Phi^I(y, \rho)$. In (9) and below we have used the notations $y = \ln(t/\Lambda^2)$, $z = \ln(1/\beta)$, $b = \ln(\mu^2/\Lambda^2)$, $y_0 = \ln(t_0/\Lambda^2)$, $g = 8/(33 - 2n_f)$ where n_f denotes a number of active flavours in a considered process. For $y < y_0$ ($t < t_0$) we have

$$\Phi^I(y, \rho) = C_1(\rho) \exp\{(y - b)(\rho/2 + R)\} + \\ (\rho/2 - R)[1 - C_1(\rho)/(\rho/2 + R)] \exp\{(y - b)(\rho/2 + R)\} , \tag{10}$$

$$C_1(\rho) = \frac{\rho/2 + R}{2\sinh(y_0 R)} \Big[\frac{R}{D(\rho)} - \exp\{-y_0 R\}\Big] , \tag{11}$$

$$D(\rho) = \Big[\frac{g}{\sqrt{\xi}} e^{\xi/2} W_{-1/2,\,0}(\xi) - \frac{w}{\rho} + \frac{\rho}{2} - \frac{g}{\sqrt{\xi}} \frac{W_{\lambda-1/2,\,0}(\xi)}{W_{\lambda,\,1/2}(\xi)}\Big] \sinh(\eta) + R\cosh(\eta) \tag{12}$$

with $R = \sqrt{\rho^2/4 - w}$, $\xi = \rho y_0$, $\eta = (y_0 - b)R$, $\lambda = g/\rho$. In (12) $W_{k,m}(x)$ denotes the Whittaker function. For $y > y_0$ the formula for $\Phi^I(y, \rho)$ reads

$$\Phi^I(y, \rho) = C_2(\rho) \frac{e^{\rho y/2}}{\sqrt{\rho y}} W_{\lambda-1/2,\,0}(\rho y) , \tag{13}$$

$$C_2(\rho) = \frac{g e^{-\rho b/2} R}{W_{\lambda,\,1/2}(\xi) D(\rho)} . \tag{14}$$

We would like to point that at large k_T ($y > y_0$) all the non-perturbative contributions are included in the function $C_2(\rho)$ in (13) and other factors correspond to the perturbative evolution (they do not depend on the parameter of the pion exchange w). This factorization property is the same as for the DGLAP evolution. From (13) it follows immediately that for any $y_1 > y_0$ and $y_2 > y_0$ we have

$$\Phi^I(y_2, \rho) = \sqrt{\frac{y_1}{y_2}} \exp\{\rho(y_2 - y_1)/2\} \frac{W_{\lambda-1/2,\,0}(\rho y_2)}{W_{\lambda-1/2,\,0}(\rho y_1)} \Phi^I(y_1, \rho) . \tag{15}$$

It follows from (9) that $\Phi^I(y, \rho)$ is a ρth moment of $A^I(y, z)$, indeed

$$\Phi^I(y, \rho) = \int_0^\infty \exp\{-\rho z\} A^I(y, z) dz = \int_0^1 A^I(t, \beta) \beta^{\rho-1} d\beta . \tag{16}$$

Measuring $A^I(y, z)$ at some $y = y_1 > y_0$ we can calculate an experimental value of $\Phi^I(y_1, \rho)$ with the aid of (16). Putting an experimental value of $\Phi^I(y_1, \rho)$ into (15) we can predict $\Phi^I(y_2, \rho)$ (and $A^I(y_2, z)$ due to (9)) for $y = y_2 > y_0$ within the framework of the perturbative QCD only.

3 Resilts of Numerical Calculations

We have calculated the structure functions f_{1NS} and g_{1NS} making use of formulae (9), (10), (11), (12), (13) and (14) for $n_f = 4$, $M = \mu = 0.35$ GeV/c and $t_0 = 1$ (GeV/c)^2. To avoid any misunderstanding we would like to point that we have not overaged the structure functions under consideration over initial quark distributions in parent hadrons. This means that we consider the structure functions of a "free" quark. Such an approach is justified as we consider the small-x asymptotic behaviour of the structure functions only and suppose that initial quark distributions in hadrons are smooth enough. Let us discuss an influence of the pion exchange effects on values H_0 and γ_0 where these coefficients determine the behaviour of f_{1NS} and g_{1NS} at $x \to 0$

$$F_{NS} \sim H_0 x^{-\gamma_0} \; . \tag{17}$$

The results of the calculation of f_{1NS} and g_{1NS} is shown in Fig. 2a and Fig. 2b. We see

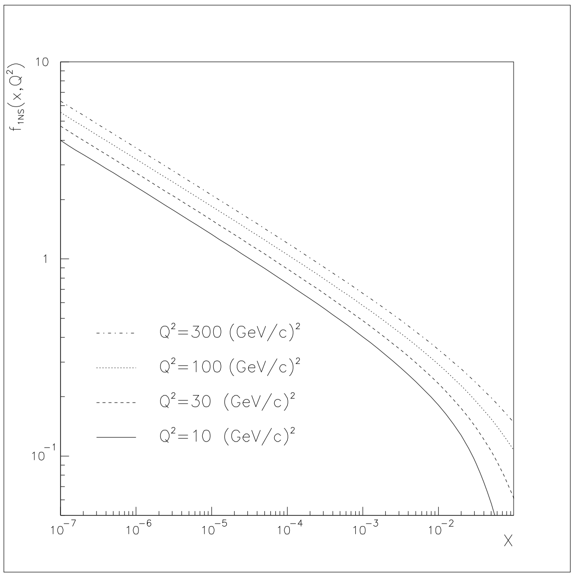

Figure 2a: Dependence of f_{1NS} on x and Q^2.

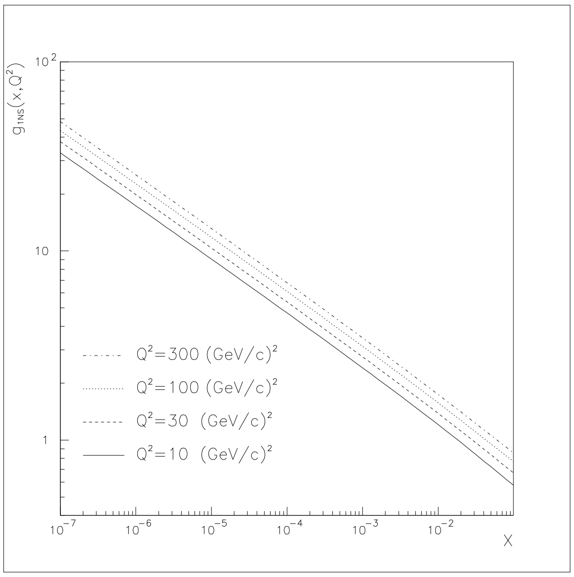

Figure 2b: Dependence of g_{1NS} on x and Q^2.

that g_{1NS} ($a = 1$) can be described by formula (17) at $x < 0.1$ with a good accuracy and H_0 and γ_0 do not depend on x. For f_{1NS} at $x \geq 0.01$ formula (17) is not applicable due to the cancellation between the gluon exchange contribution and the pion exchange one ($a = -1$) in the ladder diagram. One can see from Fig. 2a that the lower is the value of Q^2 the larger is the deviation from formula (17).

A dependence of the logarithmic derivative γ on x and Q^2 is presented in Fig. 3a and Fig. 3b where by definition

$$\gamma = \frac{\partial \ln F_{NS}(x,Q^2)}{\partial \ln(1/x)} \; . \tag{18}$$

One can see that for g_{1NS} the limit of γ for $x \to 0$ is approximately equal to $\gamma_0 = 0.28$ and for f_{1NS} $\gamma_0 = 0.25$. Let us remind that for the perturbative QCD evolution for $t_0 = \mu^2 = \Lambda^2_{QCD}$ and $n_f = 4$ we have the behaviour (17) with

$$\gamma_0 = g = 8/(33 - 2n_f) = 0.32 \tag{19}$$

(see the talk by S.Manayenkov at this workshop). We see from Fig. 3a and Fig. 3b that the dependence of γ on the non-perturbative effect contribution in the model under consideration is not crucial at $x < 10^{-3}$ and the value of γ_0 deviates from the perturbative QCD prediction less than by 20% both for f_{1NS} and g_{1NS}.

If we put in formulae (9) –(14) $w = 0$ we take into account only the perturbative QCD contribution at $k_T \geq 1$ GeV/c . Denoting such structure functions as $F_{NS}^{(0)}$ we can definite the ratio

$$R_0 = F_{NS}(x, Q^2)/F_{NS}^{(0)}(x, Q^2) \ . \tag{20}$$

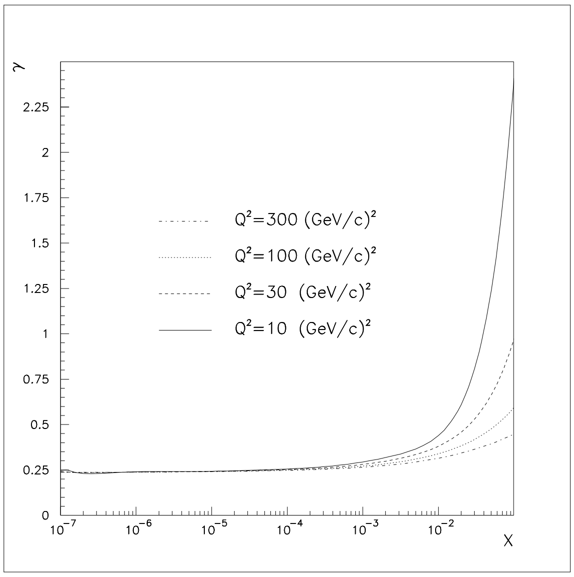

Figure 3a: Behaviour of the logarithmic derivative of f_{1NS} over $\ln(1/x)$

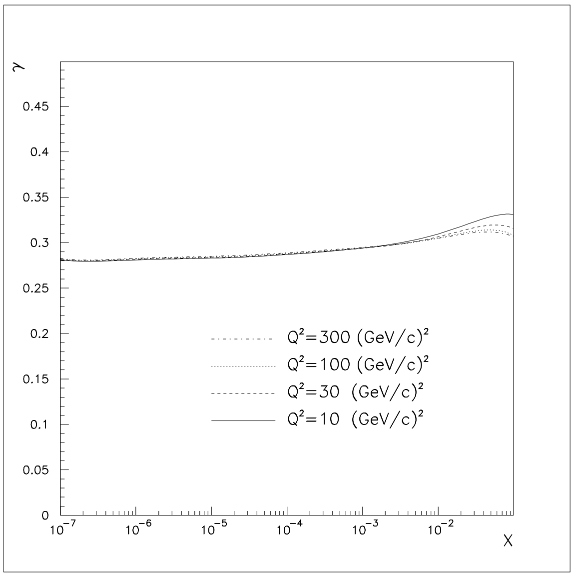

Figure 3b: Behaviour of the logarithmic derivative of g_{1NS} over $\ln(1/x)$

The dependence of R_0 on x and Q^2 is shown in Fig. 4a and Fig. 4b. We see that the pion exchange contribution to f_{1NS} ($a = -1$) decreases the value of the non-singlet structure function more than two times due to the negative sign of w in (6) and (7) which leads to a cancellation of the gluon and pion exchange contributions. The redaction under discussion is more important at large x and low Q^2. It is the decrease of f_{1NS} which can be responsible for the Gottfried sum rule violation (see eq. (2)). For g_{1NS} the pion exchange contribution is positive (as $a = 1$) and it increases a value of g_{1NS} rather significantly (see Fig. 4b). This effect can probably explain the large difference between the proton and neutron structure functions $g_{NS}^p - g_{NS}^n$ measured by the SMC [15] .

The importance of the non-perturbative effects can be expressed also by the ratio

$$R = F_{NS}(x, Q^2)/F_{NS}^{PQCD}(x, Q^2) \tag{21}$$

where $f_{NS}^{PQCD}(x, Q^2)$ denotes the structure function with the perturbative QCD evolution starting from $|k_T| \sim \mu$. We remind that in the structure function $F_{NS}(x, Q^2)$ the perturbative QCD

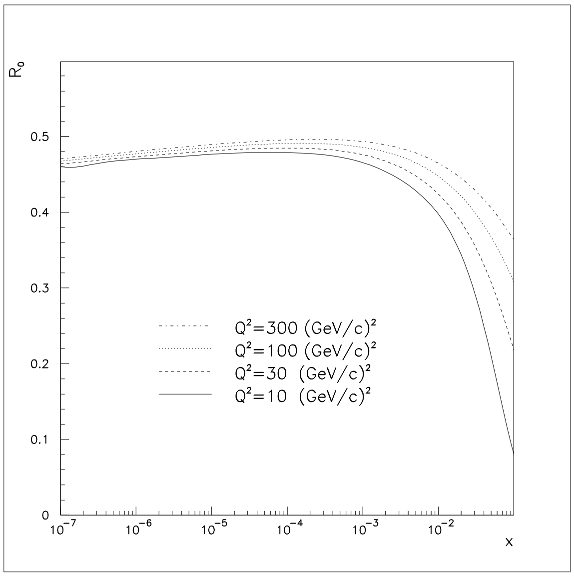

Figure 4a: Dependence of R_0 on x and Q^2 for f_{1NS}. Quantity R_0 is defined in (20).

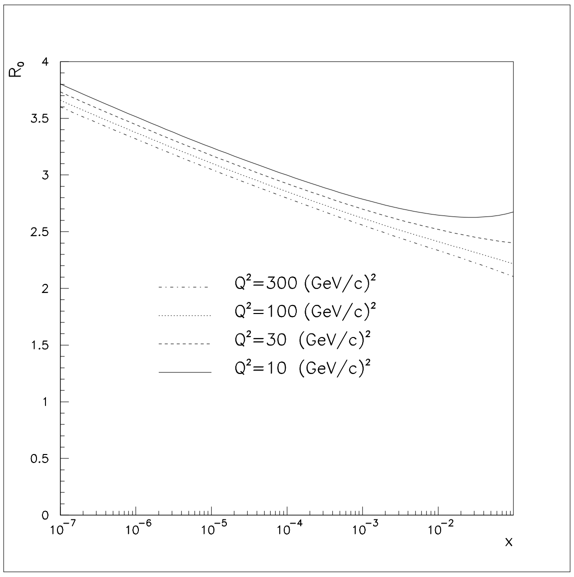

Figure 4b: Dependence of R_0 on x and Q^2 for g_{1NS}. Quantity R_0 is defined in (20).

evolution starts from $|k_T^2| \geq t_0$ and the region $\mu^2 \geq t \geq t_0$ is described by the pion exchange. The calculated ratios R for the f_{1NS} and g_{1NS} cases are presented in Fig. 5a and Fig. 5b. We see that for f_{1NS} the pion exchange contribution causes a very crucial change of a value of the non-singlet structure function. If our model of the non-perturbative effects is realistic enough the crucial change of f_{1NS} means an impossibility to consider the perturbative QCD evolution starting from low Q^2. From this point of view an unexpectedly successful description of available experimental data with parton distribution [16] in which the perturbative QCD evolution starts from $Q^2 = 0.25$ (GeV/c)^2 is an intriguing enigma.

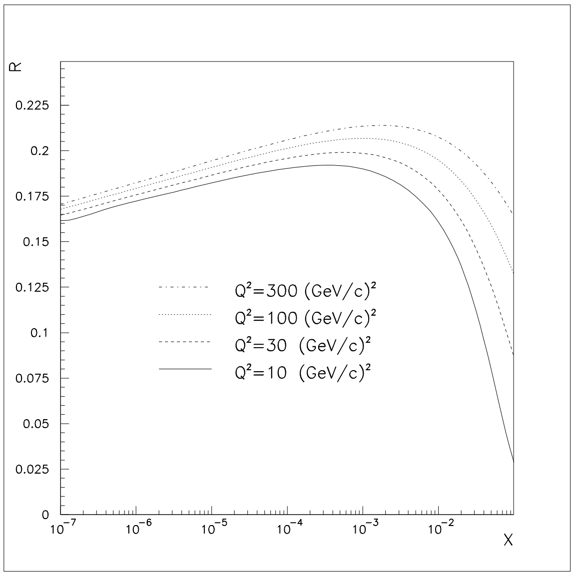

Figure 5a: Dependence of R on x and Q^2 for f_{1NS}. Quantity R is defined in (21).

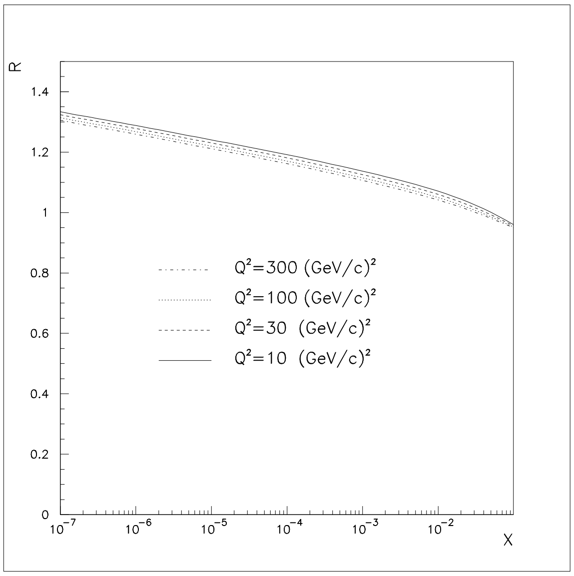

Figure 5b: Dependence of R on x and Q^2 for g_{1NS}. Quantity R is defined in (21).

4 Conclusion

A simple model for non-perturbative effect contribution to the non-singlet structure functions at small x in the framework of the double-logarithmic approximation has been considered. The double-logarithmic integral equation taking into account both perturbative and non-perturbative contributions has been proposed and its analytical solution has been found. The pion exchange contributions change the logarithmic derivatives of the non-singlet structure functions unessentially but they decrease a value of f_{1NS} and increase a value of g_{1NS} appreciably. The former effect can presumably explain the Gottfried sum rule violation measured by NMC. The latter one is probably responsible for the large difference between the proton and neutron structure functions $g^p_{NS} - g^n_{NS}$ observed by SMC.

References

[1] V. G. Gorshkov,V. N. Gribov, L. N. Lipatov and G. V. Frolov: *Yad. Fiz.* **6** (1967) 129 (*Sov. J. Nucl. Phys.* **6** (1967) 95).

[2] V.G.Gorshkov: *Usp. Fiz. Nauk* **110** (1973) 45 (*Sov. Phys. Uspekhi* **16** (1973) 322).

[3] B. I. Ermolaev, S. I. Manayenkov, M. G. Ryskin: *Z. Phys.* **C69** (1996) 259.

[4] J. Bartels, B. I. Ermolaev, M. G. Ryskin: preprint DESY 95-124, June 1995.

[5] V. N. Gribov and L. N. Lipatov: *Yad. Fiz.* **15** (1972) 781 and 1218 (*Sov. Journ. Nucl. Phys.* **15** (1972) 438 and 675).

[6] G. Altarelli and G. Parisi: *Nucl. Phys.* **126** (1977) 297.

[7] Yu. L. Dokshitser: *JETP* **73** (1977) 1216 (*Sov. Phys. JETP* **46** (1977) 641).

[8] A. A. Anselm, M. G. Ryskin: *Z. Phys.* **C68** (1995) 293.

[9] P. Amaudruz et al.: *Phys. Rev. Let.* **66** (1991) 2712.

[10] A. Manohar, H. Georgi: *Nucl. Phys.* **B234** (1984) 189.

[11] D. I. Dyakonov and V. Yu. Petrov: *Nucl. Phys.* **B272**(1986) 457.

[12] R. L. Heimann *Nucl. Phys.* **B64** (1973) 429;
B. L. Ioffe, V. A. Khoze, L. N. Lipatov: "Hard Processes" North-Holland, Amsterdam, 1984.

[13] R. Kirschner, L. N. Lipatov: *Nucl. Phys.* **B213** (1983) 122.

[14] V. V. Sudakov: JETP **30** (1956) 87.

[15] B. Adams et al.: *Phys. Lett.* **B329** (1994) 399;
B357 (1995) 248.

[16] M. Gluck, E. Reya, A. Vogt: *Z.Phys.* **C67** (1995) 433.