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DETERMINATION OF RELATIVISTIC NUCLEI LIGHT FRAGMENT CHARGES IN PHOTOEMULSION.

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ABSTRACT.

The new method of relativistic nuclei light fragment charges determination in emulsion is proposed. This method is based on blob sum length measurement and allows to control the measurement procedure and to estimate the fragment charges with high reliability. The method was sampled for light fragment charges determination in the cases of 4.1 A GeV/c neon and 160 A GeV lead projectiles.

1 Introduction.

There are several charge determination methods in nuclear emulsion: the grain number count, the count of number of gaps with length larger than predetermined one and $\delta-$electron density determination. In the emulsion chambers exposed with the light nuclei such as ^{22}Ne [1],^{6}Li [2] there were so-called "pure" decays of the projectile-nuclei. So it was easy to determine the fragment charges using the density of $\delta-$electrons. In such events the sum of the fragment charges is equal to the charge of the initial projectile. Thus, there was a possibility to calibrate the method in the whole region of the fragment charges from 1 to the projectile charge Z .

There is no such possibility for hard relativistic nuclei as uranium, lead, gold. The density of $\delta-$electrons becomes very high and some part of them disappears in the thickness of the track. So we must count the number of $\delta-$electrons with the length more than the predetermined value. The count of $\delta-$electrons becomes to be depended on the operator. It is impossible to estimate the fragment charge with an accuracy to unity.

For the light fragments ($Z = 2,\ 3$) the density of $\delta-$electrons is small , so one should count them at the length of track about several centimeters. The background contribution increases.

When we use the $\delta-$electrons density for charge estimation we round off the charge value to the nearest integer. But there is a probability for this charge value to be smaller or greater by unit. If we have the charge value about 2.6, for example, we suppose this charge to be equal to 3 charge units, but, however, there is some probability for this charge to be equal to 2 charge units. Such probability has nowhere been determined. Furthermore, one supposes sometimes [1] that the charge can be measured with the accuracy 0.5 charge unit, but it is well known that it must be the whole value only.

In this case the count of number of gaps with length larger than predetermined one becomes more effective than the count of $\delta-$electrons. The whole information about the track structure can be obtained from the blob spectrum.

It was shown in [3] that from the blob spectrum on the proton tracks with known range it is possible to estimate the ionization G of particle up to ten times larger than the minimal one for the single-charged particle. In the case of relativistic fragments the problem of charge estimations at small Z with blob spectrum is more simple compared with the problem of charge estimation resolved in [3]. Now it is enough to differ 1, 4, 9...$-$multiple ionization losses one from another. This is the typical problem of the object classification with their signs [4]. We can resolve it in such a way that the reliability of fragment charge estimation will be sufficiently high and known for every fragment. Let us show how to do this.

2 Solution of the problem.

It is well known [3], [5], that the density function of blob lengths b is exponentially distributed with the constant Λ

$$f(b) = \frac{1}{\Lambda} e^{-b/\Lambda} \ . \tag{1}$$

By measurement of K blobs on the track one can obtain the estimate of Λ with maximum likelihood method as $\Lambda = (\sum_{i=1}^{i=K} b_i)/K$ with the dispersion $\delta_\Lambda^2 = \Lambda^2/K$ [6]. In [3] it was found the empirical dependence of $Y = \ln\Lambda$ on $X = \ln G$ in polynomial form

$$Y = a_0 + a_1 \cdot X + a_2 \cdot X^2 \tag{2}$$

with coefficients $a_0 = -0.57,\ a_1 = 0.23,\ a_2 = 0.41.$ The errors of this coefficients is about 10 %. For $G = 1,\ 4,\ 9$ the estimates of $\Lambda_1,\ \Lambda_2,\ \Lambda_3$ for the fragments with charges $Z = 1,\ 2,\ 3$ will be equal to $0.566,\ 1.7,\ 6.7\ \ \mu$m respectively.

After the measurement of K blob lengths on the track of fragment, which charge Z we want to determine, the sum of blob length is $B_K = \sum_{i=1}^{i=K} b_i$. The expected value of this sum is $< B_K >= \Lambda_Z \cdot K,$ and the density function $x = B_K$ is

$$f(x, \Lambda_Z) = \frac{1}{\Lambda_Z^K \cdot \Gamma(K)} \cdot x^{K-1} \cdot e^{-x/\Lambda_Z} \ . \tag{3}$$

This is the Erlang distribution of order K [7]. There is maximum at $x = \Lambda_Z \cdot (K-1),$ but this distribution is not symmetrical with respect to this point.

The distribution function B_K is

$$P(B_K, \Lambda_Z) = \frac{1}{\Lambda_Z^K \cdot \Gamma(K)} \int_0^{B_K} x^{K-1} \cdot e^{-x/\Lambda_Z} \cdot dx \ , \tag{4}$$

where the integral from 0 till B_K is incomplete gamma-function multiplied by Λ_Z^K. It is possible to resolve any problem related to the observation probability of B_K value if one know the density function and the distribution function. If ε is some infinitesimal quantity, the inequalities $P(B_K, \Lambda_Z) < \varepsilon$ and $1 - P(B_K, \Lambda_Z) < \varepsilon$ are unlikely. According with the base classification idea [4] the most reasonable value Z is one for which the modulus of difference $B_K - \Lambda_Z \cdot K$ has the minimal value. When K (number of blobs measured) increases, the B_K value tends to its expected value nearby the mode of distribution , but the relative thickness of distribution decreases as $1/\sqrt{K}$. It follows that when K increases, the value $\varepsilon_K = 1 - P_K,$ where P_K the reliability of charge estimation, uniformly tends to zero.

It is evident, that the observation probability of B_K value in dB_K interval for any of three values $Z,\ Z-1,\ Z+1$ is equal to the ratio of it's density function (3) at $x = B_K$ to the sum of three such densities at the same x.
Denoting $c_1 = \Lambda_Z/\Lambda_{Z-1},\ c_2 = \Lambda_Z/\Lambda_{Z+1}$, one obtains for the reliability of the fragment charge estimate close to the expected value of the blob length sum a simple expression

$$P_K = \frac{1}{1 + c_1^K \cdot e^{-K\cdot(c_1-1)} + c_2^K \cdot e^{-K\cdot(c_2-1)}}\ . \tag{5}$$

It is evident that when K increases, the reliability of Z classification uniformly tends to 1. That is, for any, as small one likes value ε, there exists the K value for which $1 - P_K = \varepsilon_K$ is less than ε.

3 Calibration.

This method was sampled for light fragment charges estimation in the case of ^{22}Ne at 4.1 A GeV/c and ^{207}Pb at 160 A GeV.

The blob length, width and square spectra were obtained for the ^{22}Ne fragments with charges from 1 till 8. The integral blob length distributions for the tracks of fragments with charges 2,3,4,6 are presented at fig.1. At OY axis there is the probability that the blob length is smaller than predetermined value.

The same integral distributions for ^{207}Pb fragments with charges Z=2,3,5,6,9 are presented at figure 2. It is shown that the number of blobs decreases in the exponential form according to the blob length increasing, beginning from some L value (about 1 μm). All the fragment charges were previously determined by the count of $\delta-$ electrons and were calibrated with respect to $\alpha-$ particle charge. The estimation of Λ value with maximum likelihood method is equal to average blob length for every charge (in the case when the density function of blob length is exponentially distributed). Such Λ estimations were obtained by measuring of 500 blob length values at the fragment tracks with charges Z=1,2,3,5,6,9. Parameter Λ has a parabolic dependence of Z as :

$$\Lambda = a_0 + a_1 \cdot Z + a_2 \cdot Z^2 \tag{6}$$

The coefficients were found to be: α_0=0.000, α_1=0.327, α_2=0.074. In the case of slow protons the average blob lengths for ionization G=1,4 and 9 were found to be 0.566, 1.7 and 6.7 microns [3]. But in the case of relativistic lead fragments with charges Z=1,2,3 ones were found to be (0.475$\pm$0.016),(1.00$\pm$0.04), (1.48$\pm$0.05) μm. So one suggested that there is a different structure of tracks with the same ionization: the tracks of slow protons with β=0.41 and 0.25 and the tracks of relativistic particles with charge 2 and 3.

To verify an assumption that there is ionization energy loss into to the blob width H, the blob square and blob width distributions have also been investigated.

It is shown that the blob width distribution does not contradict to Gauss distribution and the blob width varies very slow in the fragment charges range from 2 till 10 (see fig.4). So, the mean value of blob width for the track of fragment with charge Z=2 is (0.72±0.11) μm, and for charge Z=10 one equals (0.87±0.11) μm only. The blob width increasing is due, in general, to rise of number of $\delta-$electrons but the width of "pivot" in itself grows very slow with charge increasing.

So integral distributions of blob squares are similar to ones of blob length. And the efficiency of charge separation of blob square parameter is the same as blob length one.

4 Practical usage.

Thus, we have the following experimental average blob length values in microns for the charges from 1 till 9: Λ_2=(1.00±0.04), Λ_3=(1.48±0.05), Λ_4=(2.49±0.09), Λ_5=(3.33±0.12), Λ_6=(4.87±0.17), Λ_7=(5.92±0.21), Λ_8=(7.35±0.26), Λ_9=(8.86±0.32).

The fragment track of unknown charge is set along OX axis. There is a request to operator to input the number of blobs K to be measured. Operator inputs the number of blobs, for example 30, and measures coordinates of the bounds between blobs and gaps, which are recorded in the computer memory. If K blobs are measured the sum length B_k is determined. After that the minimum of modulus of $(B_K - \Lambda_i \cdot K)$ value, which is the difference between the measured blob length sum and expected one for the charge Z_i, is found. As a result, we have on computer display the following information: the number of blobs measured (K), the average blob length and the admixtures of nearest charges. If the admixtures are great, operator continues the charge determination procedure by measuring an additional number of blobs. As a result the admixtures decrease and the reliability of charge estimation increases.

The charges of 661 fragments in 122 ^{207}Pb interactions were determined using this method. It was shown in practice that by measuring of 30 blobs for double-charged fragments the maximal admixture of charge with Z=1 is (1-3)%, and one with Z=3 is (7-8)%. For 40 blobs measured the one-charged admixture is about 0.6%, and three-charged admixture about 5%. For 50 blobs measured the one-charged admixture is only 0.005% and three-charged admixture is (2-5)%. At further increasing of number of blobs (60,80) the one-charged admixture is equal to zero, but three-charged admixture remains at the same level. Thus, for charge estimation of double-charged fragments it is enough to measure about 40 blobs at the track.

To determine the charge of three-charged fragment the optimal number of measured blobs is about 40-50. In this case the admixture of charge Z=2 does not exceed (1-3)% and admixture of Z=4 about (5-6)%. Charges of fragments with Z=4 and Z=5 would be estimated by measuring about 50-60 blobs. But the charge which equals to 6 would be determined by measuring the sum length of 80 blobs. The admixture of Z=5 varies from 0.3% till 10% and admixture of Z=7 is not greater than 10%. The further increasing of number of blobs leads to decrease of admixtures and to increase of reliability. However, it is hardly reasonable to measure more than 100 blobs because of high measurement time.

So, utilization of this method has a number of advantages over previously used ones. It does not depend on operator. It is a kind of "on-line" method and a charge of fragment is already known during the measuring procedure. Furthermore, the reliability of this charge estimate and admixtures of nearby charges are also known. And operator can control a measurement process and to exceed the charge estimate reliability (in definite limits, of course). Thus, this method is more convenient and labour-saving compared with other ones.

We should like to thank L.N. Tkatch who performed the measurements.

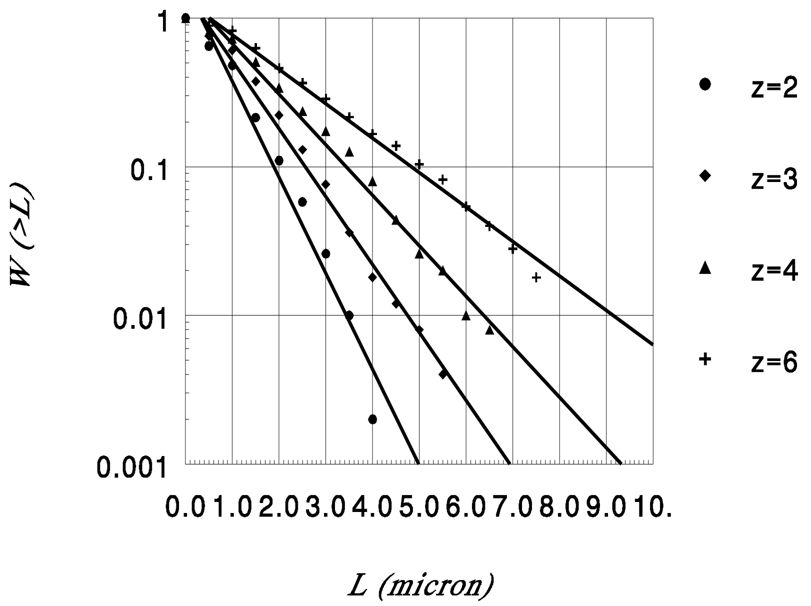

Fig.1. Integral blob length distributions for neon fragments.

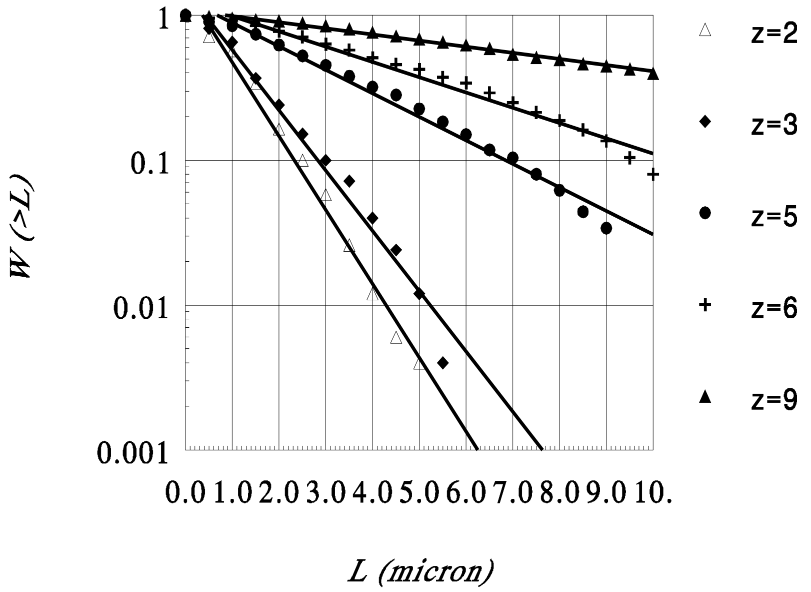

Fig.2. Integral blob length distributions for plumbum fragments.

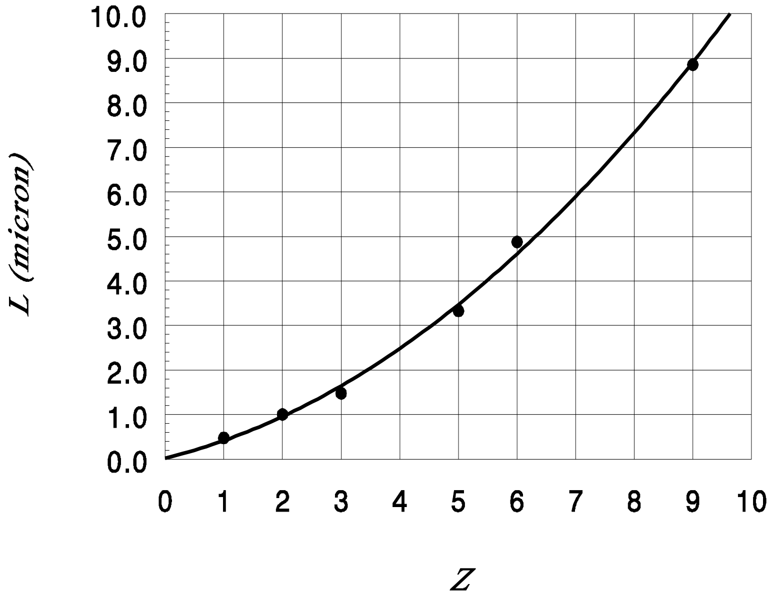

Fig.3. Blob length dependence of plumbum frarment charge.

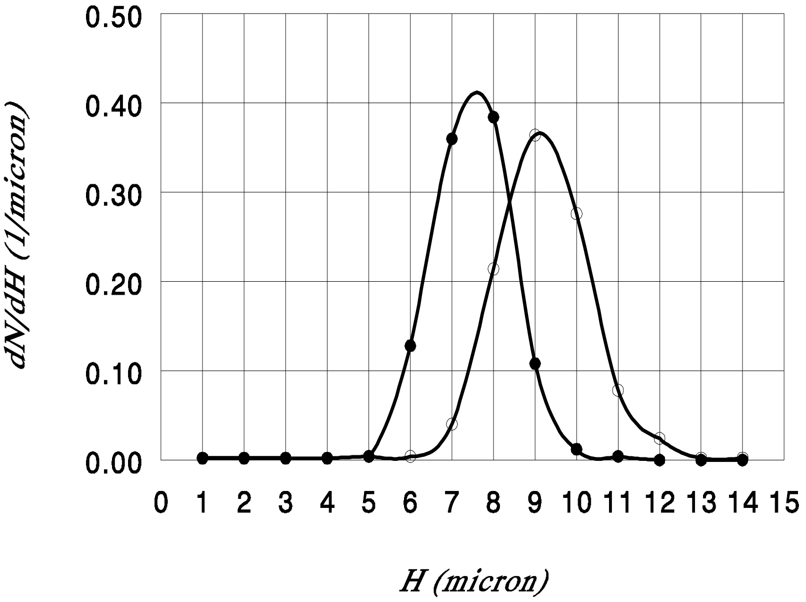

Fig.4. Blob width distributions for plumbum fragments with Z=2,10.

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