

Fidelity and Wilson loop for quarks in confinement region

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Content

Introduction

Wilson loop and stochastic vacuum in QCD

Fidelity in HQC

Gauss dominance in systems with noise

Chaos criterion in QFT, fidelity, SSB

Squeezing, entanglement, instability in evolution

Conclusion

Introduction.

Chaos intervention in physics.

- Phenomenon of chaos attracts much attention in various fields of physics.
- Originally it was associated with problems of classical mechanics and statistical physics. Substantiation of statistical mechanics initiated intensive study of chaos and uncover its basic properties mainly in classical mechanics.
- One of the main results in this direction is a creation of KAM theory and understanding of the phase space structure of Hamiltonian systems. || **A.N. Kolmogorov, DAN SSSR (1954)**
|| **V.I. Arnold, Izv. AN SSSR (1961)**
- It was clarified that the root of chaos is local instability of dynamical system. Local instability leads to mixing of trajectories in phase space and thus to non-regular behavior of the system and chaos. || **N.S. Krylov (1950)**
|| **G.M. Zaslavsky, R.G. Sagdeev, (1984)**
|| *Intr. to Nonl. Phys.*
- Large progress is achieved in understanding of chaos in semiclassical regime of quantum mechanics via analysis of the spectral properties of the system. || **T. Prosen, M. Robnik (1994)**
|| **B.V. Chirikov (1992)**
- Role of chaos in QFT and HEP is a kind of challenge

There are a lot of footprint of chaos here:

- chaotic solutions of classical Yang-Mills field equations of all fundamental interactions.

Chaos and order in classical YMH models ||**Savvidy PL (1983)**,
||**Kawabe PRD (1983)**

- Investigation of the stability of classical field solutions faces difficulties caused by infinite number of degrees of freedom. That is why authors often restrict their consideration by investigation of some model field configurations.

|| **Kawabe, Ohta (1991)**

- Chaos assisted quantum tunneling: probability of tunneling between wells increases by several orders in presence of classical driving chaotic force ||**Lin et.al. PRL (1990)**

- Chaos assisted instanton tunnelling ||**Kuvshinov, Kuzmin,**
||**Shulyakovsky (2002)**

- quantum footprints of classical chaos in nuclear physics (energy level spacing distribution) and stochastic billiards

||**Zaslavskii, Sagdeev,**
|| **Introduction in nonlinear physics (1988)**

In semiclassics Gutzwiller formula gives connection between level spacing and classical phase trajectories

- Chaos simulates confinement || **Savvidy, PL (1977)**
- Higgs field lead YMH system to order (in classics)
- Quantum fluctuations of YM field lead the system chaos-order transition || **Kuvshinov,**
|| **Kuzmin,(2001)**
- Intermittency phenomenon in HEP
- Chaos and squeezing are connected: roughly :
the more chaos – the more squeezing || **Alekseev, Perina**
|| **chao-dyn/9804041**
- Coexistence of C. and S.
(see below) || **Kuvshinov, Marmysh,**
|| **Shaparau (2002)**

chaos theory is developed in:

- classical mechanics with finite number degrees of freedom and statistical physics
- in semi-classical regime of QM

- there are papers devoted to chaos in quantum field theory. But there is no generally recognized definition of chaos for quantum fields. This fact restricts use of chaos theory in field of elementary particle physics.

|| T.S. Biro, B. Muller, S.G. Matinyan (1991)

- the definition was given in **||Kuvshinov, Kuzmin, PLA (2001)**

where new quantum chaos criterion was suggested for QFT in terms of Green function, true also for QM and which in classical limit goes to known Toda criterion.

Wilson loop and stochastic vacuum in QCD

- $W(c) = P \exp(ig \oint_c dz_\mu A_\mu^a t^a)$, $T_r W(c)$ - gauge invariant || Yu.A.Simonov 1996, 2004
 $A_\mu = A_\mu^a t^a$, - stochastic ansamble in vacuum over which we should average || "Uspekhi"

- Averaging + non-abelian Stocks theorem

$$\langle T_r W(c) \rangle = \left\langle \frac{1}{N_c} P T_r \exp(ig \oint_C A_\mu dz_\mu) \right\rangle = \frac{1}{N_c} \langle P T_r \exp(ig \int_S dr_{\mu\nu}(z) G_{\mu\nu}(z, x_0)) \rangle$$

$$G_{\mu\nu}(z, x_0) = \Phi(x_0, x) F_{\mu\nu}(x) \Phi(x, x_0); \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu A_\nu]$$

- Van Kampen decomposition $\implies \langle T_r W(c) \rangle = \exp \sum_{n=2}^{\infty} (i^n) \Delta^{(n)}[s]$

$$\Delta^{(2)}[s] = \frac{1}{2} \int_S d\sigma_{\mu\nu}(z_n) \int d\sigma_{\rho\sigma}(z_2) g^2 D_{\mu\nu\rho\sigma}^2(z_1, z_i, z_0); \quad D_{\mu\nu\rho\sigma}^2 = \langle T_r G_{\mu\nu} G_{\rho\sigma} \rangle$$

- Asumptions: Gauss dominance (stochastic vacuum)

$$\Delta^2[s] \gg \Delta^n[s], \quad n > 2, \quad \text{or} \quad gFT_g^{(2)} \ll 1; \quad T_g - \text{correlation length}$$

- $\langle T_r W(s) \rangle = \exp[-V(r)T] \implies \exp(-\sigma S_{min})$, $\sigma = \frac{1}{2} D(x) d^2x$; $V(\mu) = \sigma \mu$

"Area law" - string tension - confinement; $g^2 D_{\mu\nu\rho\sigma}^2 = () D^2(z^2) + () D_1(z^2)$; $D(z^2) \sim \exp\left(\frac{-|z|}{T_g}\right)$

- T_g - correlation length - "domen size"; σ - string tension \implies stochastic vacuum leads to confinement

Fidelity in Holonomic Quantum Computations

- $F(t)$ has been proposed first by Peres as a measure of stability of quantum motion and then measure of the loss of decoherence in QC; || **A.Peres. 1984.**
 $(F|t|)^2$ - "Loschmidt echo" (Pastawski H), "hyperseasitivity" (Shack R) || **I.L. Chuang 2000**

- $F(t) = \langle u_\delta^{-1} u^t \rangle$, δ - perturbation, $\langle \ \rangle$ in initial state, $F(t) = 1$ - stable motion,
 $0 < F(t) < 1$ - unstable motion

- HQC: Non-abelian holonomics used for quantum gates in subspace C^N on degenerate eigenvalue of isospectral Hamiltonian $H(\lambda)$, $\lambda = (\lambda_\mu)$ - control parameters: || **P.Zanardi 1999, 2000**

$$\Gamma_\gamma(A_\mu) = P \exp i \oint_\gamma A_\mu d\lambda_\mu, \quad (A_\mu)_{mn} = \int d^3x \Psi_m^*(x, \lambda) \left(-i \frac{\partial}{\partial \lambda_\mu}\right) \Psi(x, \lambda)$$

- Fidelity HQC

$$F(t) = \text{tr}(\rho \Gamma_\gamma^{-1} \Gamma_{\gamma_0}) = \text{tr}(\rho P \exp[i \oint_{\delta\gamma} A_\mu d\lambda_\mu])$$

γ_0 - adiabatic loop, γ' - actual loop with some errors, $\delta\gamma = \gamma'^{-1} \cdot \gamma_0$

FHQC - is mathematically similar to Wilson loop in QCD

"Area law" (confinement) of WL corresponds to exponential decreasing of F || **V.Kuvshinov, A.Kuzmin, 2003**

- instability of quantum system

- HQC by analogy with QCD non-abelian Stocks theorem gives

- $F(t) = T_r \rho P \exp(i \int_{\delta S} d\sigma_{\chi\rho}(z) G_{\chi\rho}(z, x_0))$

$$G_{\chi\rho} = \Phi F_{\chi\rho} \Phi; \quad F_{\chi\rho} = \partial_\chi A_\rho - \partial_\rho A_\chi - [A_\chi, A_\rho], \quad \Phi = P \exp i \int_z^{x_0} A_\mu d\lambda_\mu$$

for $\delta\lambda_\mu < \|A_\mu\|^{-1}, \quad |\delta\lambda_\chi \lambda_\rho| < \|F_{\chi\rho}\|^{-1}$

$$F(t) \approx 1 + iT_r \rho F_{\chi\rho} \delta\lambda_\chi \delta\lambda_\rho + \dots \quad \left| \frac{\delta F}{\delta S} \right| = T_r(\rho F_{\mu\nu})$$

The less curvature tensor leads the less deviation F(t) from unity = more stable

- Let γ_0 - the same, but $A_\mu \longrightarrow A'_\mu = A_\mu + \delta A_\mu \implies \delta A_\mu = F_{\mu\nu} \delta\lambda^\nu$ || **Buividovich, Kuvshinov, 2004**

$$F(t) = T_r \rho \exp \left[\int \Phi(\lambda, \lambda_0) F_{\chi\alpha} \lambda^\alpha \Phi(\lambda_0, \lambda) \frac{d\lambda^\chi}{ds} ds \right]$$

- Gauss dominance:decomposition, averaging over all possible errors

Assumptions: $\sqrt{(\delta A)^2} \cdot \lambda_{corr} \ll 1$ or $\sqrt{G^2 \delta\lambda^2} \cdot \lambda_{corr} \ll 1$, λ - correlation length

$$\overline{F}(t) = T_r \rho \exp(-\lambda_{corr} \int_{\gamma_0} \sigma^2 ds) \quad \sigma^2 = \delta A^1 \delta A^1$$

Gauss dominance in systems with noise

QCD, HQC, percolation and other systems are characterized by existence of stochastic vacuum (noise, random reservoir, ...) and have general properties and methods of analysis:

- $A = A_0 + A_s$
- Gauss dominance
- Van Kampen decomposition
- Averaging over random realisations
- $l_{corr}, \sigma^2, A_0 l_{corr} \ll 1, \sigma l_{corr} \ll 1$
- exponential decreasing of evolution operator (Wilson loop, fidelity, Green functions)
- similar behaviour of instability, squeezing, entanglement, decoherence

Example

$A = A_0 + A_s$, A_0 - non-perturbed, A_s - random perturbation

$$U(t, 0) = \prod_i U(t_{i+1}, t_i) = T \exp \int_0^t (A_0 + A_s) dt$$

$$\overline{U(t_{i+1}, t_i)} = I + A_0 \Delta t + i \int_{t_i}^{t_{i+1}} \overline{A_s} dt - \underbrace{\int_{t_i}^{t_{i+1}} dt_1 \int_{t_i}^{t_1} dt_2 \overline{A_s(t_1) A_s(t_2)}}_{\frac{A_0^2 \Delta t^2}{2}} - i \underbrace{\int_{t_i}^{t_{i+1}} dt_1 \int_{t_i}^{t_1} dt_2 \int dt_3 \overline{A_{s_1} A_{s_2} A_{s_3}}}_{+}$$

$$\overline{A_s} = 0, \quad \overline{A_s(t_1) A_s(t_2)} = \sigma^2 f(t_2 - t_1), \quad f(0) = 1, \quad A_0^2 \Delta t^2 \ll \sigma^2 \Delta t \int d\xi f(\xi), \quad (\underline{\quad}) \gg (\underbrace{\quad}).$$

If τ_{corr} - correlation time, $\tau_{corr} \ll \Delta t$

$$\int_0^\infty d\xi f(\xi) = \tau_{corr}; \quad A_0 \cdot \tau_{corr} \ll 1, \quad \sigma \tau_{corr} \ll 1 \implies \overline{U(t_{i+1}, t_i)} = I + 1 A_0 \Delta t + \frac{-\sigma^2 \tau_{corr} \Delta t}{2}$$

$$\overline{U(t, 0)} = T \exp(\int_0^t (i A_0 - \frac{\sigma^2 \tau_{corr}}{2}) dt)$$

Chaos criterion in QFT, SSB, fidelity

Local instability. Toda criterion.

If the distance between two phase space trajectories initially very close behaves with time as follows:

$$d(t) = e^{\sigma t},$$

$\sigma > 0$ — Lyapunov exponent \Rightarrow the system is locally unstable, it leads to mixing and to chaos. Regular stable motion is characterized by $\sigma = 0$

2 degrees of freedom

$$H = \frac{p_1^2}{2} + \frac{p_2^2}{2} + V(q_1, q_2), \quad \frac{d\mathbf{q}}{dt} = \frac{\partial H}{\partial \mathbf{p}}, \quad \frac{d\mathbf{p}}{dt} = -\frac{\partial H}{\partial \mathbf{q}},$$

where $\mathbf{q} = (q_1, q_2)$ and $\mathbf{p} = (p_1, p_2)$. The linearized equation of motion for the deviations are

$$\frac{d\delta\mathbf{q}}{dt} = I\delta\mathbf{p}, \quad \frac{d\delta\mathbf{p}}{dt} = -S(t)\delta\mathbf{q}, \quad S_{ij}(t) = \left. \frac{\partial^2 V}{\partial q_i \partial q_j} \right|_{\mathbf{q}=\mathbf{q}(t)},$$

The stability of the dynamical system is then determined by the eigenvalues of the 4×4 stability matrix

$$\Gamma(\mathbf{q}(t)) = \begin{pmatrix} 0 & I \\ -S(t) & 0 \end{pmatrix}.$$

If at least one of the eigenvalues of the stability matrix Γ is real, then the separation of the trajectories grows exponentially and the motion is unstable. Imaginary eigenvalues correspond to stable motion.

The eigenvalues are $\lambda = \pm[-B \pm \sqrt{B^2 - 4C}]^{\frac{1}{2}}$, $B = [\frac{\partial^2 V}{\partial q_1^2} + \frac{\partial^2 V}{\partial q_2^2}]$, $C = [\frac{\partial^2 V}{\partial q_1^2} \frac{\partial^2 V}{\partial q_2^2} - (\frac{\partial^2 V}{\partial q_1 \partial q_2})^2]$. Now, if $B > 0$ then with $C \geq 0$ the eigenvalues are purely imaginary and the motion is stable, while with $C < 0$ the pair of eigenvalues becomes real and this leads to chaotic motion. The parameter C has the same sign as the Gaussian curvature K_G of the potential–energy surface

$$K_G(q_1, q_2) = \frac{\partial^2 V}{\partial q_1^2} \frac{\partial^2 V}{\partial q_2^2} - \left(\frac{\partial^2 V}{\partial q_1 \partial q_2} \right)^2 / \left[1 + \left(\frac{\partial^2 V}{\partial q_1^2} \right)^2 + \left(\frac{\partial^2 V}{\partial q_2^2} \right)^2 \right]^2$$

Instability of classical YMH fields and quantum fluctuations.

It was analytically (Savvidy, Matinyan and others) and numerically (Kawabe and others) shown that classical gauge YM theories are inherently chaotic theories.

1) Example: the SU(2) YMH system ||**Salachnich (1997)**

describes the interaction between a scalar Higgs field ϕ and three non-Abelian Yang-Mills fields A_μ^a , $a = 1, 2, 3$, $\mu = 0, 1, 2$.

$$L = \frac{1}{2}(D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi) - \frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a},$$

$$V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4$$

We choose spatially homogeneous Yang-Mills and the Higgs fields

$$\partial_i A_\mu^a = \partial_i \phi = 0, \quad i = 1, 2$$

When $\mu^2 > 0$ the potential V has a minimum at $|\vec{\phi}| = 0$, but for $\mu^2 < 0$ the minimum is at

$$|\vec{\phi}_0| = \sqrt{\frac{-\mu^2}{4\lambda}} = v,$$

which is the non zero Higgs vacuum. This vacuum is degenerated and after spontaneous symmetry breaking the physical vacuum can be chosen $\vec{\phi}_0 = (0, 0, v)$. If $A_1^1 = q_1$, $A_2^2 = q_2$ and the other components of the Yang-Mills fields are zero, in the Higgs vacuum the Hamiltonian of the system reads ($p_1 = \dot{q}_1$, $p_2 = \dot{q}_2$)

$$H = \frac{1}{2}(p_1^2 + p_2^2) + g^2 v^2 (q_1^2 + q_2^2) + \frac{1}{2} g^2 q_1^2 q_2^2,$$

$$V(q_1, q_2) = g^2 v^2 (q_1^2 + q_2^2) + \frac{1}{2} g^2 q_1^2 q_2^2$$

At low energy the Gaussian curvature is positive and motion is periodic or quasi-periodic. If the energy is increased, the system will be, for some initial conditions, in a region of negative curvature, where the motion is chaotic. The energy E_c of chaos-order transition is

$$E_c = V_{min}(K_G = 0) = 6g^2 v^4$$

► There is a order-chaos transition by increasing the energy E of the system and a chaos-order transition by increasing the value v of the Higgs field in the vacuum.

2) Quantum fluctuations influence in $SU(2) \otimes U(1)$ model

|| **Kuvshinov, Kuzmin, JNMP 9 (2002)**

We took into account contributions of all diagrams with one loop of (W, Z, A) gauge fields and all external lines of Higgs field, with effective Hamiltonian ($q_1 = r \cos \varphi, q_2 = r \sin \varphi, p_\varphi = r^2 \dot{\varphi}$)

$$H = \frac{1}{2}(p_r^2 + p^2 + \frac{p_\varphi^2}{r^2}) + \frac{1}{8}g^2 \langle \rho \rangle^2 r^2 + u(\langle \rho \rangle)$$

We demonstrate that quantum fluctuations of non-abelian gauge fields leading to radiative corrected effective due to Coleman-Weinberg mechanism Higgs potential and spontaneous symmetry breaking can generate order region in phase space of inherently chaotic classical field system.

► QF lead to order-chaos transition

► For $p_\varphi = 0$: $E_{lr} = const \exp(2\alpha_\omega - \frac{2\lambda}{g^4}\beta_\omega) (1 + \frac{1}{2\cos^4\theta_\omega})$

► E_{cr} is liner on p_φ (numerically)

► ratio λ/g^4 has to be less then some critical value

Chaos definition in QFT

||Kuvshinov, Kuzmin, PLA (2001)

- Language of path integrals should be suitable in QM, QFT for any number of degrees of freedom in terms of classical quantities.
- quantum chaos criterion should also reduce to known quasiclassical criteria in the limit $\hbar \rightarrow 0$.

■ From statistical mechanics and ergodic theory it is known that chaos in classical systems is a consequence of the property of mixing. Mixing means rapid (exponential) decrease of correlation function with time.

■ In other words, if correlation function exponentially decreases than the corresponding motion is chaotic, if it oscillates or is constant then the motion is regular.

■ We expand criterion of this type for quantum field systems. All stated bellow remains valid for quantum mechanics, since mathematical description via path integrals is the same.

For field systems the analogue of classical correlation function is two-point connected Green function

$$G_{ik}(x, y) = -\frac{\delta^2 W[\vec{J}]}{\delta J_i(x) \delta J_k(y)} \Big|_{\vec{J}=0}.$$

Here $W[\vec{J}]$ is generating functional of connected Green functions, \vec{J} are the sources of the fields, x, y are 4-vectors.

Chaos criterion for quantum field theory and quantum mechanics is the following:

- ▶ If two-point Green function exponentially goes to zero when the distance between its arguments goes to infinity then system is chaotic.
- ▶ If it oscillates or remains constant in this limit then we have regular behavior of quantum system.
- For finite number degree of freedom in quasiclassical approximation two point Green function in real time is

$$G_i(t_1, t_2) = \frac{i}{2} \operatorname{Re} \left(\frac{e^{-\lambda_i(t_1-t_2)}}{\lambda_i} \right), \quad t_1 > t_2.$$

We see that

■ If motion is locally unstable (chaotic) then according Toda criterion there is real eigenvalue λ_i . Therefore Green function exponentially goes to zero for some i when $(t_1 - t_2) \rightarrow +\infty$. Opposite is also true. If Green function exponentially goes to zero under the condition $(t_1 - t_2) \rightarrow +\infty$ for some i , then there exists real eigenvalue of the stability matrix and thus classical motion is locally unstable.

■ If all eigenvalues of the stability matrix G are pure imaginary, that corresponds stable motion, then in the limit $(t_1 - t_2) \rightarrow +\infty$ Green function oscillates as a sine. Opposite is also true. If for any i Green functions oscillate in the limit $(t_1 - t_2) \rightarrow +\infty$ then $\{\lambda_i\}$ are pure imaginary for any i and motion is stable and regular.

Thus proposed quantum chaos criterion coincides with Toda criterion in the semi-classical limit (corresponding principle). • Instability and spontaneous symmetry breakdown

■ One of possible applications of proposed chaos criterion in field theory is an investigation of the stability of classical solutions with respect to small perturbations of initial conditions. To study the stability of certain classical solution of field equations one has to calculate (for instance, using one loop approximation) two-point Green function in the vicinity of considered classical solution.

Example: real scalar φ^4 -field

$$L = \frac{1}{2} (\partial_\mu \varphi)^2 - \frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4!} \varphi^4.$$

Here $\lambda > 0$ is a coupling constant, m^2 is some parameter which can be larger or less than zero. In both cases $\varphi = 0$ is a solution of field equations. Asymptotics of two-point Green function calculated in the vicinity of the classical solution $\varphi = 0$ in the zero order of perturbation theory is

$$G(x, y) \underset{\rho \rightarrow \infty}{\sim} \rho^{-\frac{1}{2}} e^{im\sqrt{\rho}}, \quad \rho = (x - y)^2 > 0.$$

We can study the stability of considered solution with respect to small perturbations. and have two different cases

■ Green function oscillates and slowly (non-exponentially) goes to zero when $\rho \rightarrow \infty$. According proposed chaos criterion considered solution is stable. Indeed, it follows that parameter m is real in this case. Therefore $\varphi = 0$ is a stable vacuum state.

■ Green function exponentially goes to zero in the limit $\rho \rightarrow \infty$. From proposed chaos criterion it follows that $\varphi = 0$ is an unstable solution. That is true since one can see that parameter m has to be pure imaginary. It is known that in this case state $\varphi = 0$ becomes unstable, two new stable vacuums are appeared and we obtain spontaneous symmetry breakdown.

• Correspondence between chaos criterion and confinement criterion in lattice models - Green function, propagator, cumulant go to zero exponentially.

Connection: Green function, Wilson loop

$$G(x, y) = \int_0^\infty ds \int_{z_\mu(0)=x_\mu}^{z_\mu(s)=y_\mu} Dz_\mu \exp \left[-m^2 s - \frac{1}{2} \int_0^s d\tau \left(\frac{dz_\mu(\tau)}{d\tau} \right)^2 \right] \langle T_r W(c) \rangle$$

Squeezing, Entanglement, Instability during evolution

- Gluons interacting in jets lead to quantum squeezed states || **V.Kuvshinov, V.Shaporov, 2001**
|| **Acta Phys.Pol., 2004**
- Squeezed gluon evolution leads to entangled gluons
- Connection and evolution of squeezing and instability || **V.Kuvshinov, V.Marmysh, 2003**
|| **Theor.Math.Phys**

example of quadratic Hamiltonians:

$$\frac{d}{dt}g^{mn} = \Gamma_l^m g^{ln} + \Gamma_l^n g^{lm},$$

Γ_l^n - instability matrix, g^{lm} - matrix of dispersions \implies more instability - more squeezing

- YMH fields entangle during intension || **V.Kuvshinov, V.Shaporov, Letter of JEPNP 2004**
- Entangled gluon produce entangled quarks, which interacting with stochastic vacuum give:
pairs of $q\bar{q}_s$, string tension, confinement

Conclusion

- There exists connection between stochastic confinement and chaos (instability)
 - Wilson loop is mathematically similar to fidelity (stability of quantum motion)
 - Gauss dominance and general behaviour of the systems with noise (vacuum, thermostat)
 - Gluons and quark squeeze and entangle during evolution
 - Entangled quarks interesting with stochastic vacuum give: $\bar{q}q$ pairs, chaos, decoherence spontaneous symmetry breaking, masses, string, confinement