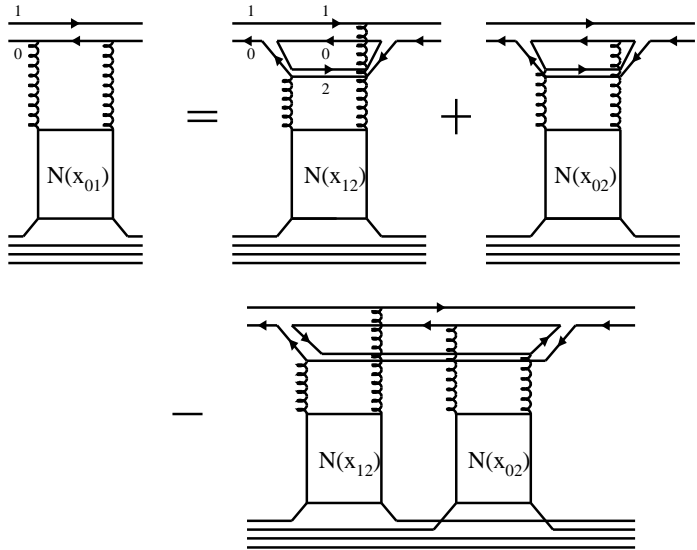


# Low $x$ Evolution beyond the Balitsky - Kovchegov Equation

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DESY/Theory

# Balitsky-Kovchegov Non-linear Evolution



**GLR (81)**  
**Mueller & Qiu (86)**

.....  
**Balitsky (95)**  
**Kovchegov (99)**  
**Braun (2000)**  
**Iancu, Leonidov & McLerran (2000)**

$$\frac{dN(x_{01}, y; b)}{dy} = -\frac{2C_F\alpha_S}{\pi} N(x_{01}, y; b) \ln \frac{x_{01}^2}{\rho^2} +$$

$$\frac{C_F\alpha_S}{\pi^2} \times \int_{\rho} d^2x_2 \frac{x_{01}^2}{x_{02}^2 x_{12}^2} \times$$

$$(2N(x_{02}, y; b) - N(x_{02}, y; b)N(x_{12}, y; b))$$

Approximations:  $L \log 1/x$ ;  $N_c \rightarrow \infty$ ;  $\alpha_s$  - const.;  $b$  - large.

$$\frac{dN}{dy} = \alpha_s \text{Ker} \otimes (N - N N). \quad \text{BKE}$$

## Theoretical successes

- LO BFKL evolution
- s-channel unitarity of partial waves (fixed impact parameter)
- Gluon saturation
- Infrared diffusion problem

## Phenomenological successes

- $F_2$  DIS data  
E. Gotsman, E. Levin, M.L, U. Maor (2002)  
E. Iancu, S. Munier and K. Itakura (2003)  
 $J/\Psi$  production  
E. Gotsman, E. Levin, M.L, U. Maor and E. Naftali (2003)
- Saturation scale  $Q_s(x)$  and geometrical scaling  
M. Braun and N. Armesto (2001);  
M.L. (2001);  
K. Golec-Biernat, L. Motyka and A. Stasto (2001)
- Diffractive Dissociation  
E. Levin and M.L. (2001)

## • Approximations

- LO BFKL: soft gluon emission approx. (fixed  $\alpha_s$ )
- Large  $N_c$  - dipole picture - mean field theory without correlations
- No target correlations

## Problems

- Violates the t-channel unitarity (Pomeron loops)
- Violates the s-channel unitarity at intermediate steps of the evolution  
A. Mueller and A. Shoshi (2004)
- Violates the Froissart bound
- Needs high order corrections including running of  $\alpha_s$

# Corrections to BKE

- NLO BFKL ( $\alpha_s$  corrections)

D.N. Triantafyllopoulos (2002) NLO BFKL + saturation boundary;

I. Balitsky and A. Belitsky (2001)  $\alpha_s$  suppressed cubic term:

$$\frac{dN}{dy} = \alpha_s \text{Ker} \otimes (N - N N) - \alpha_s^2 \tilde{\text{Ker}} \otimes N N N.$$

G. Chachamis, M.L., A. Sabio-Vera (2004?) BKE + rapidity veto

$$\frac{dN(y)}{dy} = \alpha_s \text{Ker} \otimes (N(y - \eta) - N(y - \eta) N(y - \eta))$$

- $N_c$  corrections (JIMWLK equation = Balitsky's chain)

$$N \equiv \langle W \rangle_{\text{target}}$$

$$\frac{d\langle W \rangle}{dy} = \alpha_s \text{Ker} \otimes (\langle W \rangle - \langle W W \rangle). \quad \text{Bal. 1}$$

$$\frac{d\langle W W \rangle}{dy} = \alpha_s \text{Ker} \otimes (\langle W W \rangle - \langle W W W W \rangle). \quad \text{Bal. 2}$$

Kovchegov:  $\langle W W \rangle = \langle W \rangle \langle W \rangle = N N; \quad N_c \rightarrow \infty$

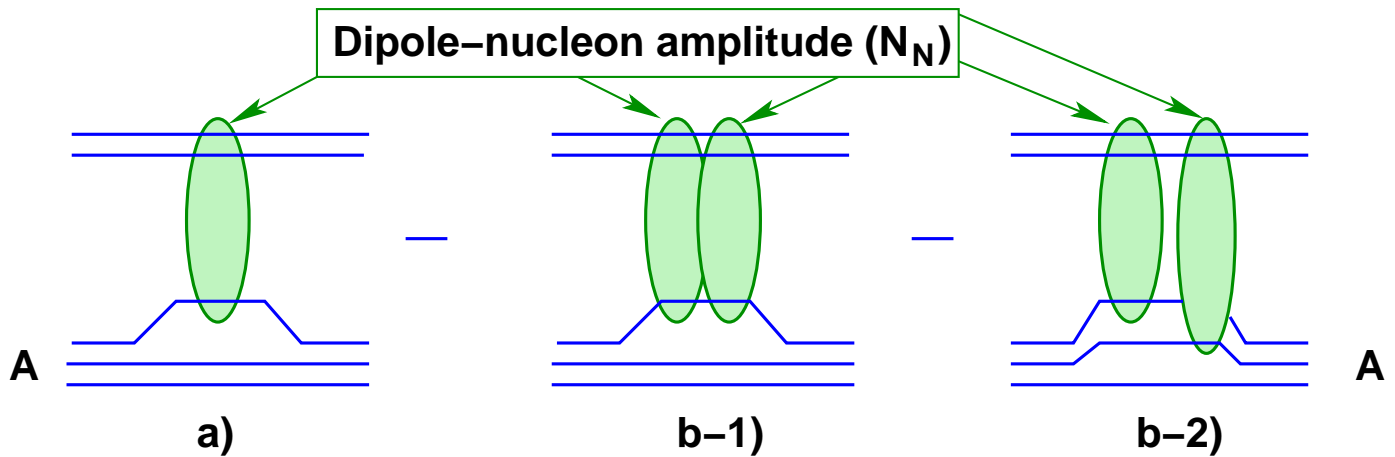
K. Rummukainen and H. Weigert (2003) numerical solution of JIMWLK;

J. Bartels, L. Lipatov, and G.P. Vacca (2004)

$N_c$  correction to triple Pomeron vertex:

$$\frac{dN}{dy} = \alpha_s \text{Ker} \otimes (N - N N - \frac{1}{N_c^2} n)$$

- Target correlations for proton and realistic (not dense) nuclei  
E. Levin and M.L. (2003) Linear functional differential equation



Model dependent modification of the BKE:

$$\frac{d N_A}{d y} = \alpha_s K e r \otimes \left( N_A - \frac{1 + \kappa}{\kappa} N_A N_A \right);$$

$$\kappa = \pi R_p^2 S_A(b) \propto A^{1/3}$$

- Pomeron loops

E. Iancu and A. Mueller (2003); M. Kozlov and E. Levin (2004)  
cannot be formulated as an equation.

$$N(Y) = 1 - e^{-c(Y - Y_0)^2}; \quad Y \rightarrow \infty \quad BKE$$

$$N(Y) = 1 - e^{-1/2 c(Y - Y_0)^2}; \quad Y \rightarrow \infty \quad PL$$

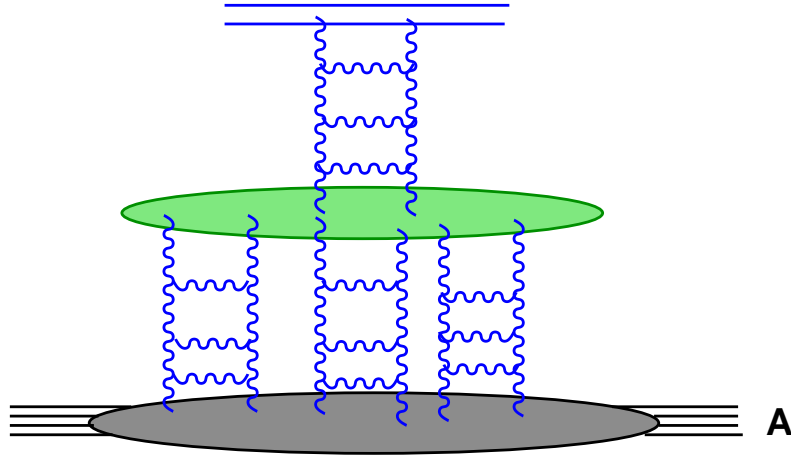
$$c = 2 \bar{\alpha}_s$$

- Multi Pomeron vertices

I. Balitsky and A. Belitsky (2001)  $\alpha_s$  suppressed cubic term;

E. Levin and M. Lublinsky (2003, 2004?)

eikonal resummation of all multi-pomeron vertices - modified BKE



$$\frac{dN}{dy} = (1 - N) \alpha_s \text{Ker} \otimes (N - N N).$$

BKE:  $N \ll 1; \quad N \sim N^2$

# Open Questions

- Impact parameter dependence
- Dipole picture vs. traditional diagrammatics
- **Simple** effective Reggeon field theory in QCD