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ELEMENTARY PARTICLES AND FIELDS
Experiment

Some Special Features of the Fragmentation of a Relativistic Nucleus ^{11}B in Photoemulsion

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Abstract—Events where two doubly charged fragments are directly formed from a fragmenting nucleus ^{11}B of momentum 2.75 GeV/c per nucleon are separated, the cascade production of such fragments via the fragmentation channel $^8\mathrm{Be} \to 2\alpha$ being avoided. Where possible, the mass numbers of these doubly charged fragments are determined by using a signal from Coulomb scattering in photoemulsion. It is found that the measured fraction of the isotope ^{6}He is $(9.6 \pm 1.5)\%$, while its calculated probability is about 12%. The transverse-momentum distributions obtained experimentally for the isotopes ^{3}He and ^{4}He are compatible with a Rayleigh distribution characterized by constant values of 110.4 ± 6.0 and 127.5 ± 6.0 MeV/c, respectively. These features of the distributions agree with a purely statistical mechanism of the fragmentation of ^{11}B nuclei.

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1. INTRODUCTION

A procedure for separating individual fragmentation events in which ^{10}B nuclei of momentum 1.7 GeV/c per nucleon fragment through the channel involving a ^{8}Be nucleus was tested in [1]. The analysis performed there revealed that the cascade mechanism of the fragmentation of relativistic light nuclei does indeed exist and that the fraction of experimentally observed doubly charged fragments emitted by their unstable combinations, referred to as prefragments [2], is not small. In order to study the mechanism of relativistic-nucleus fragmentation, it is therefore necessary to separate channels involving the cascade fragmentation of the primary nucleus from channels where direct particle emission by this nucleus is the most probable. It is obvious that, while nuclei of the isotopes ^{3}He and ^{4}He can be emitted not only by a prefragment such as ^{8}Be but also by ^{5}Li and ^{5}He nuclei, nuclei of the isotope ^{6}He are most likely to be emitted directly from a relativistic nucleus ^{11}B.

The problem of classifying helium isotopes according to their mass numbers is not new. In a number of studies, including that which was reported in [3], the classification was performed on the basis of assessing the momentum of a particle by its multiple scattering in emulsion. However, this is a very cumbersome procedure. Moreover, the momentum of relativistic-nucleus fragments are only used to separate particles in mass numbers. As was shown in [4], the mass-number classification of particles alone can be reliably performed by using a much smaller volume of measurements along particle tracks. It is precisely this circumstance that made it possible to carry out the analysis reported here. The entire body of information about the experiment being discussed is contained in [5]. Therefore, many details concerning the measurements and experimental-data treatment are omitted here.

2. DESCRIPTION OF THE EXPERIMENT

The emulsion chamber used was exposed to a beam of ^{11}B ions accelerated at the nuclotron of the Laboratory of High Energies at the Joint Institute for Nuclear Research (JINR, Dubna) to a momentum of 2.75 GeV/c per nucleon along the emulsion layer. The angle between the ion beam and the emulsion plane proved to be 7 mrad. This contributed to accurately estimating the angles α in a plane orthogonal to the emulsion plane in the events coordinate frame where the x axis is aligned with the primary-particle momentum. Events were sought by viewing relevant tracks, whereby a statistical significance of the resulting sample was ensured. In all, 1928 inelastic interactions between the primary nucleus and emulsion nuclei were found over a length of 291 m. There were two doubly charged fragments in 373 events. Their identification usually presents no difficulties to an experience operator. The fragment emission angles

φ and α in the emulsion plane and in a plane orthogonal to it were estimated in these events by using the coordinates of points on particle tracks. This was done by means of the procedure described in detail elsewhere [1, 5]. It was verified that the distributions of the angles φ and α can be approximated by normal distributions characterized by the constants σ_φ and σ_α taking close values. The distribution of the azimuthal angle $\Psi = \arctan(\varphi/\alpha)$ is compatible with a uniform distribution over the range between zero and 2π. Upon excluding events proceeding through the formation of the ^{8}Be intermediate state, there remained 281 events where two doubly charged particles are emitted with a high probability directly from the ^{11}B nucleus. The advent of the procedure for separating helium isotopes that was described in [4] opened radically new experimental possibilities for studying the fragmentation of relativistic nuclei.

The basic idea of the study reported in [4] is that the problem of estimating the momentum of a particle or, more precisely, the quantity $p\beta c/Z$ for it and the problem of classifying relativistic particles according to their mass numbers at known Z are two substantially different problems of mathematical statistics, which are to be solved differently.

In the first problem, it is necessary to determine, in an independent experiment, the constant K for the Coulomb scattering of charged particles in emulsion [6]. We then have

$$p\beta c/Z = K\frac{t^{3/2}}{\sigma(D_\mathrm{C}, t)}, \qquad (1)$$

where t is the distance along the track in the coordinate X within the Fowler method [7] (in terms of this distance, the coordinate Y along the track is measured at N points) and $\sigma(D_\mathrm{C}, t)$ is a constant characterizing the normal distribution of the second differences,

$$D_k = (Y_{k-2} - Y_{k-1}) - (Y_{k-1} - Y_k),$$

that are due to the Coulomb scattering of a particle after it travels the distance t in the emulsion. One does not measure purely Coulomb scattering in an experiment; instead, there is always the sum of this useful signal and various random noises. The second difference due to purely Coulomb scattering can be found under the assumption that

$$\sigma^2(D_\mathrm{exp}) = \sigma^2(D_\mathrm{C}) + \sigma^2(D_\mathrm{noise});$$

that is, the measured coordinate Y is the sum of at least two random variables obeying each a normal distribution, their variances taking individual values.

Having determined the quantity $p\beta c/Z$ from relation (1), we can then find the mass number of a fragment. However, the need for eliminating random noises in performing measurements and for concurrently attaining a minimum error [8], which is

$$\delta(p\beta c/Z) = \frac{0.81\, p\beta c}{\sqrt{N}},$$

requires increasing the number N of the measured segments of the track being studied. For this purpose, one needs measurements of coordinates Y over the particle-track segment of length equal to a few centimeters, frequently in several layers of the emulsion chamber (see [3]).

To a high precision, the momentum p of a relativistic-nucleus fragment is equal to the product of the fragment mass number and the primary-nucleus momentum per nucleon. Therefore, the mass-number classification of particles alone can be based on the fact that, at given Z, the dependence $f[\sigma(D_2), t]$ of the standard deviation $\sigma(D_2)$ of the second differences directly observed in an experiment and distributed normally on the cell length t for different mass numbers of doubly charged fragments is accurately parametrized in the form of a second-degree polynomial. These dependences of f^A on t can readily be obtained in a given chamber for each mass number A at given Z.

The track being considered, for which the mass number x is not known, but for which the dependence $f^x[\sigma(D_2), t]$ has been determined experimentally, can be classified according to the following procedure. First, we specify the values of the function f^x at k points for the cell-length values of $t_1, t_2, \ldots, t_k$. At the same points, we then find the values of the function f^A at known mass numbers of doubly charged fragments. After that, we calculate the sum of the squares of the differences of the values obtained for the above functions at the k points in question, these differences being normalized to the error in their estimates. The square root of this sum is the distance R in space of dimensionality k. For a given fragment, the unknown mass number x is determined by minimizing the distance

$$R = \sqrt{\sum_{i=1}^{k}\left\{\frac{f^{A,i}[\sigma_i(D_2), t_i] - f^{x,i}[\sigma_i(D_2), t_i]}{S_i}\right\}^2} \qquad (2)$$

at $A = 3$, 4, or 6, where S_i is an estimate of the accuracy of the measurements at each of the k points on the two curves $f^A[\sigma(D_2), t]$ and $f^x[\sigma(D_2), t]$.

Key: 1. GeV

In practice, the dependence $f^A[\sigma(D_2), t]$ was obtained over tracks of length about 2 cm within a single layer, while the dependence $f^x[\sigma(D_2), t]$ was found only within a 6-mm segment of the track being considered.

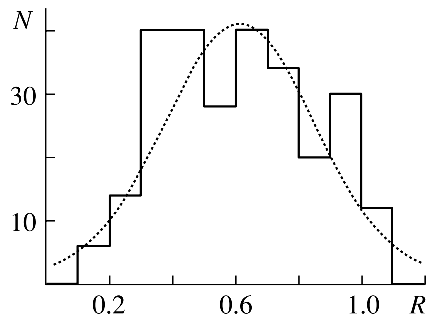

Fig. 1. Distribution of the dimensionless parameter R of the classification of doubly charged fragments for alpha particles.

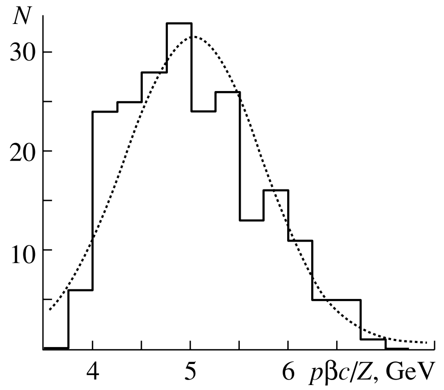

Fig. 2. Distribution of the quantities $p\beta c/Z$ for alpha particles.

The distribution of the minimum values of R, which were used to associate a doubly charged particle with one of three classes in accordance with its mass number, is given in Fig. 1. This distribution is compatible with a normal distribution at a mean value equal to the standard deviation and, hence, with an approximation by a Poisson distribution, which, in this case, yields the probability of observing, for example, an integral number of tenths of the continuous variable R. This means that the variable R is distributed at random in this experiment around its mean value.

For the already separated alpha particles at a specific value of $t = 800$ μm, the experimental estimates obtained for the known value of $p\beta c/Z = 5.0$ GeV are displayed in Fig. 2. It is clear that, if the measured value of the particle momentum is used to classify particles according to their mass numbers, then, under the conditions being considered, the distributions of the experimental estimates of the particle momenta will develop overlap regions, where these estimates for particles of different mass number will be identical. As a result, the classification according to this parameter will prove to be ambiguous.

We can now proceed to display the results obtained from the present analysis.

3. RESULTS

If the mass-number separation of doubly-charged fragments is not performed in an experiment, all of them are usually assumed to be alpha particles, even though a mixture of the isotopes ^{3}He, ^{4}He, and ^{6}He with different weights is present here. For a particle of mass number A_F, the variance of the distribution of the momentum projection onto an arbitrary direction Y in the transverse plane is given by [9]

$$\sigma^2(Y) = \sigma_0^2 \frac{A_F(A_0 - A_F)}{A_0 - 1}, \qquad (3)$$

where $\sigma_0^2 = P_{\rm F}^2/5$ is the variance of the momentum distribution of nucleons in a nucleus of mass number A_0 and Fermi momentum $P_{\rm F}$.

For our case, we obtain

$$\sigma^2(^4\text{He})/\sigma^2(^3\text{He}) = 7/6;$$

that is, the difference of $\sigma(^4\text{He})$ and $\sigma(^3\text{He})$ must not be great.

If we assume that the fraction of the isotope ^{4}He is larger than the fraction of any other helium isotope, then it is legitimate to consider all of the doubly charged fragments as alpha particles. However, this is not always so: according to [3], the fractions of the isotopes ^{3}He and ^{4}He in ^{6}Li fragmentation proved to be nearly identical.

In our experiment, the numbers of doubly charged fragments having mass numbers of $A = 3$, 4, and 6 proved to be 190, 216, and 43, respectively. The fraction of the helium isotope whose mass number is six was (9.6 ± 1.5)%. This fraction in the fragmentation of a ^{11}B nucleus can be estimated by calculating, as in [10], the probabilities of all 105 channels of the fragmentation of this nucleus. There are only three channels featuring a nucleus of the isotope ^{6}He and yet another doubly charged particle. With a probability of 2.9%, this is the channel

$$^{11}\text{B} \to ^6\text{He} + ^5\text{Li} \to ^6\text{He} + ^4\text{He} + p.$$

In this channel, ^{5}Li decays, with a high probability, to final states containing a helium isotope. The probability of observing a direct channel producing ^{6}He, ^{4}He, and a proton proves to be 3.2%.

The reason for comparatively high probabilities of these channels is that the energy threshold for them is relatively low, $\varepsilon = 15$ MeV. The kinetic energies T_1 and T_2 of secondary particles in the reference frame comoving with the center of mass of the fragmenting nucleus cannot be high since the variance of the momentum projection onto an arbitrary direction in this reference frame is given by (3). At the same scale of the proton and helium-isotope momenta, the kinetic energies of the helium-isotope nuclei are somewhat smaller on average than the proton kinetic energy.

In calculating the probability of any channel according to [10], the quantity

$$\Delta E = \varepsilon + T_1 + T_2$$

appears in the exponent of the Gibbs distribution at a temperature of about 8 MeV. As a result, the ratio of the sum of the probabilities for three channels of ^{11}B fragmentation that involve the production of the isotope ^{6}He to the sum of the probabilities of observing two doubly charged particles in the final state with allowance for their production through channels involving the formation of ^{8}Be, ^{5}Li, and ^{5}He proved to be 12%.

Only one channel involving the isotope ^{6}He is possible in the fragmentation of ^{10}B nuclei. This is the channel

$$^{10}\mathrm{B} \to ^6\mathrm{He} + ^3\mathrm{He} + ^1\mathrm{H}.$$

The energy threshold for this channel is about 28 MeV, its final state featuring three particles, including a proton. The sum of the kinetic energies in the reference frame comoving with the center of mass of the fragmenting nucleus and the reaction threshold (we denote this sum by ΔE) is so large that it reduces the probability of observing this channel to 0.63%.

The ^{11}B nucleus proved to be a unique source of the isotope ^{6}He. In all probability, no other nucleus can ensure so high a yield of this isotope.

Basic features of the resulting transverse-momentum distributions of helium-isotope nuclei are quoted in the table.

In the second row of the table, we present the constants of the normal distributions both for the angle φ and for the angle α, since these constants proved to be nearly identical in the two cases. The hypothesis that these distributions for three helium isotopes and all distributions given below are consistent with empirical distribution functions was tested by using three consistency criteria: the Kolmogorov criterion [11], the ω^2 criterion [11], and the less popular Kuiper criterion [12]. Their critical values at a 1% significance level are 1.67, 0.743, and 2.001, respectively. The experimental values for those distributions that are quoted in the present study are always smaller than these critical values (see table).

The results obtained here are also illustrated by the figures presented in this article. Figure 3 displays the transverse-momentum distribution of fragments identified as nuclei of the isotope ^{4}He, while Fig. 4 shows the distribution of the vector sum of the transverse momenta of two doubly charged particles in a single event. If the constant of the uniform distribution of two independent alpha particles in an event is $\sigma(P_\perp) = 127.0$ MeV/c, then the constant of the distribution of their vector sum in the case where the particles fly apart independently must be $\sigma(\sum P_\perp) = 180.0$ MeV/c. Its experimental value is 185 ± 9 MeV/c.

Table

Feature	^{3}He	^{4}He	^{6}He
Number of tracks	190	216	43
σ_φ, σ_α, mrad	13.4	11.6	9.9
$\sigma(P_\perp)$ for the χ^2 distribution	110.4	127.5	166.5
Kuiper consistency criterion	1.622	1.998	1.319
ω^2 consistency criterion	0.313	0.470	0.131
$\sigma(\sum P_\perp)$	154.75	184.94	247.3

An indication that particles originating from the fragmentation of relativistic nuclei are emitted independently was also obtained in [1]. It was shown there that the sum of the angles φ and α for two particles in a single event has a standard deviation $\sigma(\alpha_1 + \alpha_2 + \varphi_1 + \varphi_2)$ that is exactly twice as great as the value of σ for each of these angles, this corresponding to the case of two noncorrelated particles. However, they cannot be independent of each other if they arise in the decay of some excited system to two, three, or four particles. This fact fits well in the concept of the cold fragmentation of relativistic nuclei according to [13]. The illusion of independent emission arises because of choosing our two fragments among an indeterminate number of virtual particles, for which the momentum-conservation law holds, ensuring kinematical correlations. We merely do not see them.

If relation (3) holds, a determination of the transverse momentum and mass number of a fragment in an experiment makes it possible to find the quantity

$$X = \frac{P_0 \tan\varphi \sqrt{A_F}}{A_0 - 1}. \quad (4)$$

It must obey a normal distribution and have zero mean value and the variance $\sigma_0^2 = P_\mathrm{F}^2/5$. In [14], it was shown that the value found in this way by using more than 6000 fragments of a relativistic nucleus ^{22}Ne complies with the respective result of the experiment reported in [15] and devoted to determining the Fermi momentum in electron scattering on nuclei. The temperature of the degenerate nucleon state in a nucleus is $T = \sigma_0^2/m_N$, where m_N is the intranuclear nucleon mass.

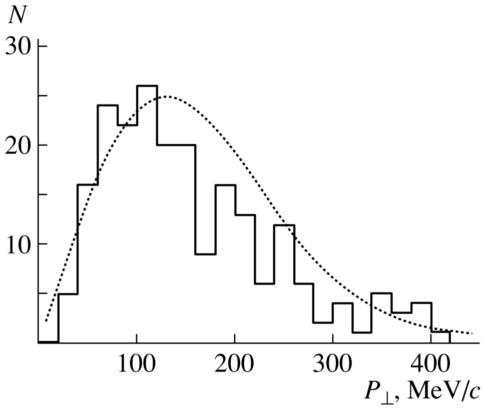

Fig. 3. Transverse-momentum ($P_\perp$) distribution of doubly charged fragments identified as alpha particles.

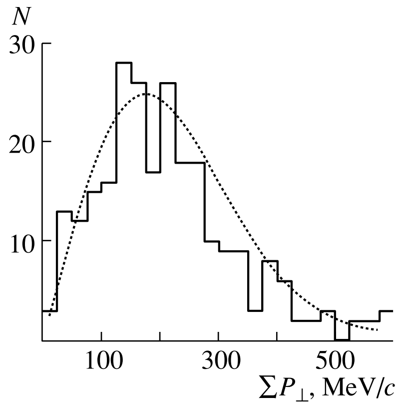

Fig. 4. Distribution of the vector sum of the transverse momenta $\sum P_\perp$ of doubly charged fragments.

The distribution of the quantities X that was obtained in the present experiment is displayed in Fig. 5. The experimental value of σ_0 proved to be 76.4 ± 2.5 MeV/c. It is used to calculate the absolute values of the probabilities of various channels of ^{11}B fragmentation according to the method developed in [10].

Since the transverse momenta and mass numbers of the helium isotopes are known in the present experiment, we can also estimate the kinetic energy of the transverse motion of the fragments. This energy is frequently used to determine the invariant mass and the excitation energy of the system emitting fragments.

4. ON THE INVARIANT MASS

In [16, 17], it is stated that the excitation energy of the system of fragments can be determined as the difference of the invariant mass of the fragmenting system and the mass of the primary nucleus. Moreover, even the excited levels of the primary nucleus

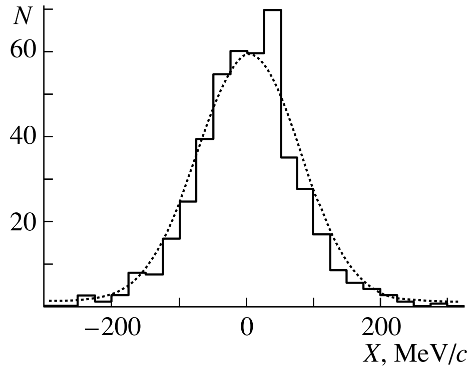

Fig. 5. Distribution of the quantity X for particles identified as ^{3}He, ^{4}He, and ^{6}He.

in the fragmentation process $^6\mathrm{Li} \to ^4\mathrm{He} + ^2\mathrm{H}$ were estimated in [17] by using this difference. The results of such investigations are presented in many articles of the BECQUEREL Collaboration [18]. Among other things, one can find there expressions for this excitation energy in terms of the invariant mass

$$M^{*2} = \sum(\mathbf{P}_i \cdot \mathbf{P}_k). \qquad (5)$$

The excitation energy is then given by

$$Q = M^* - \sum m, \qquad (6)$$

where $\sum m$ is the sum of the rest masses of the fragments in an event.

As a matter of fact, these formulas cannot be employed in photoemulsion studies. From expressions (5) and (6), it can be seen that the quantity of interest is found as the difference of two large numbers, on the order of 10^4 MeV. It follows that the relative error in the estimate of the invariant mass must be on the order of 10^{-4}. To this degree of precision, the sum of the fragment rest masses does not differ from the invariant mass in question. On the contrary, the invariant mass of the rho meson, for example, exceeds the sum of the masses of two pions, to which it decays, by a factor greater than 2.

On the basis of the value of M^* given by Eq. (5), one cannot always calculate the excitation energy by formula (6). While the invariant mass of any number of particles exists always, it is legitimate to treat the difference in (6) as the excitation energy of the system only in the case where the probability of observing the effective mass of this system is higher than the phase-space probability of observing it if the lifetime of the system in question is longer than the characteristic nuclear time. Apart from alpha-particle formation via the $^8\mathrm{Be} \to 2\alpha$ channel, whose existence was established in [1], no resonance states have so far been found in the system of fragments, although their existence cannot be ruled out conclusively.

The invariant masses determined for excited nuclei in a photoemulsion experiment have never been presented anywhere if for no other reason than the impossibility of finding them in experiments such as ours. In order to do this, it is necessary to know all four projections of the 4-momenta P_i and P_k, but, in experiments of this type, we know, for each of these 4-momenta, only two projections of the transverse momentum onto two arbitrary mutually orthogonal directions. By using these projections, one finds the transverse momentum and the kinetic energy of the transverse motion of a fragment. Instead of the sum of the total kinetic energies of the fragments in the reference frame comoving with their center of mass, one finds only the sum of their kinetic energies in the transverse plane, taking it for the excitation energy of the system of fragments.

For the decay process $^8\mathrm{Be} \to 2\alpha$, one employs, instead of the correct quantity $Q = M^*_{2\alpha} - 2m_\alpha$, the quantity

$$(P_\alpha^2/m_\alpha)\cdot\sin^2(\theta_{12}/2),$$

where θ_{12} in the angle between the momenta of two alpha particles in emulsion. It is obvious that, if $\theta_{12} = 0$, then the excitation energy defined in this way is also equal to zero. However, its true value is known to be always 92 keV, irrespective of the angle between the particle momenta in the laboratory frame.

For the reasons presented above, the excitation energy of the system of fragments has never been determined in emulsion experiments. By means of a simulation, one can easily show that, even if the system of fragments features a level or a resonance, we cannot obtain any maximum in the energy of the transverse motion of the fragments if we record only their transverse momenta. Nevertheless, groups of several events will inevitably arise in the interval extending up to 10 MeV if the number of events is commensurate with the number of histogram channels, as was the case in [17]. Since many light nuclei always have levels in this interval, a random coincidence of some of these groups with such a nuclear level is possible. Of course, all these effects will disappear with increasing statistical sample.

From the above formulas, it is clear that the integrated distribution of the probability of observing an experimental value in excess of given Q will be represented by a straight line on a logarithmic scale (it is similar to the integrated distribution of $P_\perp^2$, which is precisely the quantity that determines the value of Q in the present experiment).

In just the same way, emulsion data do not give any new information about the distance between the particles of a pair in the space of 4-velocities either

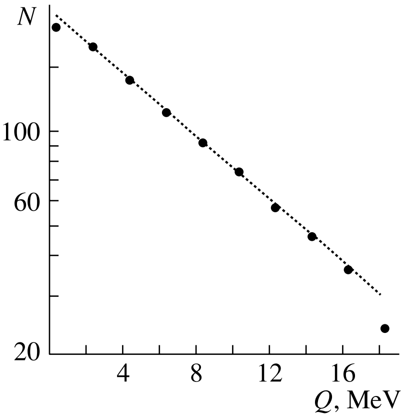

Fig. 6. Integrated distribution of the quantity Q defined as the sum of the kinetic energies of doubly charged fragments in the transverse plane for events where the mass numbers of helium isotopes have been determined.

(in [19], it was proposed to classify nuclear interactions in terms of this quantity),

$$b_{ik} = -(u_i - u_k) = 2\left(\frac{E_i\,E_k - p_i p_k \cos\theta_{12}}{m_i m_k} - 1\right), \tag{7}$$

since, in our experiment, we cannot determine the total energies or the total momenta of secondary particles. Replacing them by the respective quantities in the transverse plane, one finds the following experimental estimate of this distance for two alpha particles:

$$b_{ik} = \left(\frac{2P_\alpha}{m_\alpha}\sin\frac{\theta_{12}}{2}\right)^2. \tag{8}$$

In just the same way as in the preceding case, we arrive in practice at a distribution of the angle θ_{12} between the tracks of the fragments.

Let us return to our experiment. Its distinction from other experiments is that the mass numbers of doubly charged fragments are known here. In general, the masses of doubly charged fragments in an event are therefore different in this experiment. We estimate Q as the sum of the kinetic energies of two particles in the transverse plane. The result is presented in Fig. 6 in the form of the Q dependence of the number of particles for which the value of Q is in excess of a given value (on a logarithmic scale)—that is, the expected linear dependence is indeed confirmed by the experiment. However, this is not the excitation energy but the familiar integrated dependence for observing the square of the particle transverse momentum in excess of a given one. The last circumstance is due to the consistency of the distribution of transverse momenta themselves with the χ^2 distribution, as has already been demonstrated in Fig. 3.

In relation to the result in Fig. 3, neither the distribution with respect to the excitation energy Q nor the distribution with respect to the invariant variable b_{ik} from photoemulsion experiments contains any new information.

5. CONCLUSIONS

The use of the proposed method for classifying helium isotopes according to their mass numbers has made it possible to find the fractions of the isotopes ^{3}He, ^{4}He, and ^{6}He in a chamber irradiated with ^{11}B ions of momentum 2.75 GeV/c per nucleon. The experimental value obtained for the fraction of the isotope ^{6}He is in agreement with its calculated value.

Transverse-momentum distributions for groups of doubly charged fragments are in agreement with the distributions expected within the simplest statistical model of relativistic-nucleus fragmentation. The experimental-data set used from which we have removed events proceeding via the channel $^8\mathrm{Be} \to 2\alpha$ in the intermediate state shows no indications of kinematical or dynamical correlations of the transverse momenta of fragments. Everything looks as if two doubly charged particles escape from a ^{11}B nucleus in a transverse direction independently of each other.

We have shown that the constant σ_0 that determines the momentum distribution of nucleons in the ^{11}B nucleus prior to its interaction with an emulsion nucleus and cold fragmentation does not differ from its counterpart in the ^{10}B nucleus.

These results indicate that the method used here to classify helium isotopes according to their mass numbers is quite efficient and that our ideas of the mechanism of relativistic-nucleus fragmentation are basically correct. In order to obtain deeper insight into this mechanism, it is necessary to separate cascade fragmentation proceeding through various prefragments from direct fragmentation to observed particles.

The idea that a relativistic nucleus fragments through the stage of excited-nucleus decay is at odds with the experimental fact established here that doubly charged fragments are emitted independently. A different channel for the transfer of energy necessary for this is indicated in [13, 20].

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