(Nonlinear) $k_\perp$-factorization in Saturation Regime

N.N. Nikolaev[a,b)], W. Schäfer[a)], B.G. Zakharov[b)], V.R. Zoller[c)]

[a)] Institut f. Kernphysik, Forschungszentrum Jülich,
52425 Jülich, Germany
[b)] L.D.Landau Institute for Theoretical Physics,
142432 Chernogolovka, Russia
[c)]Institute for Theoretical and Experimental Physics,
11259 Moscow, Russia

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principal references:

N. N. Nikolaev, W. Schäfer. and G. Schwiete, Phys. Rev. D **63**, 014020 (2001)

N.N. Nikolaev, W.Schäfer. , B.G. Zakharov, V.R. Zoller, JETP Lett. 76 (2002) 195;

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★ DIS at very small $x \ll 1$ in the Breit frame:

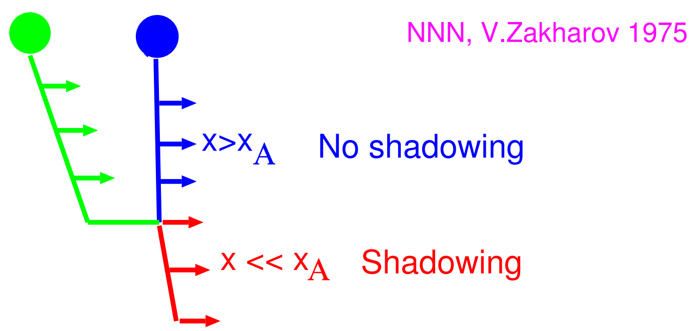
- the Lorentz-contracted ultrarelativistic nucleus becomes thinner that the longitudinal spatial localization of soft partons:

$$R_A \to R_A \frac{m_N}{p_N} < \lambda = \frac{1}{k_z} = \frac{1}{x p_N}\,.$$

- a spatial overlap of partons with

$$x \lesssim x_A = \frac{1}{R_A m_N}$$

from all the nucleons at the same impact parameter
$\Longrightarrow$ FUSION & NUCLEAR SHADOWING.



- Nuclear parton density (if it can be meaningfully defined!) is a nonlinear functional of the free nucleon parton density: the same sea is shared by two & more parent nucleons.

- Scattering theory: shadowing as a manifestation of unitarity for dipole amplitudes:

the major strategy in this talk.

★ DIS at very small $x \ll 1$ in the lab. frame:

- DIS can be viewed as an interaction of frozen multiparton Fock states of the photon with the target nucleon or nucleus. Quantum mechanics of color factorization (NNN, B.Zakharov '91)

$$\sigma_T(x,Q^2) = \langle\gamma^*|\sigma(x,\mathbf{r})|\gamma^*\rangle = \int_0^1 dz \int d^2\mathbf{r}\Psi^*_{\gamma^*}(z,\mathbf{r})\sigma(x,\mathbf{r})\Psi_{\gamma^*}(z,\mathbf{r})$$

- $z_+ = z$ and $z_- = 1-z \longrightarrow$ energy & momentum partition between q & $\bar{q}$; $\mathbf{r}$ = size & orientation of the color dipole.
- Connection to the unintegrated gluon SF of the target & equivalence between color dipole and $k_\perp$-factorization:

$$\sigma(x,\mathbf{r}) = \frac{4\pi}{N_c}\alpha_S(r)\int d^2\boldsymbol{\kappa}[1-\exp(i\boldsymbol{\kappa}\mathbf{r})]\cdot\frac{1}{\kappa^4}\cdot\frac{\partial G_N}{\partial\log\kappa^2}$$

- A convenient definition:

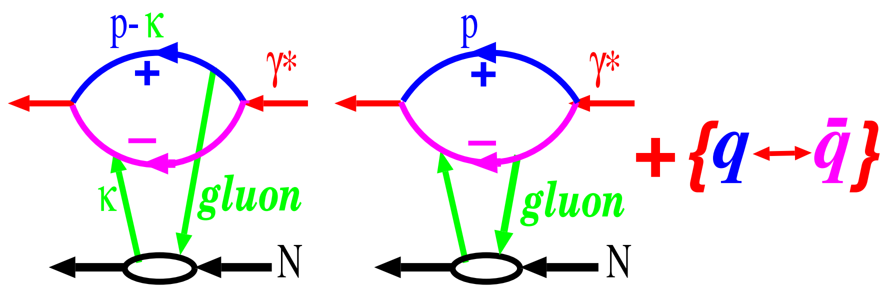
$$f(\boldsymbol{\kappa}) = \frac{4\pi}{N_c\sigma_0(x)}\cdot\frac{1}{\kappa^4}\cdot\frac{\partial G_N(x,\boldsymbol{\kappa})}{\partial\log\kappa^2}, \qquad \int d^2\boldsymbol{\kappa} f(\boldsymbol{\kappa}) = 1$$

- $$\sigma_0(x) = \sigma(x,\mathbf{r})\Big|_{r\to\infty}$$
- Color dipole BFKL equation (NNN,B.Zakharov, Zoller '94)

$$\frac{\partial\sigma(x,\mathbf{r})}{\partial\log\frac{1}{x}} = K_0\int d^2\boldsymbol{\rho}\frac{\mathbf{r}^2}{\boldsymbol{\rho}^2(\boldsymbol{\rho}-\mathbf{r})^2}[\sigma(x,\boldsymbol{\rho})+\sigma(x,\boldsymbol{\rho}-\mathbf{r})-\sigma(x,\mathbf{r})]$$

$$K_0 = \frac{1}{2\pi^2}N_c\alpha_S$$

$\langle \gamma^* | \sigma(\mathbf{r}) | \gamma^* \rangle$ sums 4 Feynman diagrams with the two-gluon tower in the t-channel



★ $f(\boldsymbol{\kappa})$ defines the Pomeron $I\!P$ for a free nucleon target

★ $f(\boldsymbol{\kappa})$ and the final states:

- Forward dijets: $x_\gamma = z_+ + z_- \approx 1$
- Forward jets have transverse momentum $\mathbf{p}$ or $\mathbf{p} - \boldsymbol{\kappa}$

- Jet-jet azimuthal decorrelation momentum $\boldsymbol{\Delta} = \mathbf{p}_+ + \mathbf{p}_-$

- $k_\perp$-factorization for the jet-jet decorrelation:

$$\frac{d\sigma_N}{dz d^2\mathbf{p}_+ d^2\boldsymbol{\Delta}} =$$

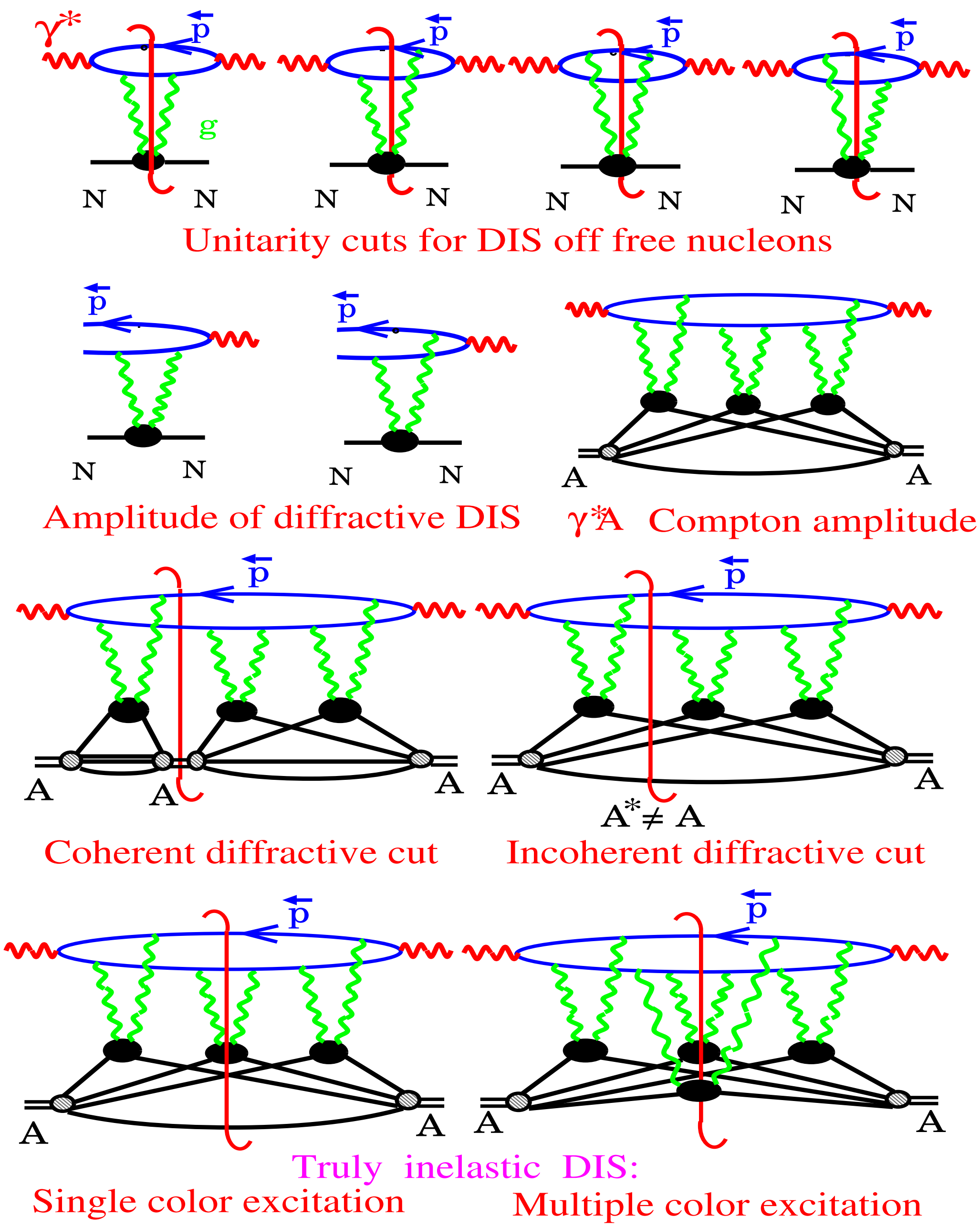
$$\frac{\sigma_0}{2} \cdot \frac{\alpha_S(\mathbf{p}^2)}{(2\pi)^2} f(\boldsymbol{\Delta}) \left| \langle \gamma^* | z, \mathbf{p}_+ \rangle - \langle \gamma^* | z, \mathbf{p}_+ - \boldsymbol{\Delta} \rangle \right|^2$$

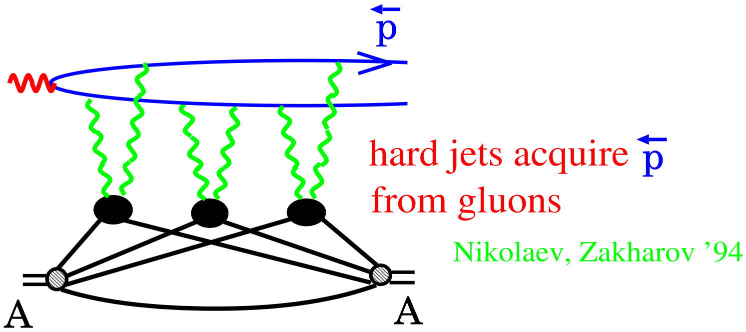
$$= \frac{\alpha_S(\mathbf{p}^2)}{2\pi N_c} \cdot \frac{\mathcal{F}(x, \boldsymbol{\Delta}^2)}{\boldsymbol{\Delta}^4} \cdot \left| \langle \gamma^* | z, \mathbf{p}_+ \rangle - \langle \gamma^* | z, \mathbf{p}_+ - \boldsymbol{\Delta} \rangle \right|^2 .$$

★ The jet-jet decorrelation distribution is a linear functional of the unintergrated gluon density of the target.

★ Diffractive and truly inelastic DIS off nuclei:
the dipole is coherent over the whole nucleus for $x \lesssim x_A$:
$\Longrightarrow$ *Glauber–Gribov formalism*:

$$\sigma_A(\mathbf{r}) = 2\int d^2\mathbf{b}\Gamma_A(\mathbf{b},\mathbf{r}) = 2\int d^2\mathbf{b}[1-\exp(-\frac{1}{2}\sigma(\mathbf{r})T(\mathbf{b}))]$$





Diffractive DIS off nuclei defines collective nuclear glue

$$\frac{d\sigma_D(\gamma^* A)}{d^2\mathbf{b}dzd^2\mathbf{p}} = \frac{1}{(2\pi)^2}\left|\langle\gamma^*|\left\{1-\exp[-\frac{1}{2}\sigma(\mathbf{r})T(\mathbf{b})]\right\}|\mathbf{p}\rangle\right|^2$$

- Nuclear thickness $T(\mathbf{b}) = \int dz n_A(\mathbf{b}, z)$ sets a new scale.

$$\Gamma_A(\mathbf{b}, \mathbf{r}) = \int d^2\boldsymbol{\kappa}\phi(\mathbf{b}, \boldsymbol{\kappa})\{1-\exp[i\boldsymbol{\kappa}\mathbf{r}]\}$$

- Nuclear flux of glue per unit area in the impact parameter space

$$\phi(\mathbf{b}, \boldsymbol{\kappa}) = \sum_{j=1}^{\infty} w_j(\mathbf{b}) f^{(j)}(\boldsymbol{\kappa})$$

$$w_j(\mathbf{b}) = \frac{\nu_A^j(\mathbf{b})}{j!}\exp\left[-\nu_A(\mathbf{b})\right], \quad \nu_A(\mathbf{b}) = \frac{1}{2}\alpha_S(r)\sigma_0 T(\mathbf{b})$$

- w_j is the probability to find j overlapping nucleons at impact parameter $\mathbf{b}$ in a Lorentz-contracted nucleus, $f^{(j)}$is a collective glue of j overlapping nucleons:

$$f^{(j)}(\boldsymbol{\kappa}) = \int \prod_i^j d^2\boldsymbol{\kappa}_i f(\boldsymbol{\kappa}_i)\delta(\boldsymbol{\kappa} - \sum_i^j \boldsymbol{\kappa}_i), \quad f^{(0)}(\boldsymbol{\kappa}) \equiv \delta(\boldsymbol{\kappa})$$

- Hard glue per bound nucleon is not shadowed: (NNN,Schäfer, Schwiete '00)

$$f_A(\mathbf{b},\boldsymbol{\kappa}) = \frac{\phi(\mathbf{b},\boldsymbol{\kappa})}{\nu_A(\mathbf{b})}$$

$$= f(\boldsymbol{\kappa})\left[1+\frac{\gamma^2}{2}\cdot\frac{\alpha_S(\boldsymbol{\kappa}^2)G(\boldsymbol{\kappa}^2)}{\alpha_S(Q_A^2)G(Q_A^2)}\cdot\frac{Q_A^2(\mathbf{b})}{\boldsymbol{\kappa}^2}\right].$$

- γ = exponent of the large–$\boldsymbol{\kappa}^2$ tail $f(\boldsymbol{\kappa}) \sim \alpha_S(\boldsymbol{\kappa}^2)/(\boldsymbol{\kappa}^2)^\gamma$
- Width of the plateau (saturation & higher twist scale)

$$Q_A^2(\mathbf{b}) \approx \frac{4\pi^2}{N_c}\alpha_S(Q_A^2)G(Q_A^2)T(\mathbf{b}).$$

(The saturation scale of Mueller '90)

- Manifestly positive-valued nuclear higher twist correction with nuclear antishadowing property. No dependence on soft parameters. (NNN,Schäfer, Schwiete '00)
- Antishadowing is an origin of the Cronin effect.
- Plateau for softer collective glue per unit area in the impact parameter plane:

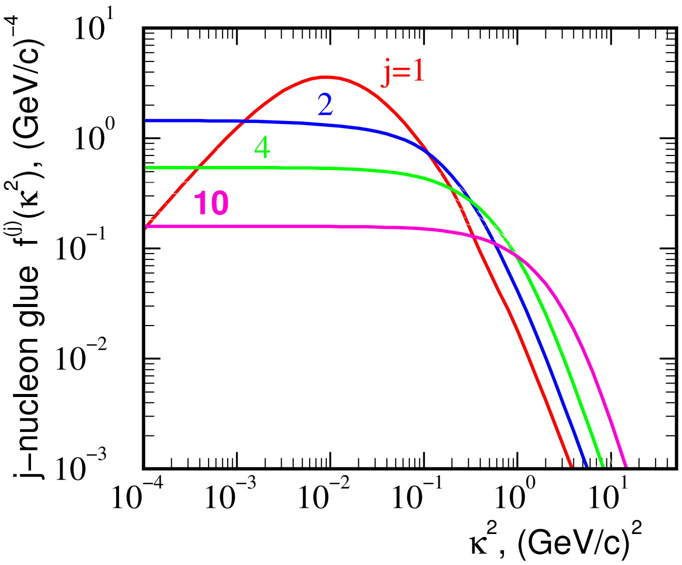
$$\phi(\mathbf{b},\boldsymbol{\kappa}) \approx \frac{1}{\pi}\frac{Q_A^2(\mathbf{b})}{(\boldsymbol{\kappa}^2+Q_A^2(\mathbf{b}))^2},$$

- Nuclear dilution of the collective glue at small $\boldsymbol{\kappa}^2 \ll Q_A^2(\mathbf{b})$:

$$\phi(\mathbf{b},\boldsymbol{\kappa}) \propto \frac{1}{\nu_A(\mathbf{b})}$$

- Numerical evaluation for $A^{1/3} = 6$ gives the saturation scale

$$\langle Q_A^2(\mathbf{b})\rangle_{Au} \approx 0.8(\mathrm{GeV/c})^2 .$$



Dilution for soft, and broadening for hard, of the collective glue of j overlapping nucleons $f^{(j)}(\boldsymbol{\kappa})$ for DIS at $x = 0.01$.

- The $k_\perp$-factorization for the total DIS cross section in terms of the *same collective glue*:

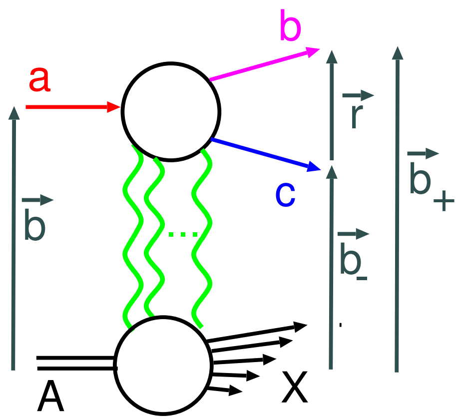
$$\sigma_{\gamma^* A} = \int d^2\mathbf{b} \int \frac{d^2\mathbf{p}}{(2\pi)^2}\alpha_S(\mathbf{p}^2) \int d^2\boldsymbol{\kappa}\phi(\boldsymbol{\kappa})|\langle\gamma^*|\mathbf{p}\rangle - \langle\gamma^*|\mathbf{p}-\boldsymbol{\kappa}\rangle|^2$$

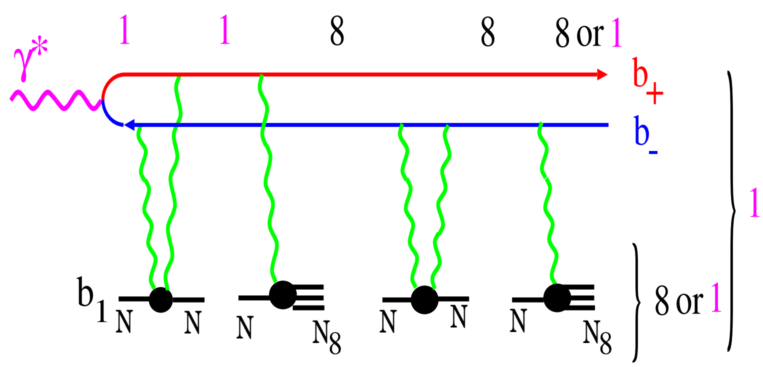
- Identical to the free nucleon case: $\phi(\boldsymbol{\kappa}) \Longleftrightarrow f(\boldsymbol{\kappa})$, the exchange by a single nuclear Pomeron.

?? Any nuclear $k_\perp$-factorization in terms of $\phi(\boldsymbol{\kappa})$??

?? Any similarity between nuclear and free-nucleon Pomerons ??

★ Non-Abelian intranuclear evolution





- 'decay' of a virtual particle $a \to bc$, with a weak, perturbative abc-coupling g: $|a\rangle_{phys} = |a\rangle_0 + g\psi(\mathbf{r})|bc\rangle_0$
- Simple action of S-matrix on bare partons. Scattering state:

$$\mathsf{S}|a\rangle_{phys} = S_a(\mathbf{b})|a\rangle_0 + gS_b(\mathbf{b}_+)S_c(\mathbf{b}_-)\psi(\mathbf{r})|bc\rangle_0$$

$$= S_a(\mathbf{b})|a\rangle_{phys} + g[S_b(\mathbf{b}_+)S_c(\mathbf{b}_-) - S_a(\mathbf{b})]\psi(\mathbf{r})|bc\rangle$$

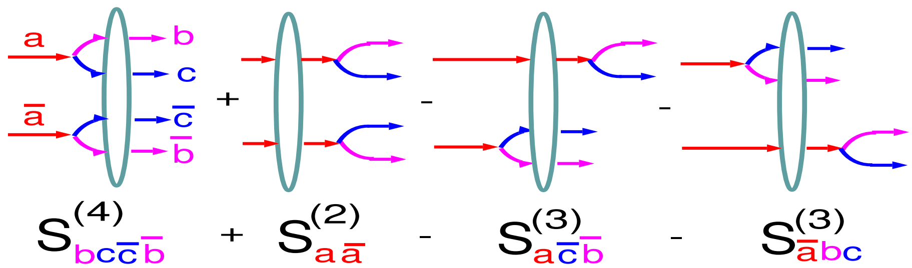
- $S_bS_c \Leftrightarrow$ scattering after and $S_a \Leftrightarrow$ scattering before the decay. Amplitude for inelastic excitation *vanishes* if $S_bS_c = S_a$.
- The nuclear S-matrix $S_A(\mathbf{b}) = \prod_{j=1}^{A} S_N(\mathbf{b} - \mathbf{b}_j)$(ordering along the longitudinal path is understood)
- 2-particle inclusive spectrum for $a \to b(\mathbf{p}_+)c(\mathbf{p}_-)$
  $\mathbf{b} = z\mathbf{b}_+ + (1-z)\mathbf{b}_-$

$$\frac{d\sigma}{dz d^2\mathbf{p}_+ d^2\mathbf{p}_-} = \frac{1}{(2\pi)^4}\int d^2\mathbf{b}_+ d^2\mathbf{b}_- d^2\mathbf{b}'_+ d^2\mathbf{b}'_-$$

$$\times \exp[i\mathbf{p}_+(\mathbf{b}_+ - \mathbf{b}'_+) + i\mathbf{p}_-(\mathbf{b}_- - \mathbf{b}_-{}')]\Psi(\mathbf{b}_+ - \mathbf{b}_-)\Psi^*(\mathbf{b}'_+ - \mathbf{b}'_-)$$

$$\{S^{(4)}(\mathbf{b}_+, \mathbf{b}_-, \mathbf{b}'_+, \mathbf{b}'_-) + S^{(2)}(\mathbf{b}, \mathbf{b}') - S^{(3)}(\mathbf{b}_+, \mathbf{b}_-, \mathbf{b}') - S^{(3)}(\mathbf{b}'_+, \mathbf{b}'_-, \mathbf{b})\}$$

- Application of closure on the nuclear side in the square of the amplitude.



$S^{(4)}$ describes scattering of a 4-body system of dipoles off the target $\rightarrow$ evaluated in Glauber-Gribov approximation. The 4-parton system is in an overall color singlet state.
Dipole states: $|R\overline{R}\rangle = |(b\bar{b})_R \otimes (c\bar{c})_{\bar{R}}\rangle$

e.g. $R = 1, 8$ for $bc = q\bar{q}$, $R = 1, 8_A, 8_S, 10 + \overline{10}, 27$ for $bc = gg$.

- $\Rightarrow$ $S^{(4)}$ is an element of the matrix $\hat{S}^{(4)} = \exp\{-\frac{1}{2}\sigma_4(\mathbf{b}_+', \mathbf{b}_-', \mathbf{b}_+, \mathbf{b}_-)T(\mathbf{b})\}$, whose eigenvalues can be expressed through the free nucleon color dipole cross section.

- The color-coupled channel aspect of intranuclear dipole propagation cannot be absorbed in a universal unintegrated glue of the nucleus. *No $k_\perp$-factorisation of two particle-spectra.*

- $\Rightarrow$ a whole density matrix of gluons in color space is called upon.

- Exceptional case of forward quark jets in DIS: back to $\sigma_{\gamma^* A}$

$\sigma_{\gamma^* A} = \int d^2\mathbf{b} \int \frac{d^2\mathbf{p}}{(2\pi)^2} \alpha_S(\mathbf{p}^2) \int d^2\boldsymbol{\kappa} \phi(\boldsymbol{\kappa}) |\langle \gamma^* | \mathbf{p} \rangle - \langle \gamma^* | \mathbf{p} - \boldsymbol{\kappa} \rangle|^2$

the single-quark $\mathbf{p}$ distribution is calculated through $\phi(\boldsymbol{\kappa})$ as if struck quarks have no FSI: an analogy between nuclear and free-nucleon pomerons for certain observables .

(similar quasi-abelian special cases of single particle spectra: Kopeliovich et al. '99, Mueller '99 , Wiedemann '00, Kovchegov '01)

- The single particle spectrum for $g^* \to Q\bar{Q}$

differential version of NNN, Piller, Zakharov '94

- On the free nucleon target:

$$\frac{d\sigma}{dz d^2\mathbf{p}} = \frac{1}{2(2\pi)^2} \int d^2\mathbf{r} d^2\mathbf{r}' \exp[i\mathbf{p}(\mathbf{r}-\mathbf{r}')]\Psi(\mathbf{r})\Psi^*(\mathbf{r}')$$

$$\{\sigma_3(z\mathbf{r}', \mathbf{r}) + \sigma_3(z\mathbf{r}, \mathbf{r}') - \sigma_{2,Q\bar{Q}}(\mathbf{r}-\mathbf{r}') - \sigma_{2,gg}(z(\mathbf{r}-\mathbf{r}'))\}$$

- Three–body cross section (Nikolaev,Zakharov '94)

$$\sigma_3(\boldsymbol{\rho}, \mathbf{r}) = \frac{C_A}{2C_F}\Big[\sigma_2(\boldsymbol{\rho}) + \sigma_2(\mathbf{r}-\boldsymbol{\rho}) - \frac{1}{N_c^2}\sigma_2(\mathbf{r})\Big]$$
$$\equiv F[\sigma_2]$$

- For the *nuclear cross section* substitute:

$$\sigma_2(\mathbf{r}) \to \sigma_{2A}(\mathbf{r}) = 2\int d^2\mathbf{b}[1-\exp(-\frac{1}{2}\sigma_2(\mathbf{r})T(\mathbf{b}))]$$

$$\sigma_3(\mathbf{r}, \boldsymbol{\rho}) \to \sigma_{3A}(\mathbf{r}, \boldsymbol{\rho}) = 2\int d^2\mathbf{b}[1-\exp(-\frac{1}{2}\sigma_3(\mathbf{r}, \boldsymbol{\rho})T(\mathbf{b}))]$$

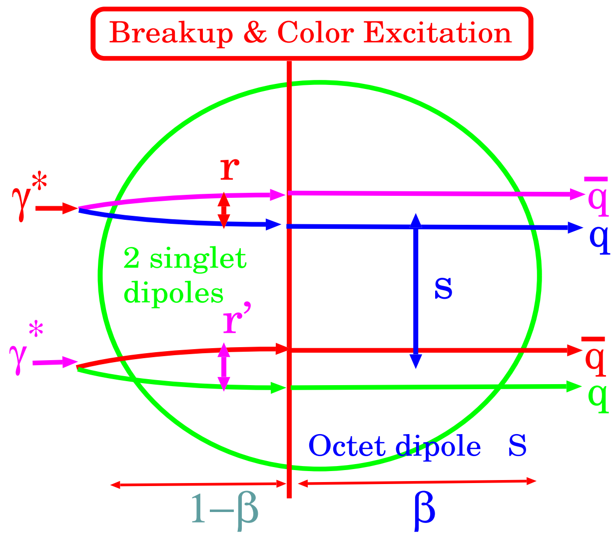
- !! But: $\sigma_{3A}(\mathbf{r}, \boldsymbol{\rho}) \neq F[\sigma_{2A}]$!!

$\Longrightarrow$ if the decaying particle a is not a passive state, *$k_\perp$–factorization is in general violated already for the single particle spectrum.*

- Breakup of virtual photon into dijets $\gamma^*(Q^2) \to q(\mathbf{p}_+)\bar{q}(\mathbf{p}_-)$ in *truly inelastic* DIS

- Coherent diffractive DIS = 50 per cent of total for heavy nucleus (NNN, Zakharov, Zoller '94). Diffractive dijets are exactly back-to-back, $\boldsymbol{\Delta} = \mathbf{p}_+ + \mathbf{p}_- \sim \frac{1}{R_A} \sim 0$

- For hard dijets $\Rightarrow$ linear dependence on the nuclear unintegrated glue in terms of: $\Phi(\nu_A(\mathbf{b}), \boldsymbol{\kappa}) = \exp(-\nu_A(\mathbf{b}))\delta^{(2)}(\boldsymbol{\kappa}) + \phi(\nu_A(\mathbf{b}), \boldsymbol{\kappa})$ for grey nuclei includes a numerically important no-rescattering term $\propto \delta^{(2)}(\boldsymbol{\kappa})$



- Nuclear dijet spectrum is a convolution of Φ with the free nucleon cross section ($\lambda_c \equiv C_A/2C_F$):

$$\frac{d\sigma_{in}}{d^2\mathbf{b}dzd^2\mathbf{p}_+d^2\boldsymbol{\Delta}} = T(\mathbf{b}) \times \int d^2\boldsymbol{\kappa} \int_0^1 d\beta \Phi(2\beta\lambda_c\nu_A(\mathbf{b}), \boldsymbol{\Delta} - \boldsymbol{\kappa}) \frac{d\sigma_N}{dzd^2\mathbf{p}_+d^2\boldsymbol{\kappa}}.$$

- Dependence on Φ is different from the $k_\perp$-factorization for the free nucleon case $\Longrightarrow$ breaking of the $k_\perp$-factorization for nuclei !

★ Semihard dijets below the saturation scale: $|\mathbf{p}_\pm|^2 \lesssim Q_A^2$
$\Longrightarrow$ *complete azimuthal decorrelation of dijets* and a highly nonlinear dependence on the nuclear unintegrated glue with no resemblance of the free-nucleon formula.

- Simple closed form in the large-N_c approximation:

$$\frac{d\sigma_{in}}{d^2\mathbf{b}dz\,d\mathbf{p}_-d\mathbf{\Delta}} = \frac{1}{2(2\pi)^2}\alpha_S\sigma_0 T(\mathbf{b})$$
$$\times \int_0^1 d\beta \int d^2\boldsymbol{\kappa}_3 d^2\boldsymbol{\kappa} f(\boldsymbol{\kappa})$$
$$\times \Phi(\beta\nu_A(\mathbf{b}), \mathbf{\Delta} - \boldsymbol{\kappa}_3 - \boldsymbol{\kappa})\Phi(\beta\nu_A(\mathbf{b}), \boldsymbol{\kappa}_3)$$
$$\times \Big| \int d^2\boldsymbol{\kappa}_1 \Phi((1-\beta)\nu_A(\mathbf{b}), \boldsymbol{\kappa}_1)$$
$$\{\langle\gamma^*|z, \mathbf{p}_- + \boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_3\rangle - \langle\gamma^*|z, \mathbf{p}_- + \boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_3 + \boldsymbol{\kappa}\rangle\} \Big|^2 .$$

- What are the universality properties of such a nonlinear $k_\perp$-factorization **?**
- The slice $(1-\beta)$ in which the dipole was in the color-singlet state $\Longrightarrow$ Initial State Interaction (distortion of the WF)
- The slice β in which the dipole is in the color-octet state $\Longrightarrow$ Final State Interaction (broadening) .

  ★ $\phi((1-\beta)\nu_A(\mathbf{b}), \boldsymbol{\kappa}), \phi(\beta\nu_A(\mathbf{b}), \boldsymbol{\kappa}) \neq \phi(\nu_A(\mathbf{b}), \boldsymbol{\kappa})\Big|_{DIS}$

  ★ Dijets are described by a "four nuclear Pomeron exchange".
- Soft dijets, $|\mathbf{p}_-|, |\mathbf{\Delta}| \lesssim Q_A$: a complete disappearance of the azimuthal correlation

Small-x evolution of the nuclear DIS cross section and nuclear Pomeron.

★ The free-nucleon target:
the effect of the $q\bar{q}g$ and higher Fock states can be reabsorbed in the linear BFKL evolution for the dipole cross section or the unintegrated glue, with the photon treated as the $q\bar{q}$ state.

★ Similar reabsorption is possible for nuclei, but without a closed-form evolution equation !!

★The first iteration of the $\log\frac{1}{x}$ evolution:

$$\frac{\partial \Delta\sigma_A(x,\mathbf{r})}{\partial \log\frac{1}{x}} = K_0 \int d^2\boldsymbol{\rho} \frac{\mathbf{r}^2}{\boldsymbol{\rho}^2(\boldsymbol{\rho}-\mathbf{r})^2}$$

$$\times 2\int d^2\mathbf{b}[\Gamma_{3A}(\mathbf{b},\boldsymbol{\rho},\mathbf{r}) - \Gamma_{2A}(\mathbf{b},\mathbf{r})] = 2\int d^2\mathbf{b}\frac{\partial\Gamma_A(x,\mathbf{b},\mathbf{r})}{\partial\log\frac{1}{x}}$$

$$\Gamma_{3A}(\mathbf{b},\boldsymbol{\rho},\mathbf{r}) = 1 - S_{3A}(\mathbf{b},\boldsymbol{\rho},\mathbf{r}) = 1-\exp[-\frac{1}{2}\sigma_3(\boldsymbol{\rho},\mathbf{r})T(\mathbf{b})]$$

- Simplified Glauber formulas in the large-N_c approximation:

$$S_{3A}(\mathbf{b},\boldsymbol{\rho},\mathbf{r}) = S_{2A}(\mathbf{b},\boldsymbol{\rho}-\mathbf{r})S_{2A}(\mathbf{b},\boldsymbol{\rho})$$

$$\Gamma_{3A}(\mathbf{b},\boldsymbol{\rho},\mathbf{r}) - \Gamma_{2A}(\mathbf{b},\mathbf{r}) =$$

$$\Gamma_{2A}(\boldsymbol{\rho}-\mathbf{r}) + \Gamma_{2A}(\mathbf{b},\boldsymbol{\rho}) - \Gamma_{2A}(\mathbf{b},\mathbf{r}) - \Gamma_{2A}(\mathbf{b},\boldsymbol{\rho}-\mathbf{r})\Gamma_{2A}(\mathbf{b},\boldsymbol{\rho})$$

- $\partial\Gamma_A(x,\mathbf{b},\mathbf{r})/\partial\log\frac{1}{x}$ is a nonlinear functional of Γ_{2A}, the identification of $\Gamma_A(x,\mathbf{b},\mathbf{r})$ with $\Gamma_{2A}(x,\mathbf{b},\mathbf{r})$, and the extension of the first iteration to the closed-form nonlinear equation, would be utterly erroneous.

The momentum-space analysis is illuminating:

- Define nuclear transparency for large dipoles:
$S_A(\mathbf{b},\sigma_0) = \exp[-\frac{1}{2}\sigma_0 T(\mathbf{b})] = 1 - \int d^2\mathbf{k}\phi(\mathbf{k})$
- The first iteration for unintegrated nuclear glue:

$$\frac{\partial \Delta\phi_A(x,\mathbf{b},\mathbf{\Delta})}{\partial \log\frac{1}{x}} = S_A(\mathbf{b},\sigma_0)\mathcal{K}_{BFKL}\otimes\phi(\mathbf{\Delta})$$

$$+K_0\int d^2\mathbf{p}d^2\mathbf{k}\phi(\mathbf{k})\Big\{K(\mathbf{\Delta}+\mathbf{p},\mathbf{\Delta}+\mathbf{k})\phi(\mathbf{p})$$

$$-K(\mathbf{p},\mathbf{p}-\mathbf{\Delta})-\mathbf{k})\phi(\mathbf{\Delta})\Big\}$$

$$= S_A(\mathbf{b},\sigma_0)\mathcal{K}_{BFKL}\otimes\phi(\mathbf{\Delta}) + \mathcal{K}_{NonLin}\Big[\phi(\mathbf{\Delta})\Big]$$

- Contains an absorption suppressed linear BFKL term.
- For central DIS off heavy nuclei $S_A \to 0$. The evolution is entirely driven by the nonlinear term quadratic in $\phi(\mathbf{k})$.
- The connection to the $\mathcal{K}_{BFKL}$:

$$K(\mathbf{p},\mathbf{k}) = \frac{(\mathbf{p}-\mathbf{k})^2}{\mathbf{p}^2\mathbf{k}^2}$$

$$\mathcal{K}_{BFKL}\otimes f(\mathbf{\Delta}) =$$

$$K_0\int d^2\mathbf{p}\Big[2K(\mathbf{\Delta},\mathbf{\Delta}-\mathbf{p})f(\mathbf{p}) - K(\mathbf{p},\mathbf{p}-\mathbf{\Delta})f(\mathbf{\Delta})\Big]$$

★ Hard gluons, $\mathbf{\Delta}^2 > Q_A^2$:

$$\mathcal{K}_{NonLin}\Big[\phi(\mathbf{\Delta})\Big] = \Big[\int d^2\mathbf{p}\phi(\mathbf{p})\Big] \cdot \mathcal{K}_{BFKL} \otimes \phi(\mathbf{\Delta})$$

$$\frac{\partial \Delta \phi_A(x, \mathbf{\Delta})}{\partial \log \frac{1}{x}} = \mathcal{K}_{BFKL} \otimes \phi(\mathbf{\Delta}) \quad !!$$

- A remarkable recovery of the linear BFKL evolution for hard gluons from the nonlinear component of the nuclear X-section. The related color dipole derivation to all orders of DLLA by B.Zakharov (Zakharov '98).
- An expected result in view of unshadowed hard collective glue: $\phi(\mathbf{\Delta}) = \nu(\mathbf{b}) f(\mathbf{\Delta})$.

★ Heavy nucleus, soft gluons , $\mathbf{\Delta}^2 \ll Q_A^2$:

$$\frac{\partial \phi_A(x, \mathbf{b}, \mathbf{\Delta})}{\partial \log \frac{1}{x}} = -2C\pi K_0 \phi(x, \mathbf{b}, 0)$$

- $C \sim 1$, depends on the form of the collective nuclear glue.
- Because $\phi(x, \mathbf{b}, 0) \sim 1/Q_A^2(\mathbf{b})$, this entails an expanding plateau with the rising saturation scale

$$Q_A^2(\mathbf{b}) \Longrightarrow Q_A^2(\mathbf{b}) \left[1 + 2C\pi K_0 \log \frac{1}{x}\right]$$

- The full-fledged nonlinear evolution for $\mathbf{\Delta}^2 \sim Q_A^2$.

★ Diffractive DIS: still a marginal $\lesssim 10\%$ of total DIS at HERA, but 50% for heavy nuclei (NNN,Zakharov,Zoller '94). A manifestly nonlinear observable. First a free nucleon target:

- $\gamma^* \to q\bar{q}$: excitation of low diffractive masses, $M^2 \sim Q^2$:

$$\left.\frac{d\sigma_D}{dt}\right|_{t=0} = \frac{1}{16\pi}\langle\gamma^*(q\bar{q})|\sigma^2(x,\mathbf{r})|\gamma^*(q\bar{q})\rangle$$

- $\gamma^* \to q\bar{q}g$: excitation of high diffractive masses, $M^2 \gg Q^2$,

$$\beta = \frac{x}{x_{\mathbf{P}}} = \frac{Q^2}{Q^2+M^2} \ll 1$$

$$M^2\left.\frac{d\sigma_D}{dM^2dt}\right|_{t=0} = \frac{\partial}{\partial \log\frac{1}{\beta}}\left.\frac{d\sigma_D}{dt}\right|_{t=0}$$

$$= \frac{1}{16\pi}\langle\gamma^*(q\bar{q}g)|\Big(\sigma_3(x_{\mathbf{P}},\boldsymbol{\rho},\mathbf{r}) - \sigma(x_{\mathbf{P}},\mathbf{r})\Big)^2|\gamma^*(q\bar{q}g)\rangle$$

- Color dipole factorization representation & definition of unintegrated diffractive glue:

$$\left.\frac{d\sigma_D}{dt}\right|_{t=0} = \frac{\sigma_0(x_{\mathbf{P}})}{16\pi}\langle\gamma^*(q\bar{q})|\sigma_D(\mathbf{r})|\gamma^*(q\bar{q})\rangle$$

$$\sigma_D(\mathbf{r}) = \sigma_0(x_{\mathbf{P}})\int d^2\boldsymbol{\Delta} f_D(\boldsymbol{\Delta})[1-\exp(i\boldsymbol{\Delta}\mathbf{r})]$$

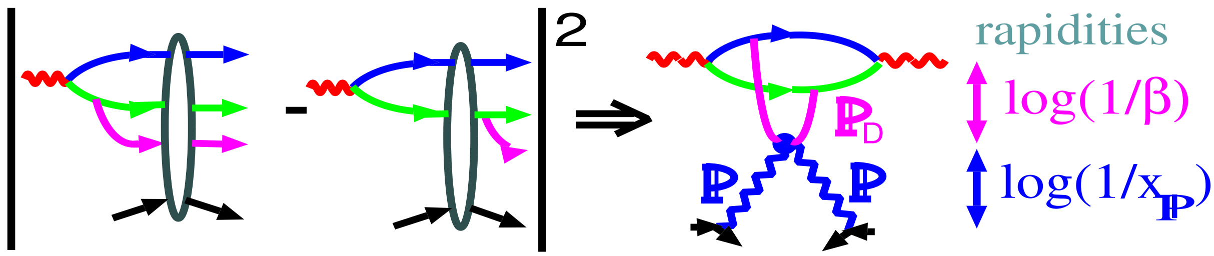
- The $q\bar{q}$ excitation: the manifestly nonlinear

$$f_{2D}(\boldsymbol{\Delta}) = 2f(\boldsymbol{\Delta}) - f\otimes f(\boldsymbol{\Delta}) = 2f(\boldsymbol{\Delta}) - f^{(2)}(\boldsymbol{\Delta})$$

★ Diffraction as inclusive DIS off a pomeron: the $q\bar{q}g$ excitation defines a gluon density in the valence gg state of the diffractive Pomeron.

★ Also a definition of the triple-pomeron vertex $A_{3\mathbb{P}}$.

★ The diffractive glue $f_D(\beta, \mathbf{b}, \mathbf{\Delta})$ is related to the diffractive Pomeron $\mathbb{P}_D$ in precisely the same way as $f(\mathbf{\Delta})$ to the Pomeron $\mathbb{P}$ of inclusive DIS.



$$\frac{\partial f_D(\beta, \mathbf{b}, \mathbf{\Delta})}{\partial \log \frac{1}{\beta}} = \mathcal{K}_{BFKL} \otimes f_{2D}(\mathbf{\Delta})$$

$$+2K_0 \int d^2\mathbf{p} d^2\mathbf{k} \Big\{ \Big[2K(\mathbf{\Delta}+\mathbf{p}, \mathbf{\Delta}+\mathbf{p}+\mathbf{k})$$

$$+K(\mathbf{\Delta}+\mathbf{p}, \mathbf{\Delta}+\mathbf{k}) \Big] f(\mathbf{p}) f(\mathbf{k})$$

$$+ \Big[K(\mathbf{\Delta}+\mathbf{p}, \mathbf{\Delta}+\mathbf{k}) - K(\mathbf{p}, \mathbf{p}-\mathbf{k}) \Big] f(\mathbf{\Delta}) f(\mathbf{k})$$

$$-K(\mathbf{p}, \mathbf{p}-\mathbf{k}) f(\mathbf{\Delta}-\mathbf{k}) f(\mathbf{k}) \Big\}$$

$$= \mathcal{K}_{BFKL} \otimes f_{2D}(\mathbf{\Delta}) + \mathcal{K}^{(\mathcal{D})}{}_{NonLin} \Big[f(\mathbf{\Delta}) \Big]$$

- Large $\mathbf{\Delta}^2$: the contribution from $\mathcal{K}^{(\mathcal{D})}{}_{NonLin}$ vanishes faster than the usual perturbative $1/\mathbf{\Delta}^4$.

- The DLLA regime of large $\mathbf{\Delta}^2$: the linear BFKL contribution matches the DLLA to DGLAP

$$\mathcal{K}_{BFKL}\otimes f_{2D}(\mathbf{\Delta})\Big|_{DLLA} = \frac{2K_0}{\Delta^4}\int^{\mathbf{\Delta}^2} d^2\mathbf{p}\,\mathbf{p}^2 f_{2D}(\mathbf{p})$$

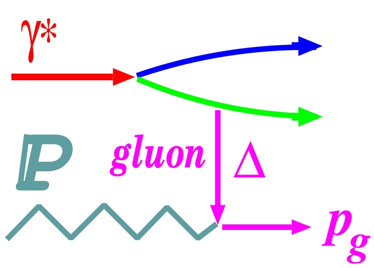
- The standard case: $\int^{\mathbf{\Delta}^2} d^2\mathbf{p}\,\mathbf{p}^2\, f(\mathbf{p}) \propto G(\mathbf{\Delta}^2) \propto \log \mathbf{\Delta}^2$.
- Diffraction is a special case: $f_{2D}(\mathbf{\Delta})$ is not positive-defined, satisfies the sum rule

$$\int d^2\mathbf{p}\,\mathbf{p}^2 f_{2D}(\mathbf{p}) = 0.$$

Hard diffractive glue $f_D(\beta, \mathbf{\Delta})$ has a vanishing leading twist component $\propto 1/\Delta^4$. Recall valence quarks in the nucleon before DGLAP evolution.

- The effect of depletion of $f^{(2)}(\mathbf{p})$ at small $\mathbf{p}^2$ is compensated for by higher twist effects at large $\mathbf{p}^2$.
- The related absence of $\log \frac{1}{\mathbf{r}^2}$ in $\sigma_D(\mathbf{r})$ has been noticed earlier in the color dipole DLLA treatment of high-mass diffraction (NNN,Zakharov '93).
- $\star$ The single-particle inclusive spectrum of diffractive gluons from $\gamma^* \to q\bar{q}g$ excitation (NNN' 99):

$$\frac{dN_g}{d^2\mathbf{p}_g} \propto \frac{G^2(x_{\mathbf{P}}, \mathbf{p}_g^2)}{p_g^4}$$



A naive treatment of the Pomeron as a particle suggests:

$p_g = \Delta$

and is at best misleading

★ High-mass diffractive DIS off heavy nuclei: $S_A = 0$.

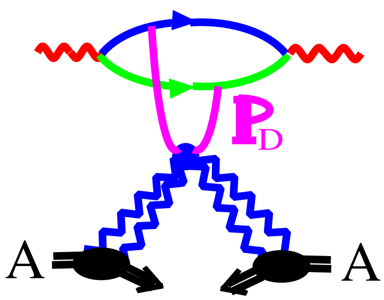
$$\frac{\partial \Delta \phi_D(\beta, \mathbf{b}, \mathbf{\Delta})}{\partial \log \frac{1}{\beta}} = 2K_0 \left[\int d^2\boldsymbol{\kappa}\phi(\boldsymbol{\kappa})\right] \int d^2\mathbf{p} d^2\mathbf{k} d^2\mathbf{q} \phi(\mathbf{k})\phi(\mathbf{q})$$

$$\times \Big\{ K(\mathbf{\Delta} + \mathbf{p} + \mathbf{k}, \mathbf{\Delta} + \mathbf{q} + \mathbf{k})\phi(\mathbf{p})$$

$$-K(\mathbf{p}, \mathbf{p} - \mathbf{k} - \mathbf{q})\phi(\mathbf{\Delta} - \mathbf{q})$$

$$+K_0 \int d^2\mathbf{p} d^2\mathbf{k} \phi^{(2)}(\mathbf{k}) \Big\{ K(\mathbf{p}, \mathbf{p} - \mathbf{k} - \mathbf{q})\phi^{(2)}(\mathbf{\Delta})$$

$$-K(\mathbf{p}, \mathbf{p} - \mathbf{k} - \mathbf{\Delta})\phi^{(2)}(\mathbf{p}) \Big\}$$



Reinterpretation with certain reservations: the pomeron in nuclear high-mass diffraction evolves from four nuclear pomerons

- High mass diffraction off heavy nuclei can not be described by the same triple pomeron coupling as in the free nucleon case.
- Color dipole representation for the low-mass diffraction off nuclei is identical to that for a free nucleon target:

$$f_{2D}(\mathbf{\Delta}) \Longrightarrow \phi_{2D}(\mathbf{\Delta})$$

$$= 2\phi(\mathbf{\Delta}) - \phi \otimes \phi(\mathbf{\Delta}) = 2\phi(\mathbf{\Delta}) - \phi^{(2)}(\mathbf{\Delta})$$

- Heavy nuclei, hard diffractive glue: $\mathbf{\Delta}^2 > Q_A^2$.

$$\frac{\partial \Delta \phi_D(\beta, \mathbf{b}, \mathbf{\Delta})}{\partial \log \frac{1}{\beta}} \simeq K_0 \quad d^2\mathbf{p} d^2\mathbf{k} K(\mathbf{\Delta} - \mathbf{p}, \mathbf{\Delta} - \mathbf{k})$$

$$\times \left[2\phi(\mathbf{p})\phi(\mathbf{k}) - \phi^{(2)}(\mathbf{p})\phi^{(2)}(\mathbf{k}) \right]$$

$$\simeq \mathcal{K}_{BFKL} \otimes \phi_{2D}(\mathbf{\Delta})$$

- Coincides with the free nucleon case subject to

$$f(\mathbf{\Delta}) \Longleftrightarrow \phi(\mathbf{\Delta})$$

- The sum rule $\int d^2\mathbf{p} \phi_{2D}(\mathbf{p}) = 0$ holds for nuclei too: entails a vanishing leading twist component, $\propto 1/\mathbf{\Delta}^4$, of nuclear diffractive unintegrated glue.

- Experience with color dipole DLLA for diffraction : DGLAP properties of hard diffractive glue will be recovered to higher orders of evolution (NNN, Zakharov '93).

  ★ Soft gluons, $\mathbf{\Delta}^2 > Q_A^2$:

$$\frac{\partial \Delta \phi_D(\beta, \mathbf{b}, 0)}{\partial \log \frac{1}{\beta}} = 2\pi \left[B_D \phi(\mathbf{b}, 0) + C_D \phi^{(2)}(\mathbf{b}, 0) \right]$$

$B_D, C_D \sim 1$ are model-dependent factors.

Conclusions

- Saturation $\Longleftrightarrow$ opacity of heavy nuclei to large color dipoles.
- Expansion of nuclear unintegrated glue in terms of *collective glue of overlapping nucleons* in the Lorentz-contracted ultrarelativistic nucleus.
- A framework for the *non-abelian* aspects of intranuclear evolution of color dipoles/multiparton systems.
- Collective nuclear glue (*nuclear Pomeron*) is a useful concept, but color coupled channels *destroy $k_\perp$ factorization* for two-particle spectra, and for certain single-particle spectra .
- Nonlinear $k_\perp$-factorization for forward dijet production off nuclei in terms of the collective nuclear glue: a semblance of the *four nuclear Pomeron exchange*.
- Complete first step of $\log \frac{1}{x}, \log \frac{1}{\beta}$ evolution for nuclear DIS and diffraction. No closed-form evolution for (*nuclear Pomeron*) because of broken $k_\perp$-factorization.
- *Recovery of linear BFKL evolution* for hard collective nuclear gluons (nuclear Pomeron) beyond the saturation scale even for strongly absorbing nuclei.
- *Recovery of linear BFKL evolution* for unintegrated diffractive gluon density (diffractive Pomeron) in the hard regime.
- Unusual sign-indefinite boundary condition for diffractive unintegrated glue (diffractive Pomeron).