Analytical solution of the double-logarithmic integral equation with QCD running coupling

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Abstract
The analytical solution of the double-logarithmic integral equation with QCD running coupling describing small-x behaviour of the non-singlet structure function $f_{NS}(x,Q^2)$ has been found for any cut-off parameter μ. Analytical properties of the solution and a position of the rightmost singularity in the complex ρ-plane which determines the asymptotics of $f_{NS}(x,Q^2)$ at small x have been studied. The asymptotical formula $f_{NS}(x,Q^2) = C_1 x^{-\lambda_1}\{\ln^{\kappa_1}(Q^2/\Lambda^2) - \ln^{\kappa_1}(\mu^2/\Lambda^2) + \kappa_1 \ln^{\kappa_1-1}(Q^2/\Lambda^2)[\psi(1) - \psi(\lambda_1)]\}$ valid if $x \ll 1$ and $\ln(Q^2/\Lambda^2) \gg 1$ has been obtained where C_1, λ_1 are constants, $\kappa_1 = g/\lambda_1$, $\lambda_1 < g = 8/(33 - n_f)$, n_f is a number of active flavours and $\psi(\xi)$ denotes the digamma function.

1 Introduction

The behaviour of the non-singlet quark structure function $f_{NS}(x, Q^2)$ in deep inelastic scattering (DIS) at small x is important for the description of the quark distributions in hadrons. For instance, to check a validity of the Ellis-Jaffe and Bjorken sum rules, one has to extrapolate experimental quark distributions into the small-x region. The most popular way to predict the behaviour of $f_{NS}(x, Q^2)$ for $x \ll 1$ in the framework of the perturbative QCD consists in extrapolating into this region the Altarelli-Parisi equations which describe DGLAP evolution of the structure functions with respect to $\ln(Q^2)$ [1], [2], [3]. These equations are valid for $x \sim 1$ where they sum up all the leading contributions of the Feynman graphs of the ladder type [3]. In this case the transverse momenta k_{jT} of virtual quarks and gluons are strongly ordered (see Fig. 1):

$$|k_{nT}^2| \ll \; ... \; \ll |k_{2T}^2| \; \ll \; |k_{1T}^2| \ll Q^2 \tag{1}$$

where $q^2 = -Q^2 < 0$ denotes the square of the momentum transferred in DIS. Ordering (1) , however, is not valid for summing up the leading double-logarithmic (DL) contributions at $x \ll 1$ where condition (1) should be replaced by the ordering of the longitudinal Sudakov variables α_j, β_j [4] for momenta k_j as follows:

$$\begin{aligned} \alpha_n \ll \; ... \; \alpha_2 \ll \; \alpha_1 \ll 1, \\ 1 \gg \beta_n \gg \; ... \; \gg \beta_1. \end{aligned} \tag{2}$$

To sum up all the double-logarithmic contributions in QED, one can apply the integral equation proposed in [5] (see also the review [6]). The analytical solution of the double-logarithmic integral equation for constant couplings has been found in [5] and can be expressed through the Infeld functions. It coincides with the solution found in [7] by the infrared evolution equation method. This method has first been applied in [8] to the description of the elastic scattering of quarks. The infrared evolution equation method consists in solving the evolution equation with respect to the infrared cut-off in the transverse momentum space. The generalization of the double-logarithmic integral equation for QCD, when the running coupling constant α_s, depends on a parton virtuality, has been done in [7] where properties of numerical solutions have been discussed. In the present paper we find a general solution of the DL integral equation describing the QCD evolution of the non-singlet quark structure function. We consider also the behaviour of the solutions at $x \to 0$.

The paper is organized as follows. In Section 2 we find the general solution of the DL integral equation for any infrared cut-off parameter μ^2. In Section 3 we study analytical properties of the solution in the complex ρ-plane and consider a number and positions of poles especially the position of the rightmost singularity determining the small-x asymptotics of $f_{NS}(x, Q^2)$. In Section 4 we obtain the asymptotical formula for $f_{NS}(x, Q^2)$ at $x \ll 1$ and $\ln(Q^2/\Lambda^2) \gg 1$ and discuss the dependence of the non-singlet structure function on Q^2, x and μ. We summarize our results in Section 5.

2 General solution of double-logarithmic integral equation with QCD running coupling

To take correctly into account all the DL contribution to the non-singlet structure function $f_{NS}(x,Q^2)$ for the QCD running coupling case one can apply the following integral equation [7]

$$A(t,\beta)=\frac{C_F}{2\pi}\alpha_s(t/\beta)+\frac{C_F}{2\pi}\int_\beta^1\frac{d\beta'}{\beta'}\int_{\mu^2}^{t_m}\frac{dt'}{t'}A(t',\beta')\alpha_s(t_m) \tag{3}$$

where $C_F=(N^2-1)/2N$ for the colour group $SU(N)$ and α_s denotes the QCD coupling constant. Integral equation (3) corresponds to the graphs in Fig. 2. Straight lines correspond to the quarks and wavy ones we assign to the gluons. All the intermediate s-channel particles are on mass shell. For the momentum k we have used the Sudakov's parameterization

$$k=-\alpha\tilde{q}+\beta p+k_T, \quad -k_T^2\equiv t\geq 0\ , \tag{4}$$

with

$$\tilde{q}_\mu=q_\mu-\frac{q^2p_\mu}{2pq} \tag{5}$$

and the analogous parameterization for k'. As it has been discussed in [7] the argument of $\alpha_s(t_m)$ in (3) is equal to

$$t_m=t\beta'/\beta\ . \tag{6}$$

The normalization of $A(t,\beta)$ in (3) differs from that one used in [7] by the factor $8\pi^2$, therefore $A(t,\beta)$ is related with the non-singlet structure function f_{NS} by the relation

$$f_{NS}(x,Q^2)=\delta(1-x)+\int_{\mu^2}^{S_m}A(t,\tilde{\beta})\frac{dt}{t} \tag{7}$$

where for small x $S_m=Q^2/(4x)$ and

$$\tilde{\beta}=x(1+t/Q^2)\ . \tag{8}$$

The QCD coupling constant in the one loop approximation looks like [9], [10]

$$\alpha_s(P^2)=\frac{12\pi}{(33-2n_f)\ln(P^2/\Lambda^2)} \tag{9}$$

where n_f is a number of active quark flavours for a process under consideration. The QCD parameter Λ has been chosen in our calculations equal to 200 MeV/c.

It is convenient to apply the following variables:

$$\begin{gathered} y=\ln(t/\Lambda^2), \quad y'=\ln(t'/\Lambda^2), \\ z=\ln(1/\beta), \quad z'=\ln(1/\beta'), \quad b=\ln(\mu^2/\Lambda^2) \end{gathered} \tag{10}$$

then equation (3) can be rewritten in the form

$$B(y,z)=\frac{g}{y+z}+\int_0^z dz'\frac{g}{y+z-z'}\int_b^{y+z-z'}dy'B(y',z') \tag{11}$$

where $B(y,z)=A(t,\beta)$ and

$$g=8/(33-2n_f)\ . \tag{12}$$

First of all, we are going to get some useful unequality with using iteration procedure. It is obvious that for $y\geq b$ we have

$$g/(y+z)\leq g/b\ . \tag{13}$$

Representing $B(y,z)$ in the form

$$B(y,z)=\sum_{n=1}^{\infty} g^n B_n(y,z) \tag{14}$$

and taking into account (13) we get

$$B_1\leq g/b\ , \tag{15}$$

$$B_2\leq \int_0^z dz'\frac{g}{y+z-z'}\int_b^{y+z-z'} dy'\frac{g}{b}=\frac{g}{b}\int_0^z dz'\frac{g(y+z-z'-b)}{y+z-z'}\leq \frac{g}{b}\frac{gz}{1!}\ , \tag{16}$$

$$B_3\leq \int_0^z dz'\frac{z'}{1!}\frac{g}{y+z-z'}\int_b^{y+z-z'} dy'\frac{g^2}{b}\leq \frac{g}{b}\frac{g^2z^2}{2!}\ ,\ \dots\ . \tag{17}$$

It is easy to see that for $B(y,z)$ we have the unequality of interest

$$B(y,z)\leq \frac{g}{b}\sum_{n=0}^{\infty}\frac{g^n z^n}{n!}=\frac{g}{b}\ \exp\{gz\}\ . \tag{18}$$

To get a differential equation instead of integral equation (11), it is convenient to make use of the following variables

$$\xi=y+z,\quad \xi'=y'+z',\quad \eta=z,\quad \eta'=z'\ . \tag{19}$$

Then we get from (11)

$$G(\xi,\eta)=\frac{g}{\xi}+\int_0^{\eta} d\eta'\frac{g}{\xi-\eta'}\int_{b+\eta'}^{\xi} d\xi' G(\xi',\eta') \tag{20}$$

where

$$G(\xi,\eta)=B(y,z)=B(\xi-\eta,\eta)\ . \tag{21}$$

It follows from (20) that

$$\frac{\partial G(\xi,\eta)}{\partial\eta}=\frac{g}{\xi-\eta}\int_{b+\eta}^{\xi} d\xi' G(\xi',\eta) \tag{22}$$

therefore we have the boundary condition $\partial M/\partial\eta=0$ at $\xi=b+\eta$, which for $B(y,z)$ in accordance with (19) and (21) looks like

$$\Big[\frac{\partial B(y,z)}{\partial y}-\frac{\partial B(y,z)}{\partial z}\Big]\Big|_{y=b}=0\ . \tag{23}$$

Differential equation for $G(\xi,\eta)$ can be easily obtained from (22)

$$\frac{\partial}{\partial\xi}\Big\{(\xi-\eta)\frac{\partial G(\xi,\eta)}{\partial\eta}\Big\}=gG(\xi,\eta) \tag{24}$$

which for $B(y,z)$ has the form

$$y\frac{\partial^2 B}{\partial y^2}-y\frac{\partial^2 B}{\partial y\partial z}+\frac{\partial B}{\partial y}-\frac{\partial B}{\partial z}+gB=0\ . \tag{25}$$

To solve equation (25) with boundary condition (23), we consider the Laplace transformation

$$\phi(y,\rho)=\int_0^\infty e^{-\rho z}B(y,z)dz\ . \tag{26}$$

Owing to unequality (18), the integral in (26) is convergent for complex ρ when $Re(\rho)>g$ and we can get an estimate

$$\phi(y,\rho)\le\frac{g}{b}\int_0^\infty\ |\exp\{-(\rho-g)z\}|dz=\frac{g}{b(Re(\rho)-g)}\ . \tag{27}$$

It follows immediately from (11) another boundary condition

$$B(y,0)=g/y\ , \tag{28}$$

therefore we have

$$\begin{aligned}\int_0^\infty\frac{\partial B(y,z)}{\partial z}e^{-\rho z}dz=B(y,z)e^{-\rho z}\Big|_0^\infty+\rho\int_0^\infty e^{-\rho z}B(y,z)dz=\\ =\rho\phi(y,\rho)-g/y\ .\end{aligned} \tag{29}$$

Applying the Laplace transformation to equation (25) and using (29) we get

$$y\frac{\partial^2\phi}{\partial y^2}+(1-\rho y)\frac{\partial\phi}{\partial y}-(\rho-g)\phi=0\ . \tag{30}$$

For the function $\omega(v,\rho)\equiv\phi(y,\rho)$ with $v=\rho y$, we have instead of (30) the differential equation for the confluent hypergeometric function

$$v\frac{\partial^2\omega}{\partial v^2}+(1-v)\frac{\partial\omega}{\partial v}-(1-\kappa)\omega=0 \tag{31}$$

with $\kappa=g/\rho$.

The solution of equation (31) can be represented in the form [11]

$$\omega(v,\rho)=v^{-1/2}e^{v/2}\{C(\rho)W_{\kappa-1/2,\,0}(v)+S(\rho)W_{1/2-\kappa,\,0}(-v)\} \tag{32}$$

where $W_{k,\,m}(\zeta)$ denotes the Whittaker function. For large y when $v=y\rho\gg1$, we make use of the well known asymptotic formula

$$W_{k,\,m}(\pm x)=(\pm x)^k e^{\mp x/2}[1+O(1/x)]\ . \tag{33}$$

With using (33), we conclude that an asymptotic behaviour of (32) at $y\rho \gg 1$ is compatible with unequality (27) when $S(\rho) \equiv 0$. The boundary condition (23) gives the relation for the function $\omega(v,\rho)$ at $v = \rho b$

$$\omega - \frac{\partial \omega}{\partial v} = g/v \ . \tag{34}$$

We have taken into consideration relation (29). Substituting (32) into (34) we get

$$C(\rho) = \frac{g}{\zeta}[H(\zeta,\kappa) - \frac{\partial H(\zeta,\kappa)}{\partial \zeta}]^{-1} \equiv g/D(\zeta,\kappa) \tag{35}$$

where $\zeta = \rho b$. We have applied in (35) the short notation

$$H(x,\kappa) = x^{-1/2} e^{x/2} W_{\kappa-1/2,\,0}(x) \ . \tag{36}$$

With using the well known representation for the Whittaker function $W_{k,\,m}(x)$ valid for $Re(k - m - 1/2) \leq 0$ [11] (For our case this condition means that $Re(g/\rho) < 1$)

$$W_{k,\,m}(x) = \frac{x^k e^{-x/2}}{\Gamma(1/2 - k + m)} \int_0^\infty t^{-k-1/2+m} (1 + t/x)^{k-1/2+m} e^{-t} dt \tag{37}$$

we can get the final formulae for the solution of equation (31) combining (32), (35), (36) and substituting $t = x\tau$ in (37)

$$\omega(v,\rho) = gH(v,\kappa)/D(\zeta,\kappa) \ , \tag{38}$$

$$H(v,\kappa) = \frac{1}{\Gamma(1-\kappa)} \int_0^\infty \tau^{-\kappa} (1+\tau)^{\kappa-1} e^{-v\tau} d\tau \ , \tag{39}$$

$$D(\zeta,\kappa) = e^{\zeta/2} W_{\kappa,\,1/2}(\zeta) = \frac{\zeta}{\Gamma(1-\kappa)} \int_0^\infty (1 + 1/\tau)^\kappa e^{-\zeta\tau} d\tau \ . \tag{40}$$

Here $\Gamma(x)$ denotes the gamma function. We would like to remind, that $v = \rho y$, $\kappa = g/\rho$, $\zeta = \rho b$ and y, z, b, g have been defined in (10) and (12). The function $B(y,z)$ can be obtained from $\omega(\rho y, \rho)$ through the inverse Laplace transformation

$$B(y,z) = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} d\rho e^{\rho z} \omega(\rho y, z) \ . \tag{41}$$

The parameter a in (41) should be chosen so that all singularities of $\omega(\rho y, \rho)$ in a complex plane of ρ are to the left from the contour of the integration.

3 Analytical properties of solution in ρ-plane

To study analytical properties of $\omega(v,\rho)$ we are to investigate analytical properties of $H(v,\kappa)$ and $D(\zeta,\kappa)$ as $\omega(v,\rho)$ is a ratio of these two functions in accordance with (38).

Let us first consider the properties of $H(v,\kappa)$. Remembering definition (36) of $H(v,\kappa)$ and decomposing the Whittaker function into a power series we have

$$H(v,\kappa) = \frac{M(1-\kappa,1,v)}{\Gamma(1-\kappa)}[2\psi(1)-\psi(1-\kappa)-\ln v] - \\ -\sum_{n=1}^{\infty}\frac{v^n}{(n!)^2}\frac{\Gamma(n+1-\kappa)}{\Gamma^2(1-\kappa)}\sum_{m=1}^{n}\Big(\frac{1}{m-\kappa}-\frac{2}{m}\Big) \tag{42}$$

where $M(\alpha,\gamma,\xi)$ in (42) denotes the Kummer function [11]

$$M(\alpha,\gamma,\xi) = 1+\frac{\alpha}{\gamma}\frac{\xi}{1!}+\frac{\alpha(\alpha+1)}{\gamma(\gamma+1)}\frac{\xi^2}{2!}+\frac{\alpha(\alpha+1)(\alpha+2)}{\gamma(\gamma+1)(\gamma+2)}\frac{\xi^3}{3!}\ ... \tag{43}$$

and $\psi(z)$ is the digamma function [11]. It is easy to see from (42) that $H(v,\kappa)$ has a branching point at $v=0$ as it contains $\ln v$. For $Re(\kappa)<1$ the function $H(v,\kappa)$ is an analytical function of κ. Applying the well known formulae [11]

$$\Gamma(1-\kappa)\Gamma(\kappa) = \frac{\pi}{\sin(\pi\kappa)}\ , \quad \psi(\kappa)-\psi(1-\kappa) = -\pi\cot(\pi\kappa) \tag{44}$$

we get from (42) the relation

$$H(v,\kappa) = \frac{\Gamma(\kappa)}{\pi}\Big\{M(1-\kappa,1,v)\Big[[2\psi(1)-\psi(\kappa)-\ln v]\sin(\pi\kappa)-\pi\cos(\pi\kappa)\Big] - \\ -\sin(\pi\kappa)\sum_{n=1}^{\infty}\frac{v^n}{(n!)^2}\Big[\prod_{m=1}^{n}(m-\kappa)\sum_{j=1}^{n}\Big(\frac{1}{j-\kappa}-\frac{2}{j}\Big)\Big]\Big\} \tag{45}$$

which makes it obvious that $H(v,\kappa)$ has no poles as a function of κ at $Re(\kappa)\geq 1$ and $H(v,\kappa)$ is an analytical function of v for all $v\neq 0$. Remembering that $v=\rho y$ and $\kappa = g/\rho$ we see that at $\rho=0$ $H(\rho y, g/\rho)$ has an essential singularity as it contains in accordance with (45) $\Gamma(g/\rho)$, $\sin(\pi g/\rho)$, $\cos(\pi g/\rho)$, $\psi(g/\rho)$ and the branching point due to $\ln(\rho y)$. Hence we conclude that $H(\rho y, g/\rho)$ is an analytical function of ρ in the complex ρ-plane with the cut from $-\infty$ up to $\rho=0$.

Making use of definition (40) for $D(\zeta,\kappa)$ and decomposing $W_{\kappa,1/2}(\zeta)$ into a power series we have for $Re(\kappa)<1$

$$D(\zeta,\kappa) = \frac{\zeta}{\Gamma(1-\kappa)}[M(1-\kappa,1,v)-(1-\kappa)M(2-\kappa,2,v)] \\ [2\psi(1)-\psi(1-\kappa)-\ln\zeta]+\frac{M(1-\kappa,1,\zeta)}{\Gamma(1-\kappa)}+\frac{\zeta(2\kappa-1)}{\Gamma(1-\kappa)} - \\ -\sum_{n=2}^{\infty}\frac{\Gamma(n-\kappa)}{\Gamma^2(1-\kappa)}\frac{\zeta^n}{n!(n-1)!}\{1+\kappa[\psi(n-\kappa)-\psi(1-\kappa)+2\psi(1)-2\psi(n+1)]\}\ . \tag{46}$$

For $Re(\kappa)\geq 1$ we apply (44) and get instead of (46) the formula

$$D(\zeta,\kappa) = \frac{\Gamma(\kappa)}{\pi}\Big\{\zeta[M(1-\kappa,1,\zeta)-(1-\kappa)M(2-\kappa,2,\zeta)] \\ [\sin(\pi\kappa)(2\psi(1)-\psi(\kappa)-\ln\zeta)-\pi\cos(\pi\kappa)]+\sin(\pi\kappa)[M(1-\kappa,1,\zeta)+\zeta(2\kappa-1)] - \\ -\sin(\pi\kappa)\sum_{n=1}^{\infty}\frac{\zeta^{n+1}}{n!(n+1)!}\prod_{m=1}^{n}(m-\kappa)\Big[1+\kappa\sum_{l=1}^{n}\Big(\frac{1}{l-\kappa}-\frac{2}{l}\Big)\Big]\Big\}\ . \tag{47}$$

Again, remembering that $\zeta = \rho b$ and $\kappa = g/\rho$ we come easily to the conclusion (as in the case of $H(v, \kappa)$), that $D(\zeta, \kappa)$ has a branching point at $\rho = 0$ in the terms containing $\ln(\zeta)$ in (46), (47) and an essential singularity at $\rho = 0$ in the terms with $\Gamma(g/\rho)$, $\sin(\pi g/\rho)$, $\cos(\pi g/\rho)$, $\psi(g/\rho)$. Hence $D(\rho b, g/\rho)$ is an analytical function in the complex plane of ρ with the cut from $-\infty$ up to 0. It follows from (38) and the properties of $H(\rho y, g/\rho)$ and $D(\rho b, g/\rho)$ that $\omega(\rho y, \rho)$ is an analytical function of ρ in the complex plane of ρ with the cut from $-\infty$ up to 0. It can have poles at some $\rho = \rho_n$ ($n = 1,\ 2,\ \dots$.) where $D(\rho b, g/\rho)$ is equal to zero.

To study the pole position and their number let us first consider the limit $b \to 0$ ($\zeta \to 0$) for which we have from (47)

$$D(0, \kappa) = \frac{\Gamma(\kappa)}{\pi} \sin(\pi\kappa) = \frac{1}{\Gamma(1-\kappa)} \; . \tag{48}$$

The zeros of $D(0, \kappa)$ coincide with the poles of $\Gamma(1-\kappa)$ which are

$$\kappa_n = g/\rho_n = n \tag{49}$$

with $n = 1,\ 2,\ \dots$. A value of a in (41) is to be chosen so that $a > \rho_1 = g$. One can use $a = a_n$ in the region $\rho_{n+1} < a_n < \rho_n$ but add the contributions of the first n poles so one has from (41), (38), (45), (48)

$$B(y, z) = \frac{1}{2\pi i} \int_{a_n - i\infty}^{a_n + i\infty} d\rho e^{\rho z} \omega(\rho y, \rho) + \sum_{m=1}^{n} (\frac{g}{m})^2 \exp\{gz/m\} M(1-m, 1, gy/m) \; . \tag{50}$$

For $n = 1$ the pole contribution to $B(y, z)$ does not depend on y. Indeed, we get from (50) and (43) that

$$B_1(y, z) = g^2 \exp\{gz\} \; . \tag{51}$$

For large z the term (51) dominates in (50) and gives the main contribution to the non-singlet structure function at small x.

To estimate the contributions of all terms in (50) except the first pole contribution (51) we substitute (38), (39), (48) into (41) and changing the order of integration over ρ and τ get for $B(y, z)$ the closed expression [12]

$$B(y, z) = \frac{g}{y+z} + g^2 \int_{\theta}^{\infty} \frac{s\, ds}{e^s - 1} \sum_{n=0}^{\infty} \frac{g^n z^n}{n!(n+1)!} \Big[s - \frac{y}{z} \frac{s}{e^s - 1}\Big]^n \tag{52}$$

where $\theta = \ln(1 + y/z)$. Relation (51) can be obtained if one puts in (52) $y/z = 0$ (which means that $\theta = 0$), changes $(\exp\{s\} - 1)^{-1}$ by $\exp\{-s\}$ and ignores the first term. Making use of (52) one can prove that (51) is applicable if the conditions

$$\exp\{gz/2\} \gg 1 \; , \quad \exp\{gz - 2\sqrt{gy}\} \gg 1 \; , \quad y \le z \tag{53}$$

are valid [12].

To calculate the pole contribution to $B(y, z)$ for more general case when $b = \ln(\mu^2/\Lambda^2) \neq 0$ we are to find all values of ρ obeying the equation

$$D(\rho b, g/\rho) = 0 \; . \tag{54}$$

Let us denote these values as λ_n $(n = 1,\ 2,\ \dots .)$ and let λ_n be ordered $\lambda_n > \lambda_{n+1}$. It is easy to see that equation (54) has an infinite number of solutions. Indeed, for $\rho_m = g/m$ $(m = 1,\ 2,\ \dots .)$ we have from (47)

$$D(gb/m, m) = (-1)^{m+1}\Gamma(m)\frac{gb}{m}[M(1-m, 1, gb/m) + (m-1)M(2-m, 2, gb/m)]\ . \quad (55)$$

It is easy to get from (43) that for $m \to \infty$

$$\lim_{m\to\infty} M(1-m, 1, gb/m) = J_0(2\sqrt{gb})\ , \quad (56)$$

$$\lim_{m\to\infty} M(2-m, 2, gb/m) = \frac{J_1(2\sqrt{gb})}{\sqrt{gb}}\ , \quad (57)$$

where $J_n(z)$ denotes the Bessel function of the first kind of order n [11]. Hence we get from (55), (56), (57) the formula valid for large m

$$D(gb/m, m) = (-1)^{m+1}\Gamma(m)\sqrt{gb}J_1(2\sqrt{gb})[1 + O(1/m]\ . \quad (58)$$

Relation (58) shows that the function $D(\rho b, g/\rho)$ has different signs at the ends of the region $g/(m+1) \leq \rho \leq g/m$ and hence has at least one zero in this region. So we have proven that the number of solutions of equation (54) is infinite and $\lambda_n \to 0$ when $n \to \infty$. The behaviour of the function $\pi D(\rho b, g/\rho)/\Gamma(g/\rho)$ is presented in Fig. 2 for different values of the infrared cut-off μ $(b = \ln(\mu^2/\Lambda^2))$. We see that the value of λ_1 (the rightmost pole of $\omega(\rho y, \rho)$) which determines the asymptotical behaviour of $f_{NS}(x, Q^2)$ at small x decreases when μ increases. Hence in the double-logarithmic approximation λ_1 reaches its maximal value at $\mu = \Lambda$ and the unequality

$$\lambda_1 \leq g = \frac{8}{33 - 2n_f} \quad (59)$$

is valid. Denoting the derivative of $D(\rho b, g/\rho)$ over ρ at $\rho = \lambda_n$ as $D_n(b)$ we have for $B(y, z)$ instead of (50) the relation

$$B(y, z) = \frac{1}{2\pi i}\int_{a_n - i\infty}^{a_n + i\infty} d\rho e^{\rho z}\omega(\rho y, \rho) +$$

$$+ \sum_{m=1}^{n} \frac{g}{D_m(b)\sqrt{y\lambda_m}} \exp\{\lambda_m(z + y/2)\} W_{\kappa_m - 1/2,\, 0}(y\lambda_m) \quad (60)$$

with $\kappa_m = g/\lambda_m$, and $\lambda_{n+1} < a_n < \lambda_n$.

4 Small-x behaviour of $f_{NS}(x, Q^2)$

Let us consider the first term in a sum in (60) which dominates at large z

$$B_1(y, z) = \frac{g}{D_1(b)}H(\lambda_1 y, \kappa_1)\exp\{\lambda_1 z\} = \frac{g}{D_1(b)}H(\lambda_1 \ln(t/\Lambda^2), \kappa_1)\beta^{-\lambda_1}\ . \quad (61)$$

Substituting (61) into (7) and taking into account (8) we can predict the asymptotical behaviour for small x

$$f_{NS}(x,Q^2) = x^{-\lambda_1}\frac{g}{D_1(b)}\int_{\mu^2}^{Q^2/(4x)}\frac{H(\lambda_1\ln(t/\Lambda^2))}{(1+t/Q^2)^{\lambda_1}}\frac{dt}{t}\ . \tag{62}$$

We see from (62) that $f_{NS}(x,Q^2)$ at small x looks like

$$f_{NS}(x,Q^2) = x^{-\lambda_1}A_1(Q^2) \tag{63}$$

with λ_1 obeying (59). We can get an asymptotical relation for $A_1(Q^2)$ at large Q^2 when the main contributions to the integral in (62) give large t. Combining relation (33) with (36) and substituting the asymptotical expression for $H(\lambda_1 y,\kappa_1)$ at large y into (62) we get

$$A_1(Q^2) = \frac{g(\lambda_1)^{\kappa_1-1}}{D_1(b)}[h_1+h_2] \tag{64}$$

where

$$h_1 = \int_{\mu^2}^{Q^2}\ln^{\kappa_1-1}(t/\Lambda^2)(1+t/Q^2)^{-\lambda_1}\frac{dt}{t}\ , \tag{65}$$

$$h_2 = \int_{Q^2}^{Q^2/(4x)}\ln^{\kappa_1-1}(t/\Lambda^2)(1+t/Q^2)^{-\lambda_1}\frac{dt}{t}\ . \tag{66}$$

Decomposing $(1+t/Q^2)^{-\lambda}$ into a power series with respect to t/Q^2 and integrating by part we get

$$\begin{aligned} h_1 = \frac{1}{\kappa_1}[\ln^{\kappa_1}(Q^2/\Lambda^2) - \ln^{\kappa_1}(\mu^2/\Lambda^2)] + \\ +\sum_{n=1}^{\infty}\frac{(-1)^n(t/Q^2)^n}{n(n!)}\prod_{m=0}^{n-1}(m+\lambda)\ln^{\kappa_1-1}(t/\Lambda^2)|_{\mu^2}^{Q^2} - \\ -(\kappa_1-1)\sum_{n=1}^{\infty}\frac{(-1)^n}{n(n!)}\prod_{m=0}^{n-1}(m+\lambda)\int_{\mu^2}^{Q^2}(t/Q^2)^n\ln^{\kappa_1-2}(t/\Lambda^2)\frac{dt}{t}\ . \end{aligned} \tag{67}$$

We can see from (67) that the first term is $\sim \ln^{\kappa_1}(Q^2/\Lambda^2)$ the second one is proportional to $\sim \ln^{\kappa_1-1}(Q^2/\Lambda^2)$. Integrating by parts we can check that the third term in (67) is $\sim \ln^{\kappa_1-2}(Q^2/\Lambda^2)$ and we shall neglect it. Omitting the small contributions in the second term containing $(\mu^2/Q^2)^n$ we get for h_1 the relation

$$\begin{aligned} h_1 &= \frac{1}{\kappa_1}[\ln^{\kappa_1}(Q^2/\Lambda^2) - \ln^{\kappa_1}(\mu^2/\Lambda^2)] + \sum_{n=1}^{\infty}\frac{(-1)^n}{n(n!)}\prod_{m=0}^{n-1}(m+\lambda)\ln^{\kappa_1-1}(Q^2/\Lambda^2) = \\ &= \frac{1}{\kappa_1}[\ln^{\kappa_1}(Q^2/\Lambda^2) - \ln^{\kappa_1}(\mu^2/\Lambda^2)] + \ln^{\kappa_1-1}(Q^2/\Lambda^2)\int_0^1[(1+\xi)^{-\lambda_1}-1]\frac{d\xi}{\xi}\ . \end{aligned} \tag{68}$$

Introducing the variable $\nu = Q^2/t$ we have for h_2 the expression

$$h_2 = \int_{4x}^{1}\nu^{\lambda_1-1}\ln^{\kappa_1-1}(Q^2/\nu\Lambda^2)\frac{d\nu}{(1+\nu)^{\lambda_1}} = \frac{\nu^{\lambda_1}}{\lambda_1}\ln^{\kappa_1-1}(Q^2/\nu\Lambda^2)|_{4x}^{1} +$$

$$+\sum_{n=1}^{\infty}\frac{(-1)^n(\nu)^{n+\lambda_1}}{n!(n+\lambda_1)}\prod_{m=0}^{n-1}(\lambda_1+m)\ln^{\kappa_1-1}(Q^2/\nu\Lambda^2)|_{4x}^{1}+$$
$$+(\kappa_1-1)\sum_{n=1}^{\infty}\int_{4x}^{1}\frac{(-1)^n(\nu)^{n+\lambda_1}}{n!(n+\lambda_1)}\prod_{m=0}^{n-1}(\lambda_1+m)\ln^{\kappa_1-2}(Q^2/\nu\Lambda^2)\frac{d\nu}{\nu}\,. \tag{69}$$

As $\lambda_1 > 0$ we can put $x = 0$ in (69) at small x. It is easy to check that the third term in (69) is proportional to $\ln^{\kappa_1-2}(Q^2/\Lambda^2)$ and we can neglect it compared to the first and second terms. So we obtain for h_2 the expression

$$h_2=\frac{1}{\lambda_1}\ln^{\kappa_1-1}(Q^2/\Lambda^2)+\sum_{n=1}^{\infty}\frac{(-1)^{n+\lambda_1}}{n!(n+\lambda_1)}\prod_{m=0}^{n-1}(\lambda_1+m)\ln^{\kappa_1-1}(Q^2/\Lambda^2)=$$
$$=\ln^{\kappa_1-1}(Q^2/\Lambda^2)\int_0^1\frac{\nu^{\lambda_1-1}d\nu}{(1+\nu)^{\lambda_1}}=\ln^{\kappa_1-1}(Q^2/\Lambda^2)\int_1^{\infty}\frac{ds}{s(1+s)^{\lambda_1}}\,. \tag{70}$$

Substituting (68) and (70) into (64) we can write

$$A_1(Q^2)=\frac{g\lambda_1^{\kappa_1-1}}{D_1(b)}\{\frac{1}{\kappa_1}[\ln^{\kappa_1}(Q^2/\Lambda^2)-\ln^{\kappa_1}(\mu^2/\Lambda^2)]+\ln^{\kappa_1-1}(Q^2/\Lambda^2)\varphi(\lambda_1)\} \tag{71}$$

where $\varphi(\lambda_1)$ is

$$\varphi(\lambda_1)=\int_0^1\Big[\frac{1}{(1+\xi)^{\lambda_1}}-1\Big]\frac{d\xi}{\xi}+\int_1^{\infty}\frac{d\xi}{\xi(1+\xi)^{\lambda_1}}\,. \tag{72}$$

It is easy to check that $\varphi(\lambda_1 = 1) = 0$ hence we can write

$$\varphi(\lambda_1)=\varphi(\lambda_1)-\varphi(1)=\int_0^{\infty}\Big[\frac{1}{(1+\xi)^{\lambda_1}}-\frac{1}{(1+\xi)}\Big]\frac{d\xi}{\xi}=$$
$$=\int_0^{\infty}\Big[e^{-\xi}-\frac{1}{(1+\xi)}\Big]\frac{d\xi}{\xi}-\int_0^{\infty}\Big[e^{-\xi}-\frac{1}{(1+\xi)^{\lambda_1}}\Big]\frac{d\xi}{\xi}=\psi(1)-\psi(\lambda_1)\,. \tag{73}$$

We have used in (73) the well known Dirichlet formula [11]

$$\psi(z)=\int_0^{\infty}\Big[e^{-\xi}-\frac{1}{(1+\xi)^z}\Big]\frac{d\xi}{\xi} \tag{74}$$

for the digamma function. The final formula follows from (63), (71), (73)

$$f_{NS}(x,Q^2)=C_1x^{-\lambda_1}\{\ln^{\kappa_1}(Q^2/\Lambda^2)-\ln^{\kappa_1}(\mu^2/\Lambda^2)+\kappa_1\ln^{\kappa_1-1}(Q^2/\Lambda^2)[\psi(1)-\psi(\lambda_1)]\} \tag{75}$$

with the constant C_1 equal to

$$C_1=\frac{\lambda_1^{\kappa_1}}{D_1(b)}\,. \tag{76}$$

It is easy to generalize formula (75). The leading logarithmic contribution to $f_{NS}(x,Q^2)$ of the nth pole (if $\ln(Q^2/\Lambda^2)\gg 1$) looks like

$$f_{NS}^{(n)}(x,Q^2)=C_nx^{-\lambda_n}\ln^{\kappa_n}(Q^2/\Lambda^2) \tag{77}$$

with λ_n being a solution of equation (54) and $\kappa_n = g/\lambda_n$.

Formula (77) is applicable if the relation

$$\lambda_n \ln(Q^2/\Lambda^2) \gg 1 \tag{78}$$

is valid. But even for $n = 1$ a value of $\lambda_1 \ln(Q^2/\Lambda^2)$ in (78) is not too large. Indeed, for $Q = 10$ GeV/c, $\Lambda = 200$ MeV/c, $n_f = 4$ we can apply an estimate $\lambda_1 \sim g = 0.32$ in accordance with (12) and get

$$g \ln(Q^2/\Lambda^2) = 2.5\ . \tag{79}$$

Nevertheless formula (75) can be used for rough estimates. The comparison of $f_{NS}(x, Q^2)$ calculated with the help of formula (75) and precise formulae (7), (38), (39), (40), (41) shows that the difference at $x < 10^{-2}$ is less than 15% as one can see from Fig. 4. We can also see from Fig. 4 that the behaviour $f_{NS}(x, Q^2) \sim x^{-\lambda_1}$ is valid at $x < 10^{-2}$.

If we consider the logarithmic derivative of $f_{NS}(x, Q^2)$ over $\ln(1/x)$

$$\lambda = \frac{\partial f_{NS}(x, Q^2)}{\partial \ln(1/x)} \tag{80}$$

we get from (75) $\lambda = \lambda_1$. One can see from Fig. 5 that at small x the logarithmic derivative λ approach λ_1 and is approximately constant for small μ but for $\mu \geq 1$ GeV/c and at $x \geq 0.001$ it deviates appreciably from λ_1 and depends both on x and Q^2.

We can see from Fig. 3 or Fig. 5 that the value of λ_1 depends strongly on the infrared cut-off parameter μ. Indeed, for $\mu = \Lambda$ λ_1 is equal to g (for $n_f = 4$ $g = 0.32$) and for $\mu = 1$ GeV/c $\lambda_1 = 0.222$. This means the importance of small k_T contributions ($k_T < 1$ GeV/c) to λ. It is reasonable to expect that at small k_T contributions of non-perturbative QCD effects are large and can appreciably change the value of λ at small x. Hence our prediction (75) obtained in the framework of the perturbative QCD can be violated by non-perturbative contributions. This problem will be considered in the next paper. It will be shown that behaviour (75) and the relation $\kappa_n = g/\lambda_n$ are valid as in the perturbative QCD. It will be shown that if the value of the isospin I in the t-channel in the ladder graph shown in Fig. 1 is equal to 1 a value of λ_1 does not differ significantly from its perturbative value. For $I = 0$ a value of λ_1 changes appreciably.

Up to now we considered the structure function of a "free" quark $f_{NS}(x, Q^2)$ where $x = Q^2/2(pq)$ and p denotes the initial quark momentum in the ladder graph shown in Fig. 1. To get for example, the non-singlet structure function of the nucleon $F_{NS}(x_B, Q^2)$ we are to convolute the quark structure function with the initial (at low Q^2) quark distribution $\Phi(\xi)$ in the nucleon. Indeed, the relation for $F_{NS}(x_B, Q^2)$ looks like

$$F_{NS}(x_B, Q^2) = \int_{x_B}^{1} f_{NS}(x_B/\xi, Q^2)\Phi(\xi)\frac{d\xi}{\xi} \tag{81}$$

where Bjorken x_B is given by the well known formula $x_B = Q^2/2(P_N q)$ with P_N denoting the nucleon momentum. The quantity ξ in (81) is the ratio of the quark to the nucleon longitudinal momentum in the infinite momentum system of frame. It is easy to see that if the initial quark distribution has a singularity at small ξ and the small-ξ contribution $\xi \sim x_B$ to $F_{NS}(x_B, Q^2)$ dominates in (81) then the first argument of $f_{NS}(x, Q^2)$ is large ($x = x_B/\xi \sim 1$). In this case one can use $f_{NS}(x, Q^2)$ calculated with the aid of the Altarelli-Parisi equation. Our precise solution is applicable at small x only where we sum up double-logarithmic terms having large parameters $\alpha_s \ln^2 x$ and $\alpha_s \ln(1/x)\ln(Q^2/\mu^2)$ and neglect all single logarithmic terms containing parameters $\alpha_s \ln(Q^2/\mu^2)$ and $\alpha_s \ln(1/x)$. We see that for the case under consideration the main contributions give $x \sim 1$, and our solution gives small correction to the Altarelli-Parisi prediction. But if the initial quark distribution $\Phi(\xi)$ is smooth enough or is localized at $\xi \sim 1$ (say, at $\xi \approx 1/3$ as in the constituent quark model) then a region $\xi \sim 1$ gives an appreciable contribution to the integral in (81). In this case the first argument of $f_{NS}(x,Q^2)$ is small and we should use for $f_{NS}(x,Q^2)$ the precise relations (7), (38), (39), (40), (41). The increase of the double -logarithmic contribution in the latter case, compared with the former one when the initial distribution is singular at small ξ, has recently been discussed in detail in [13]. For smooth initial quark distributions we need precise solution for $f_{NS}(x,Q^2)$ as its decomposition into power series of $\ln(1/x)$ converges slowly. For example, for $x = 10^{-3}$ and $\mu = 1$ GeV/c λ_1 is equal to 0.222 so we have to calculate six terms in decomposition into a power series

$$f_{NS}(x,Q^2) \sim x^{-\lambda_1} = \sum_{n=0}^{\infty} \frac{(\lambda_1)^n \ln^n(1/x)}{n!} \tag{82}$$

to get precision $\sim 10\%$.

Usually phenomenological distribution $\Phi(\xi)$ for the non-singlet structure function looks like

$$\Phi(\xi) = \xi^{-a} U(\xi) \tag{83}$$

where $U(\xi)$ denotes some smooth function without any singularities at small ξ. Substituting (82) and (83) into (81) we get the obvious relation

$$F_{NS}(x_B,Q^2) \sim \Big(\frac{x_B^{-\lambda}}{\lambda - a} + \frac{x_B^{-a}}{a-\lambda}\Big) U(\xi_0) \tag{84}$$

with $0 \le \xi_0 \le 1$. We see from (84) that for the smooth distribution when $a < \lambda$ the singularity of $F_{NS}(x_B,Q^2)$ ($\sim x_B^{-\lambda}$) is due to the singularity of the "free" quark structure function $f_{NS}(x,Q^2)$. When $\Phi(\xi)$ is more singular than $f_{NS}(x,Q^2)$ the singularity of $F_{NS}(x_B,Q^2)$ ($\sim x_B^{-a}$) coincides with the singularity of the initial quark distribution. For the latter case the origin of the singularity (ξ^{-a}) is out of the theoretical consideration in the framework of QCD. The most popular way is to put $a \approx 0.5$ for the unpolarized valence quark distributions in accordance with the Regge pole phenomenology. A fit of deep inelastic scattering data itself gives different values for a. Indeed, fit R2 performed in [14] gives, for the unpolarized distributions, $a_u = 0.4$ (for valence u-quarks) and $a_d = 0.76$ (for valence d-quarks). The result of [15] is $a_u = 0.457$, $a_d = 0.684$. Values of $a_u = a_d = 0.497$ have been obtained in the STEQ3M set [16] of the parton distribution parameters but for solution 2M values $a_u = 0.722$ and $a_d = 0.479$ [16] have been found. The fitting procedure gives $a = 0.21 \pm 0.01$ [17] for the structure function $F_3(x_B,Q^2)$. For the helicity-dependent distributions of the valence quarks in the proton, a value of $a = 0.12$ has been used in [18]. Values $a_u = 0.607$ and $a_d = 0.285$ (set 1) have been found in [19] for the spin-dependent valence quark distributions $\Delta q_v(x)$ as a result of fitting the world data on the nucleon structure function $g_1(x,Q^2)$. In the present paper we are trying to predict the type of the singularity in the framework of the perturbative QCD.

As in accordance with (59) $\lambda_1 < g = 0.32 < 0.5$, such an attempt at first glance does not look promising, as λ_1 is smaller than typical values obtained in phenomenological analysis [14], [15], [16], [19] of existing experimental data. In the next paper we shall consider the contribution of non-perturbative effects to the ladder graph which contains double-logarithmic terms. In this case a value of λ_1 depends on the value of the isospin in the t-channel and it is different for unpolarized and spin-dependent quark structure function. Some values of λ_1 obey (59) as in the perturbative QCD case, but one of them becomes approximately equal to 0.5. This gives a possibility to attribute the singularities of the non-singlet structure functions measured in deep inelastic lepton-nucleon scattering to the singularities of the "free" quark structure functions.

5 Conclusion

We have found the analytical solution of the integral equation proposed in [7] to sum up all the double-logarithmic contributions to the non-singlet structure function $f_{NS}(x, Q^2)$ for the QCD running coupling. In general case the solution is given by the inverse Laplace transformation of the ratio of two Whittaker functions (see formulae (38), (39), (40), (41)). Analytical properties of the solution and a position of the rightmost singularity in the complex ρ-plane which determines the asymptotics of $f_{NS}(x, Q^2)$ at small x have been studied. The simple asymptotical formula (75) describing the small-x behaviour of the non-singlet structure function $f_{NS}(x, Q^2)$ valid when $\ln(Q^2/\Lambda^2) \gg 1$ has been obtained. It has been shown that λ (the logarithmic derivative of $f_{NS}(x, Q^2)$ over $\ln(1/x)$) goes at $x \to 0$ to some constant λ_1 not depending on Q^2. This prediction coincides with our previous numerical result [7]. For infrared cut-off parameter μ equal to Λ the constant $\lambda_1 = g = 8/(33 - 2n_f)$ and for $\mu > \Lambda$ λ_1 is smaller than g. The contribution of small transversal momenta ($|k_T| < 1$ GeV/c) to λ_1 is appreciable which points presumably to an importance of non-perturbative contributions to the small-x asymptotics of $f_{NS}(x, Q^2)$. This problem will be studied in the next paper.

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Figure captions

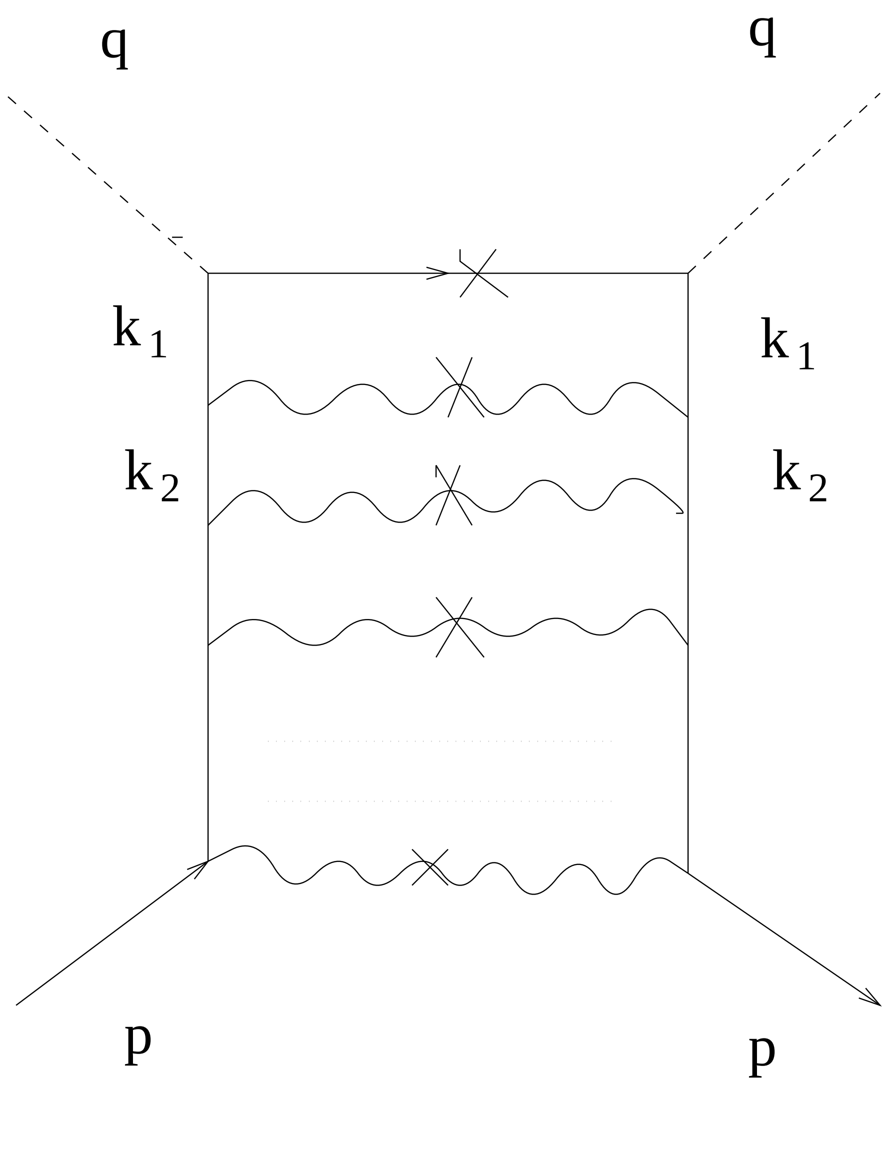
Fig. 1. Ladder graph for $f_{NS}(x, Q^2)$. Solid, wavy and dashed lines correspond to quarks, gluons and photons, respectively. Particles on mass shell are marked by crosses.

Fig. 2. Integral equation for $A(t, \beta)$. Solid and wavy lines correspond to quarks and gluons, respectively.

Fig. 3. Solutions of equation (54). For all curves $\Lambda = 0.2$ GeV/c, $n_f = 4$.

Fig. 4. Behaviour of non-singlet structure function at small x. Figures 4a and 4b show dependence of $f_{NS}(x, Q^2)$ on x and Q^2 for $\mu = 0.2$ GeV/c and $\mu = 1$ GeV/c, respectively. Figures 4c ($\mu = 0.2$ GeV/c) and 4d ($\mu = 1$ GeV/c) show ratios of $f_{NS}(x, Q^2)$ given by (75) to the non-singlet structure functions obtained from precise formulae (7), (38), (39), (40), (41). Solid, dashed, dotted and dash-dotted curves have been calculated for $Q^2 =$ 10, 30, 100, 300 $(\mathrm{GeV/c})^2$, respectively. For all curves $\Lambda = 0.2$ GeV/c, $n_f = 4$.

Fig. 5. Dependence of $\lambda = \partial \ln[f_{NS}(x, Q^2)]/\partial \ln(1/x)$ on Q^2, infrared cut-off parameter μ and x. Solid, dashed, dotted and dash-dotted curves have been calculated for $Q^2 = 10,\ 30,\ 100$ and 300 $(\mathrm{GeV/c})^2$, respectively. For all curves $\Lambda = 0.2$ GeV/c, $n_f = 4$.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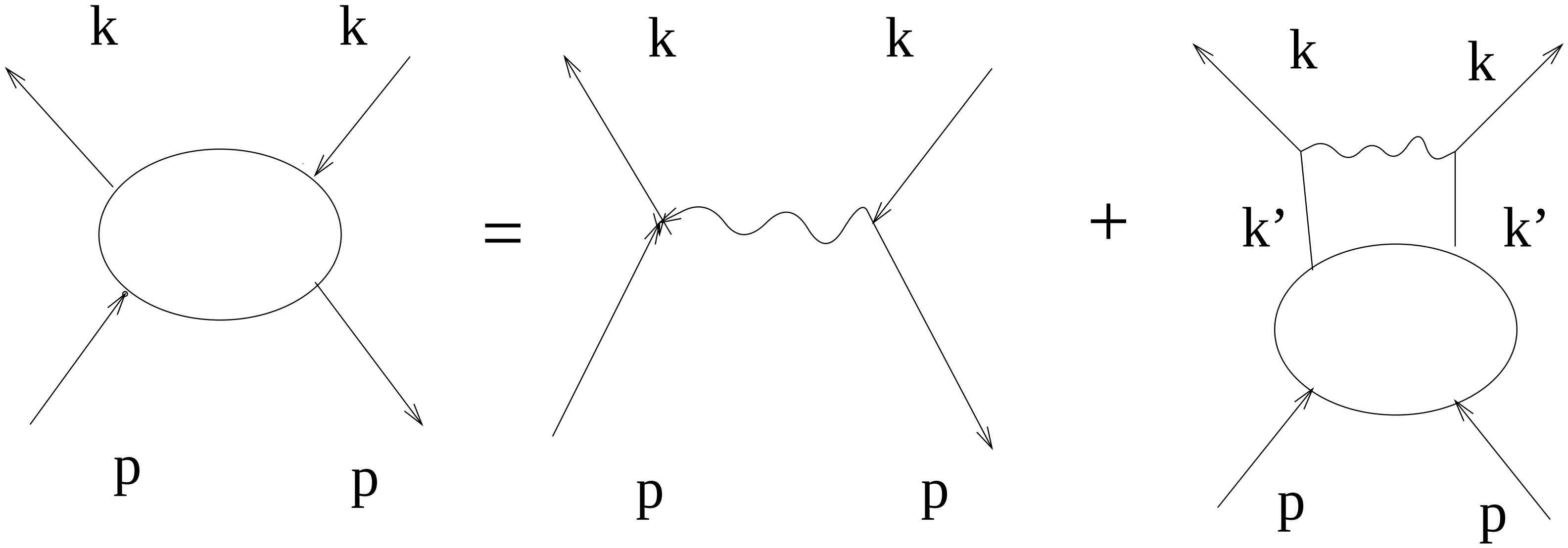
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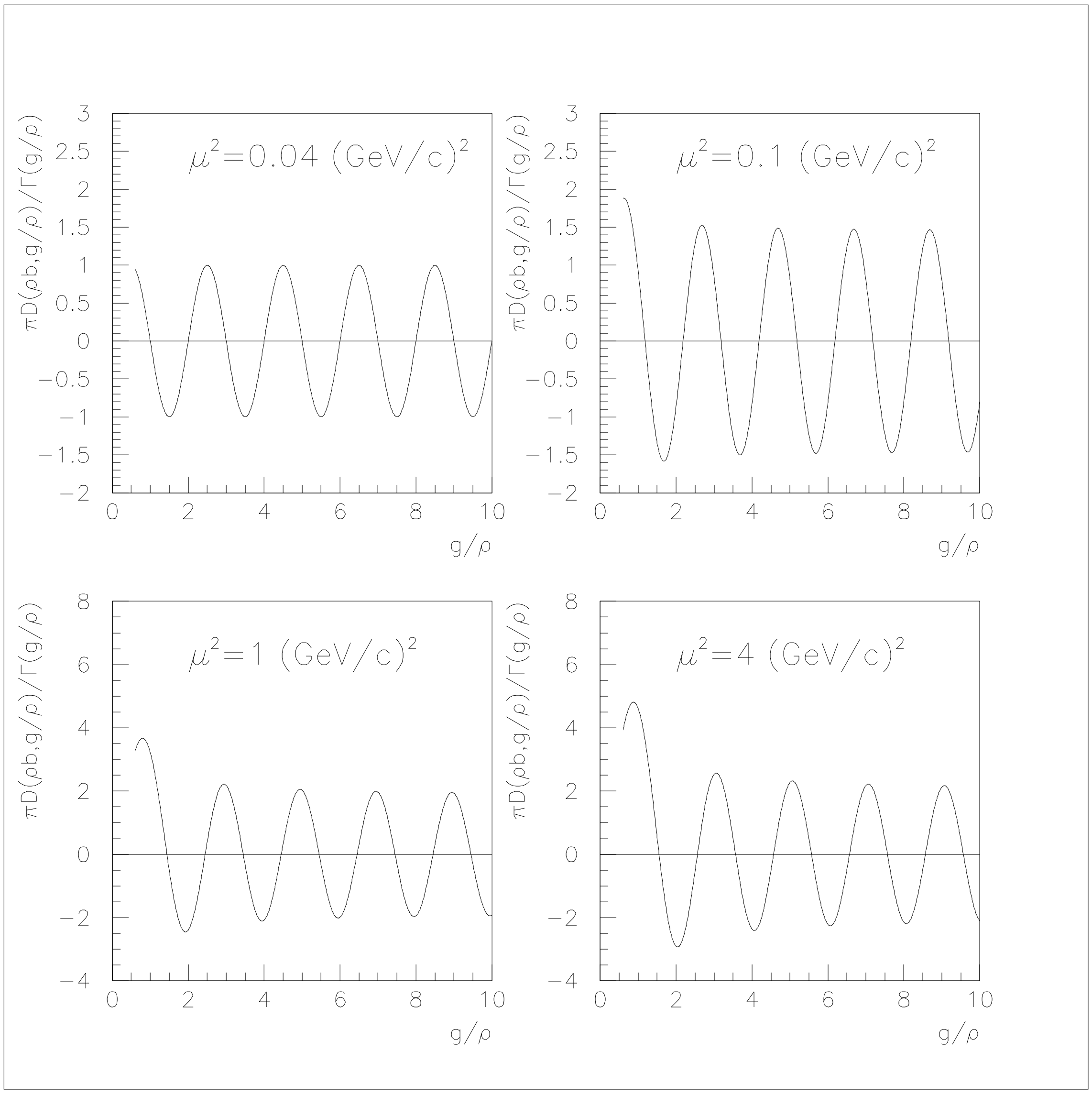
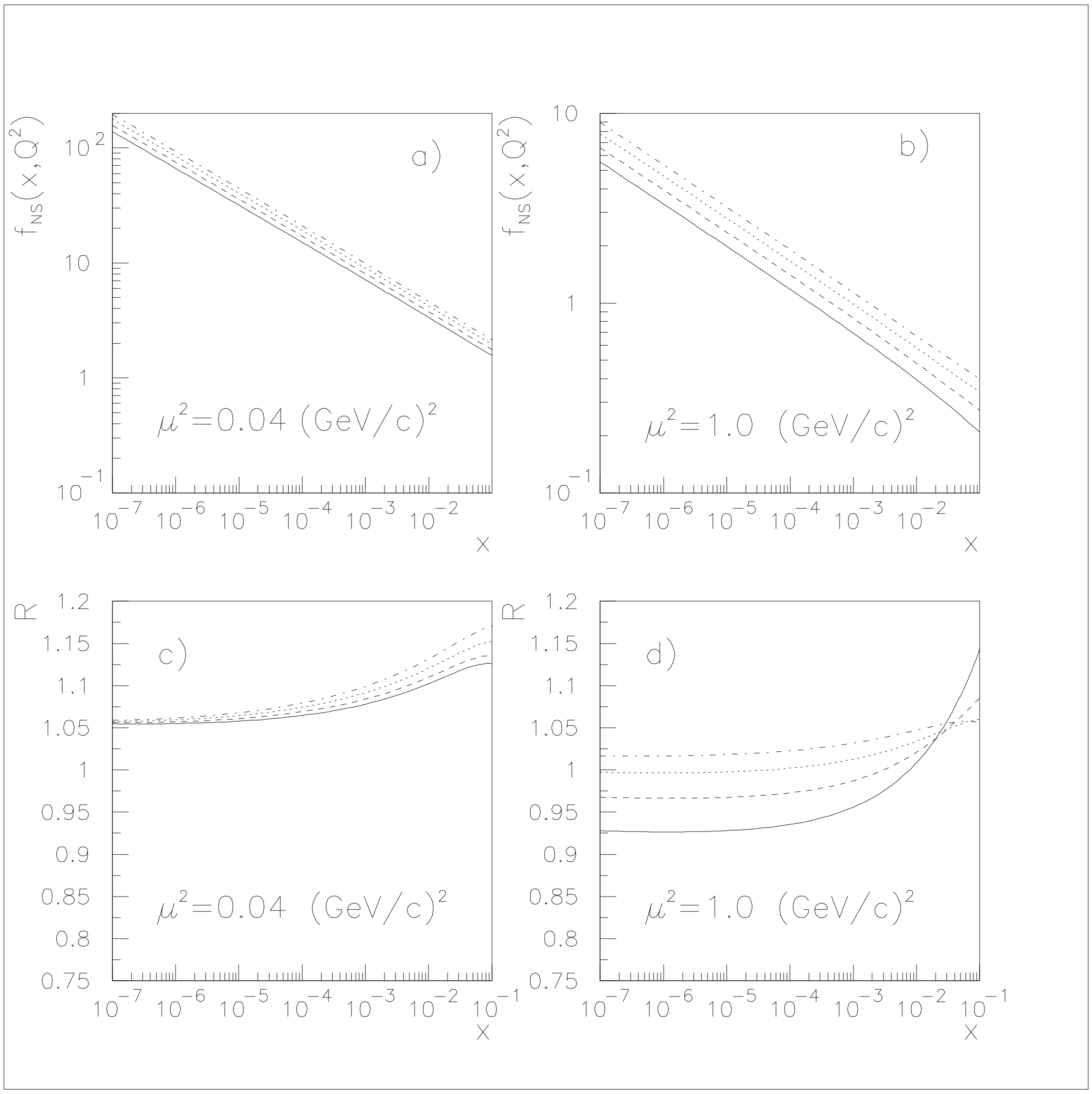
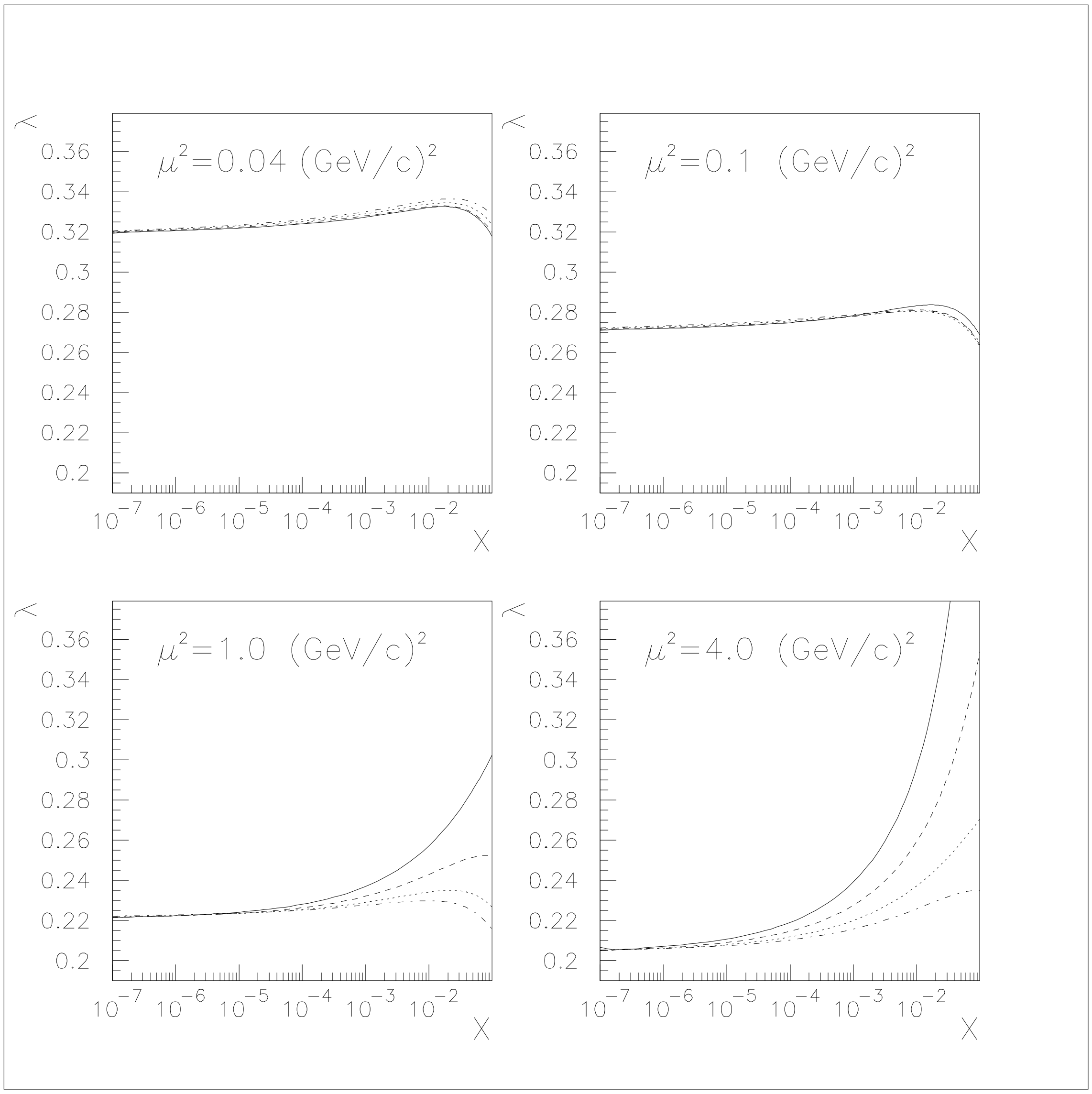


μ^2=0.04 (GeV/c)^2
μ^2=0.1 (GeV/c)^2
μ^2=1 (GeV/c)^2
μ^2=4 (GeV/c)^2
$\pi D(\rho b, g/\rho)/\Gamma(g/\rho)$
g/ρ



$f_{NS}(x,Q^2)$
a)
μ^2=0.04 (GeV/c)^2
b)
μ^2=1.0 (GeV/c)^2
R
c)
d)
x



λ
μ^2=0.04 (GeV/c)^2
μ^2=0.1 (GeV/c)^2
μ^2=1.0 (GeV/c)^2
μ^2=4.0 (GeV/c)^2
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2}
X