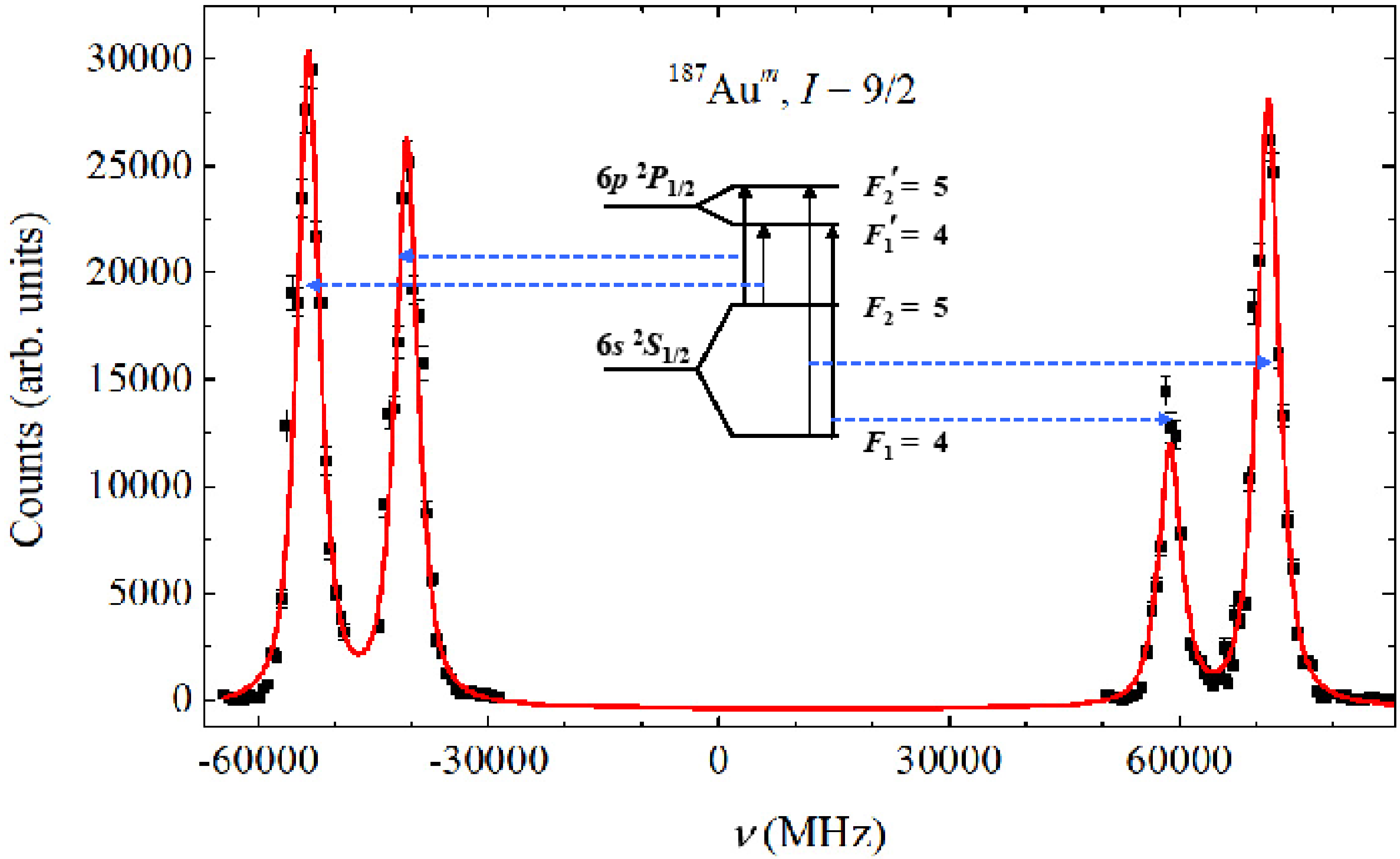
Сверхтонкая аномалия: статус, проблемы и перспективы

Hyperfine anomaly: status, problems and perspectives

A. Barzakh
Petersburg Nuclear Physics Institute, Gatchina, Russia
on behalf of Windmill-ISOLTRAP-RILIS collaboration

ISOLDE-RILIS

In-source laser spectroscopy of the neutron-deficient Au isotopes: shape coexistence and shape evolution studies



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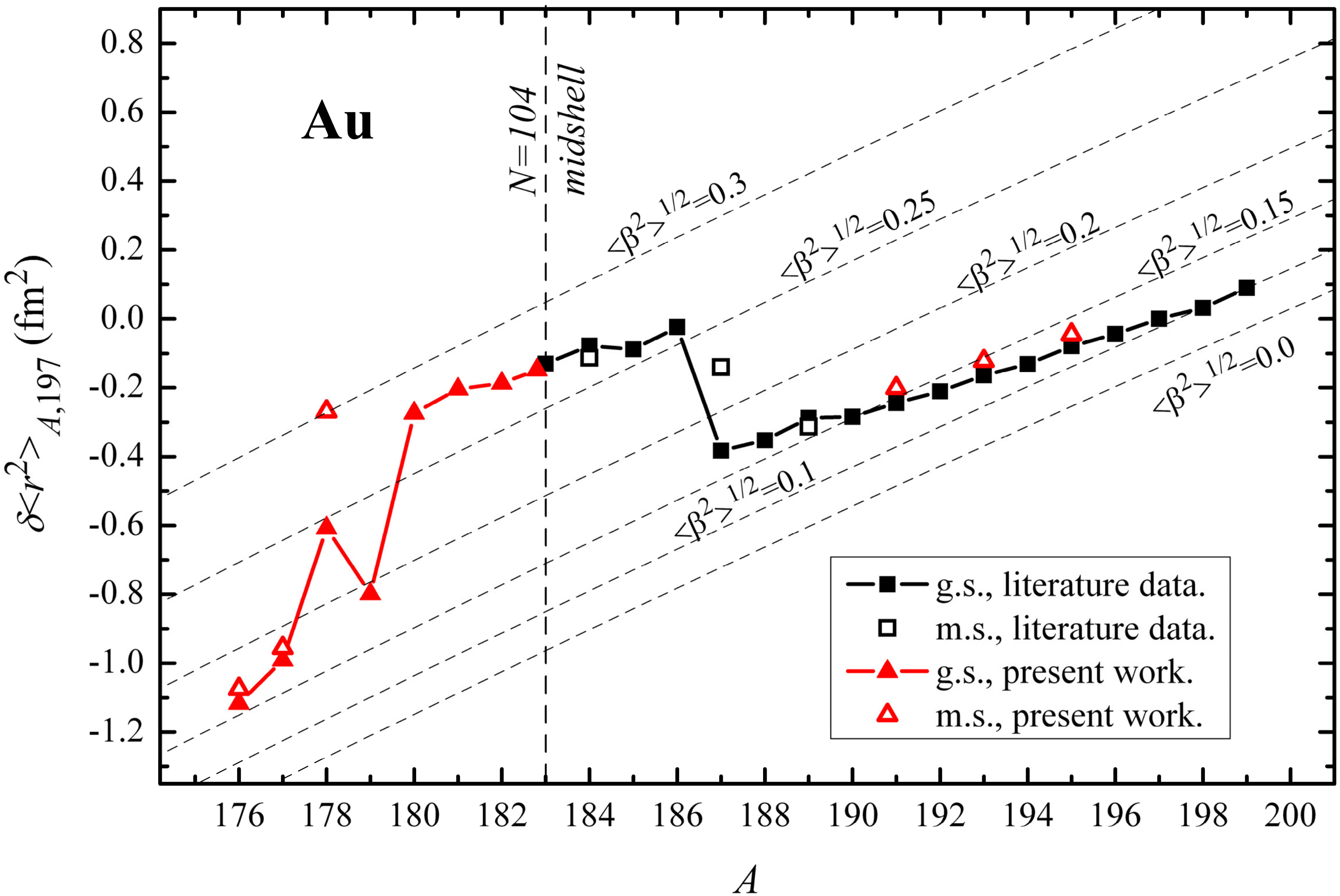


REMINDER:

$$\frac{a_1 \cdot I_1}{\mu_1} = const \quad \text{for point-like nucleus}$$

Due to the spatial distribution of the magnetic moment and the charge inside a finite-size nucleus this ratio changes from one nucleus to another

$$a = a_{point}(1+\varepsilon_A)(1+\delta_A)$$

ε — BW correction (magnetization distribution);
δ — BR correction (charge distribution)

$$^{A_1}\Delta^{A_2} = \frac{a_1 \cdot I_1}{\mu_1} \cdot \frac{\mu_2}{a_2 \cdot I_2} - 1 = {}^{A_0}\Delta^{A_1}_{BW} + {}^{A_0}\Delta^{A_1}_{BR} \approx (\varepsilon_0 - \varepsilon_1) + (\delta_0 - \delta_1)$$

For heavy atoms Δ_{BR} is expected to be negligible compared to Δ_{BW}.

$^{A_0}\Delta^{A_0+2}_{BR}(Au) \sim 1 \cdot 10^{-4},\ \Delta_{BW}(Au) \sim 1-10\%$ Accordingly, below we will ignore Δ_{BR} contribution to RHFA for Au.

REMINDER:

$$^{A_1}\Delta^{A_2} = \frac{a_1 \cdot I_1}{\mu_1} \cdot \frac{\mu_2}{a_2 \cdot I_2} - 1 \approx (\varepsilon_{A_1} - \varepsilon_{A_2}) \qquad \text{RHFA}$$

To obtain experimental values of RHFA one should measure μ and a independently → ABMR, NMR-ON etc. methods of the independent μ measurement may be applied only for stable and comparatively long-lived nuclei with high yield due to low efficiency of these techniques.

HFA, THEORY: BOHR & WEISSKOPF

A. Bohr and V. Weisskopf, 1950; A. Bohr, 1951

extreme single-particle model in nuclear sector

one-electron approximation in atomic sector

pure atomic part

pure nuclear part

$$\varepsilon = -b\left[\left(1+\frac{2}{5}\zeta\right)\alpha_s + \frac{3}{5}\alpha_L\right]\frac{\left\langle r^2\right\rangle_{nlj}}{R_0^2}$$

$$\alpha_s = \frac{g_s}{g_I}\frac{g_I - g_L}{g_s - g_L}, \ \alpha_s = 1-\alpha_L, \ \zeta \propto \zeta\left(I\right)$$

Factorization: ratio of anomalies for the different atomic states is independent from nuclear properties

WHY SHOULD WE STUDY HFA?

1. Accurate magnetic moment values

$$\mu_A = \mu_{A_0} \cdot \frac{I_A}{I_{A_0}} \cdot \frac{a_A}{a_{A_0}} \cdot (1 + {}^{A_0}\Delta^{A})$$

${}^{A_1}\Delta^{A_2}$ is of order of 10^{-2} for heavy nuclei; however for Au ~ 10%!

$$b \sim (Z \cdot A^{1/3})^{2(1-\rho)}, \; \rho = (1 - (\alpha Z)^2)^{1/2}$$

This means that for Z~90 HFA is larger nearly 2 times than for Z=79

WHY SHOULD WE STUDY HFA?

1. Accurate magnetic moment values

2. Application to QED test. Benchmark for atomic and molecular calculations in search for parity violation and other effects in atomic/molecular systems

WHY SHOULD WE STUDY HFA?

1. Accurate magnetic moment values

2. Application to QED test. Benchmark for atomic and molecular calculations

3. Information on the magnetization distribution;
   sensitivity to configurations mixture (other than in μ)

“We should use the HFA data in conjunction with the values of the magnetic moment”

$$^{A_1}\Delta^{A_2} = \frac{a_1 \cdot I_1}{\mu_1} \cdot \frac{\mu_2}{a_2 \cdot I_2} - 1$$

MOSKOWITZ-LOMBARDI RULE

$$\varepsilon = \alpha / \mu$$

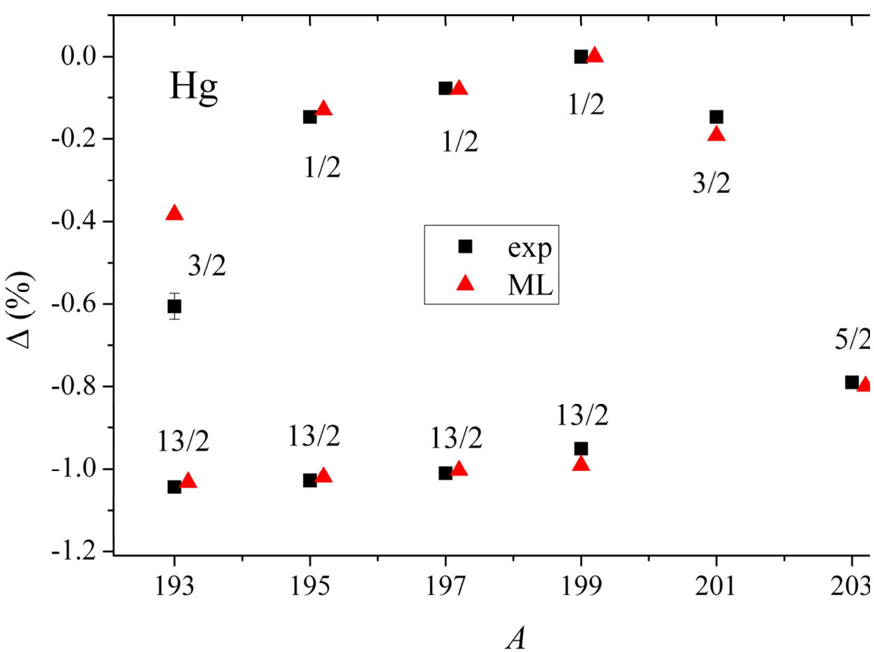
P. A. Moskowitz and M. Lombardi, Phys. Lett. 46B (1973) 334

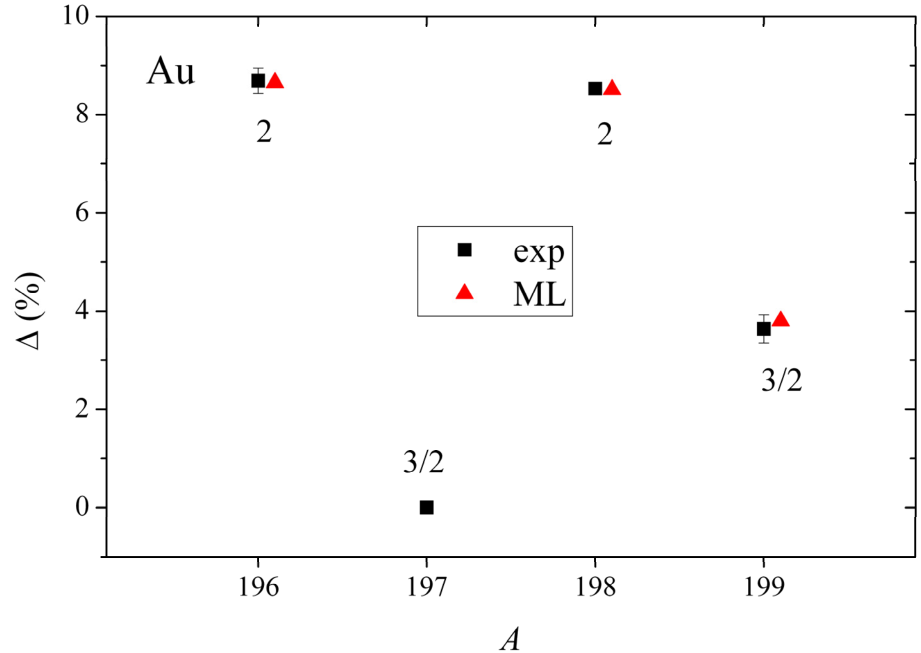
$$^{1}\Delta^{2} = \alpha\left(\frac{1}{\mu_1} - \frac{1}{\mu_2}\right), \quad \alpha = \pm 1.0 \cdot 10^{-2}\,(\mathrm{Hg}),$$

C Ekström et al., Nucl. Phys. A 348, (1980) 25

$$i = l \pm 1/2$$

$$\alpha = \pm 1.2 \cdot 10^{-2}\,(\mathrm{Au}),$$





$^{191}\Delta^{193}(\mathrm{Ir})_{ML} = 1.13\,[1/\mu(191) - 1/\mu(193)] = 0.63\%$

$^{191}\Delta^{193}(\mathrm{Ir})_{exp} = 0.64(7)\%$

S. Büttgenbach et al.,
Z. Physik A 286, 333-340 (1978)

MOSKOWITZ-LOMBARDI RULE

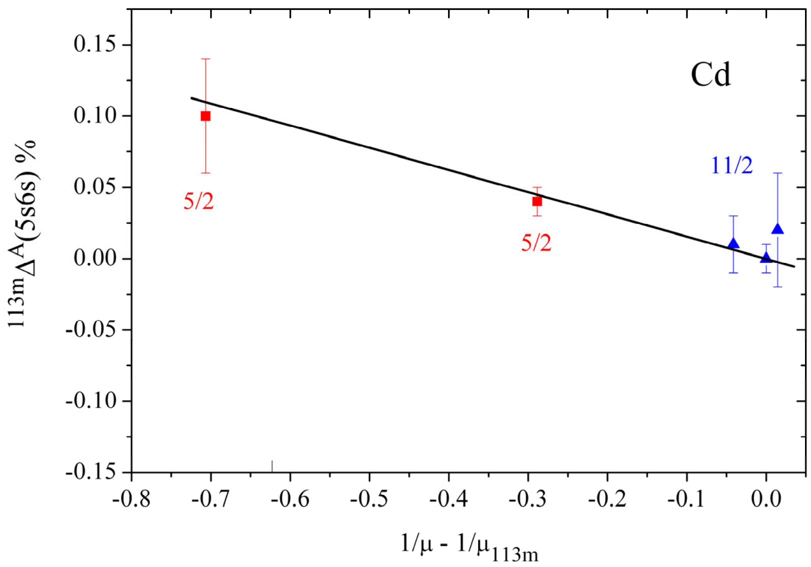
T. Fujita, A. Arima, Nucl. Phys. A254, 513 (1975)
(theoretical calculations in the framework of the core-polarization model)

$$\varepsilon = c_1 + \frac{\alpha}{\mu} \Longrightarrow$$

ML rule. However, α in theory is strongly state dependent and success of ML rule for dozen configurations with the same α remains unexplainable

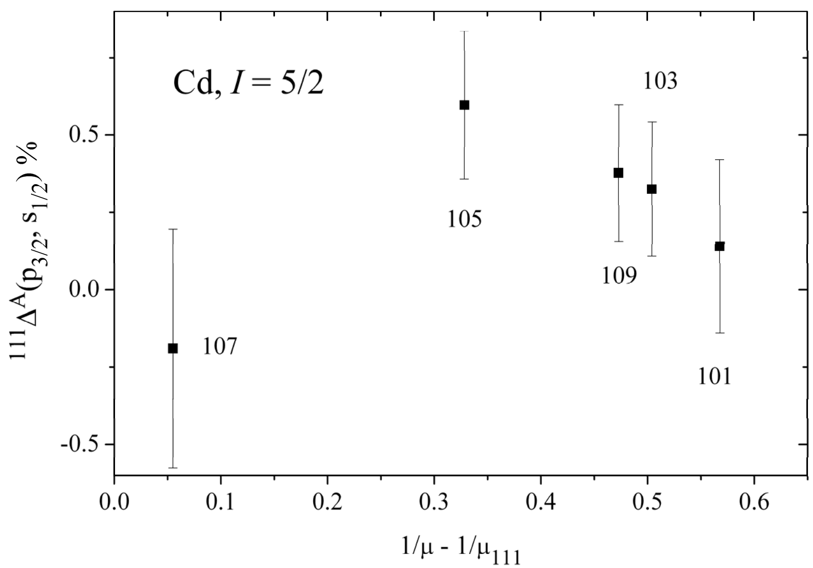
It was shown recently that this rule is (at least) not universal and does not work for rare-earths: J. R. Persson, Hfi 162, 139 (2005).

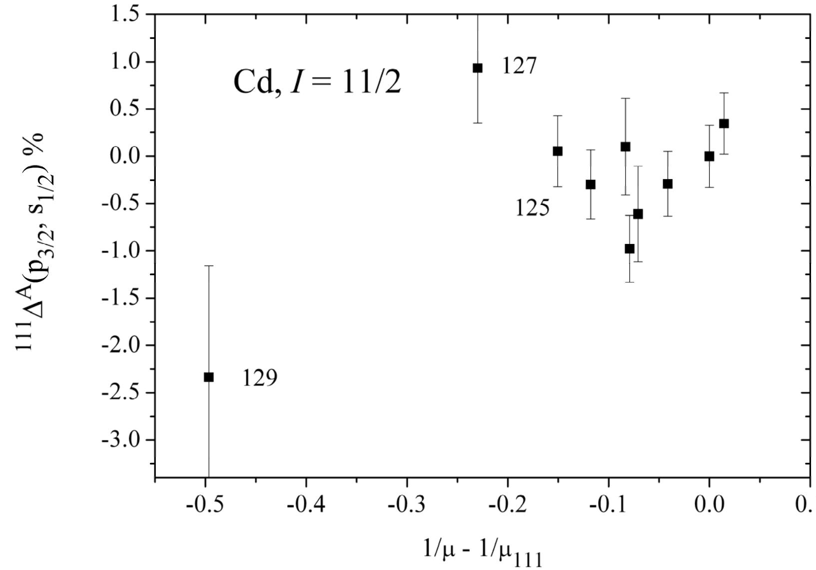
ML RULE in Cd



"Weak" ML rule (FA rule): linear dependence of ${}^{ref}\Delta^{A}$ on $(1/\mu - 1/\mu_{ref})$

However, ML rule (and its modifications) remains the most popular for at least qualitative estimation of HFA





N. Frömmgen et al., Eur. Phys. J. D 69, 164 (2015);

EKSTRÖM PRESCRIPTION for GOLD

$$^{197}\Delta^{195} = \alpha\left(\frac{1}{\mu_{197}} - \frac{1}{\mu_{195}}\right), \quad \alpha = 1.2\cdot 10^{-2}$$

$$\Downarrow$$

$$\varepsilon_{197} = \frac{\alpha}{\mu_{197}} = 8.21\%$$

$$a_{197} = a_{point}(1+\varepsilon_{197})$$

$$\Rightarrow$$

$$\mu = \frac{aI}{29005} \pm 0.012$$

$$\varepsilon = \mp\frac{1.2}{\mu}\%$$ for shell-model single-particle states with $I = l \pm 1/2$

$$\mu = \frac{aI}{29005(1+\varepsilon)}$$ in general case

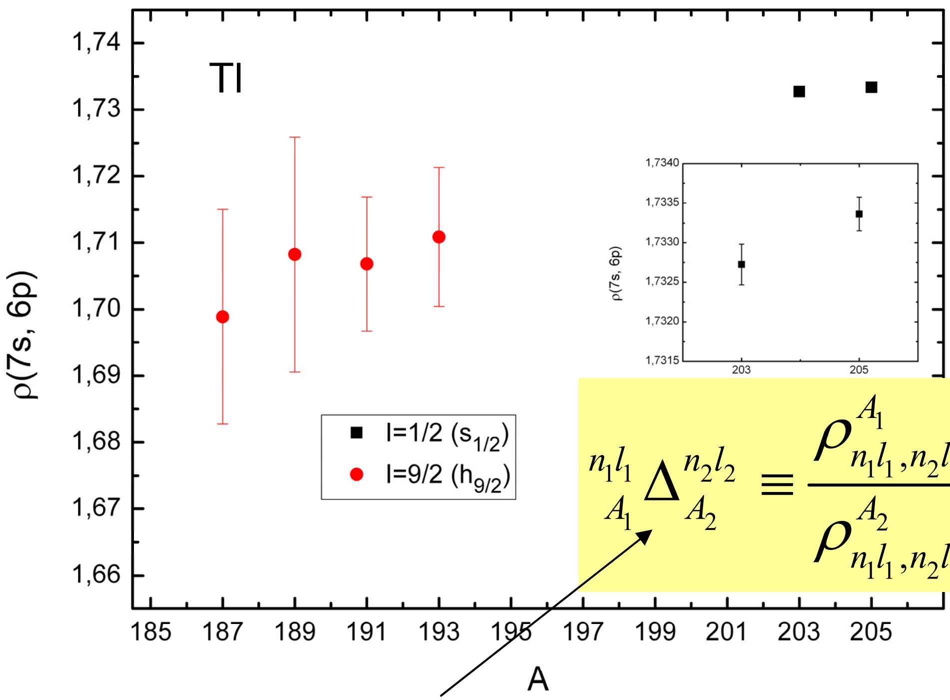
Drawbacks of the Ekström prescription: $\varepsilon = c_1 + \dfrac{\alpha}{\mu},\ c_1 \neq 0\,!$

1. c_1 from FA relation is in the region of -0.02÷0.02 → additional ~3% error
2. unknown ε gives additional ~3% error

DIFFERENTIAL HYPERFINE ANOMALY

$$\rho^{A}_{n_1 l_1, n_2 l_2} = \frac{a^{A}_{n_1 l_1}}{a^{A}_{n_2 l_2}}$$

Ratio $\rho^{A}_{n_1 l_1, n_2 l_2}$ may have a different value for different isotopes because the atomic states with different n, l have different sensitivity to the nuclear magnetization distribution.



Tl: $6p_{1/2}$ and $7s_{1/2}$ states

$$ {}^{n_1 l_1}_{A_1}\Delta^{n_2 l_2}_{A_2} \equiv \frac{\rho^{A_1}_{n_1 l_1, n_2 l_2}}{\rho^{A_2}_{n_1 l_1, n_2 l_2}} - 1 = {}^{A_1}\Delta^{A_2}(n_1 l_1) - {}^{A_1}\Delta^{A_2}(n_2 l_2) $$

DHFA

DHFA $\Longrightarrow$ RHFA $\Longrightarrow$ μ correction

$$\eta \equiv \frac{{}^{A_1}\Delta^{A_2}(n_1 l_1)}{{}^{A_1}\Delta^{A_2}(n_2 l_2)} \quad \Longrightarrow \quad {}^{A_1}\Delta^{A_2}(n_2 l_2) = \frac{{}^{n_1 l_1}_{A_1}\Delta^{n_2 l_2}_{A_2}}{\eta - 1}$$

factorization: η is independent of A

determination of RHFA without independent high-accuracy μ-measurements

$$\mu_A = \mu_{A_0} \cdot \frac{I_A}{I_{A_0}} \cdot \frac{a_A}{a_{A_0}} \cdot (1 + {}^{A_0}\Delta^{A})$$

This new method of RHFA determination was proposed in:
V. J. Ehlers *et al*., Phys. Rev. 176, 25 (1968).
J. R. Persson, Eur. Phys. J. A **2**, 3 (1998)

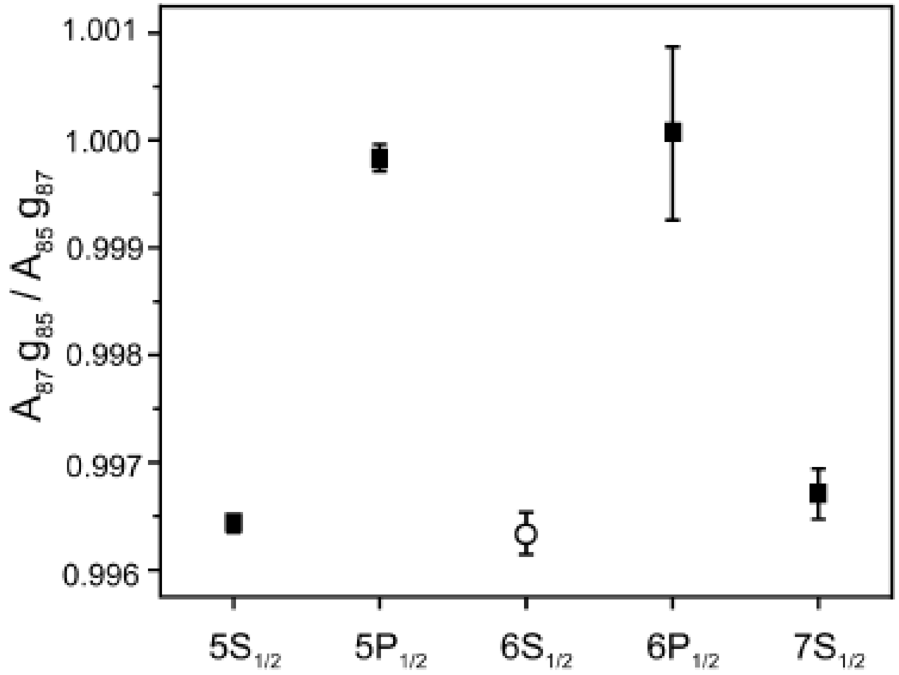
and applied for far from stability Tl nuclei in:
A. E. Barzakh *et al.* Phys. Rev. C **86**, 014311 (2012)
and for ^{208}Bi in: S. Schmidt *et al*., Phys. Lett. B 779 (2018) 324

INDEPENDENCE of the PRINCIPAL QUANTUM NUMBER n

^{85}Rb — ^{87}Rb

state	Δ (%)
5s $^2S_{1/2}$	0.35142(30)
6s $^2S_{1/2}$	0.36(2)
7s $^2S_{1/2}$	0.33(2)

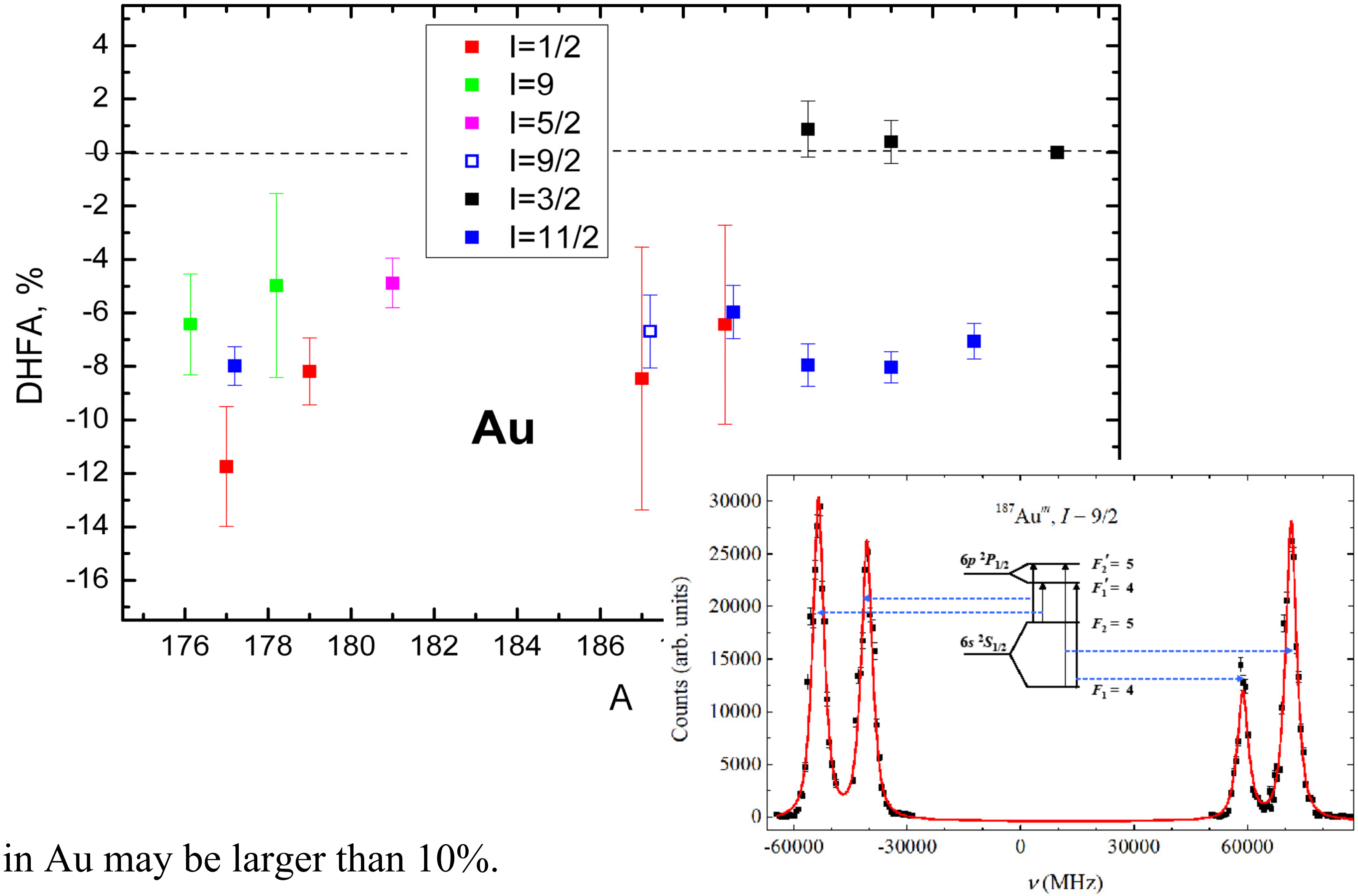
independence of n is proven with the accuracy of ~6%



A. Pérez Galván et al., Phys. Lett. B 655 (2007) 114

DHFA in GOLD

A. E. Barzakh et al., Phys. Rev. C **101**, 034308 (2020)
Yu. A. Demidov et al., Phys. Rev. A **103**, 032624 (2021)



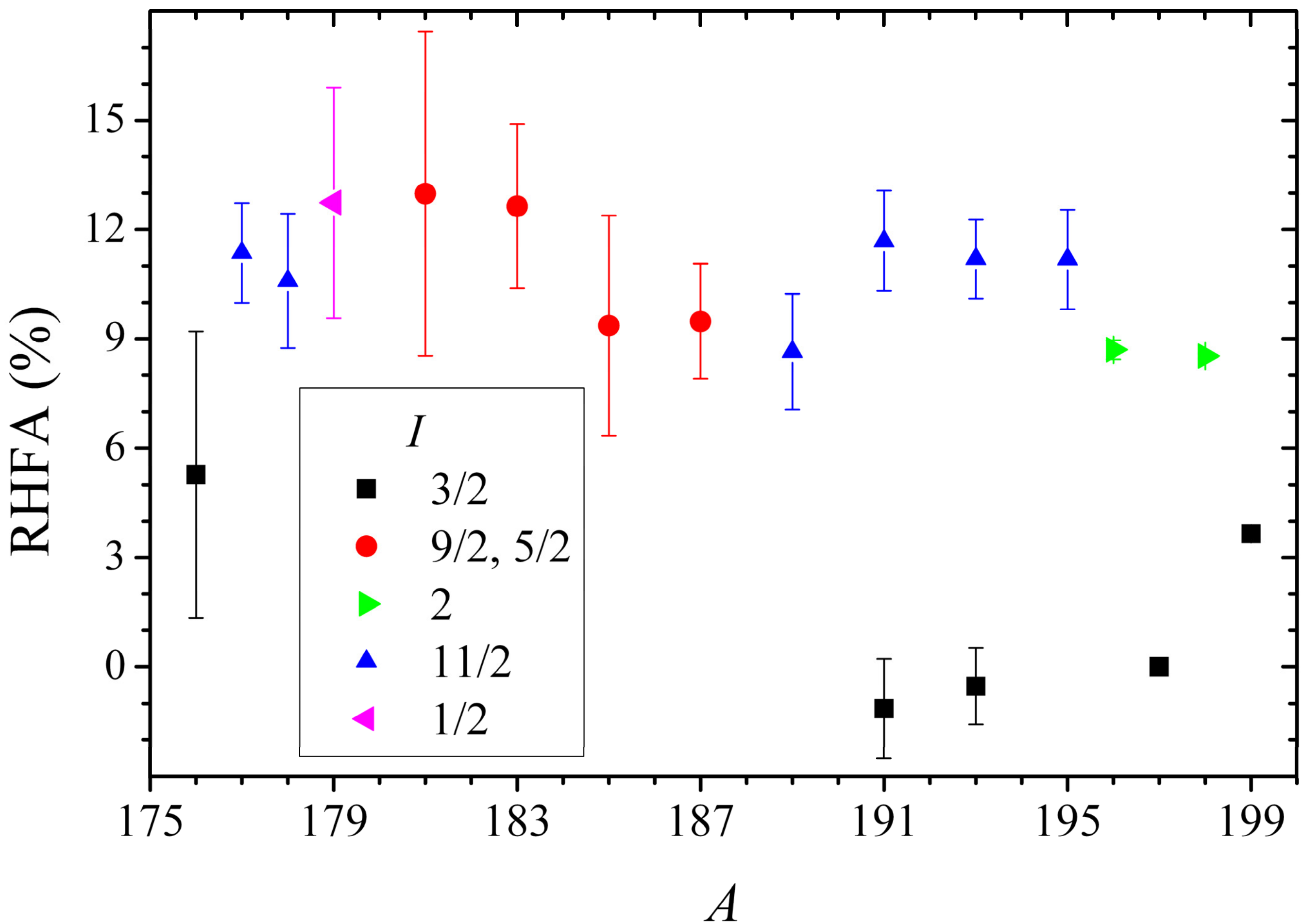
RHFA in Au may be larger than 10%.

To extract μ properly one needs calculation/measurement of η-factor.

Calculations give: $\eta = 4.03(30)$

η (Au; 6s, $6p_{1/2}$) = 4.03(30)

3.19	A. Bohr and V. F. Weisskopf (1950)
3.51	J. Eisinger and V. Jaccarino (1958)
4.50	H. H. Stroke, R. J. Blin-Stoyle, and V. Jaccarino (1961)
3.33	S. Song, G. Wang, A. Ye, and G. Jiang (2007)



RHFA AND CORRECTED μ FOR NEUTRON-DEFICIENT Au ISOTOPES

η (Au; 6s, $6p_{1/2}$) = 4.03(30)

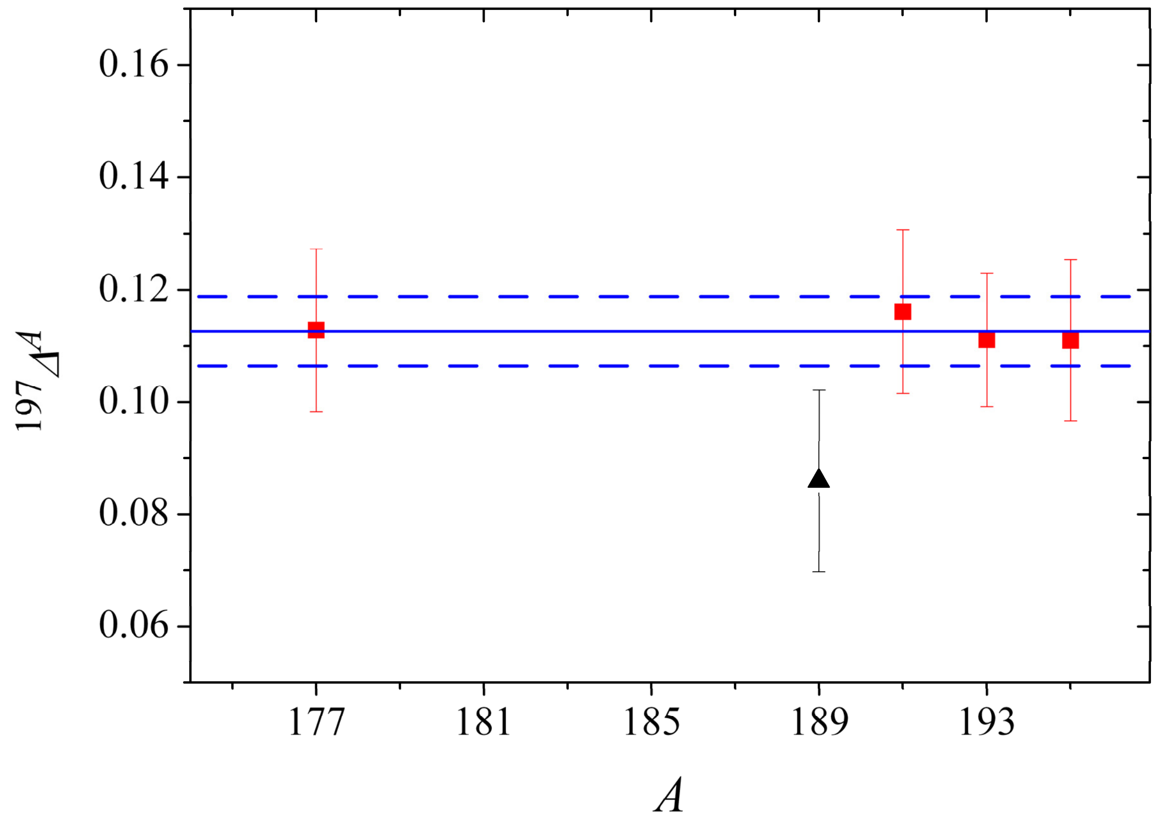
A	I	$^{197}\Delta^A$	$\mu(\mu_N)$
176m1	2	0.04(18)	-0.767(43)
176m2	9	0.053(39)	5.19(20)
177	1/2	0.178(63)	1.257(67)
177m	11/2	0.114(14)	6.519(38)
178	3	0.11(8)	-0.962(77)
178m	8	0.106(18)	4.895(82)
179	1/2	0.127(32)	1.050(30)
180	1	0.21(14)	-0.826(94)

A	I	$^{197}\Delta^A$	$\mu(\mu_N)$
181	3/2	0.130(45)	1.232(49)
182	2	0.174(65)	1.659(93)
183	5/2	0.126(23)	2.066(42)
187m	9/2	0.095(16)	3.529(53)
189m	11/2	0.087(16)	6.365(38)
191m	11/2	0.117(14)	6.326(37)
193m	11/2	0.112(11)	6.320(37)
195m	11/2	0.112(14)	6.316(37)

RECALCULATION OF THE PREVIOUSLY MEASURED Au MAGNETIC MOMENTS

A	I	$\mu_{\mathrm{lit}}(\mu_N)$	${}^{197}\Delta^{A}(6s)$ (%)	$\mu_{\mathrm{recalc}}(\mu_N)$
194	1	0.0763(13)	1.8(33)	0.0754(25)
193	3/2	0.1396(5)	–0.5(11)	0.1398(15)
191	3/2	0.1369(9)	–1.2(14)	0.1363(19)
189	1/2	0.494(14)	9.4(59)	0.499(27)
189m	11/2	6.186(20)	8.6(16)	6.365(38)
187	1/2	0.535(15)	12.7(84)	0.557(41)
186	3	–1.263(29)	3.1(51)	–1.202(60)
185	5/2	2.170(17)	9.4(30)	2.193(61)

RHFA for 11/2⁻ ISOMERS in GOLD

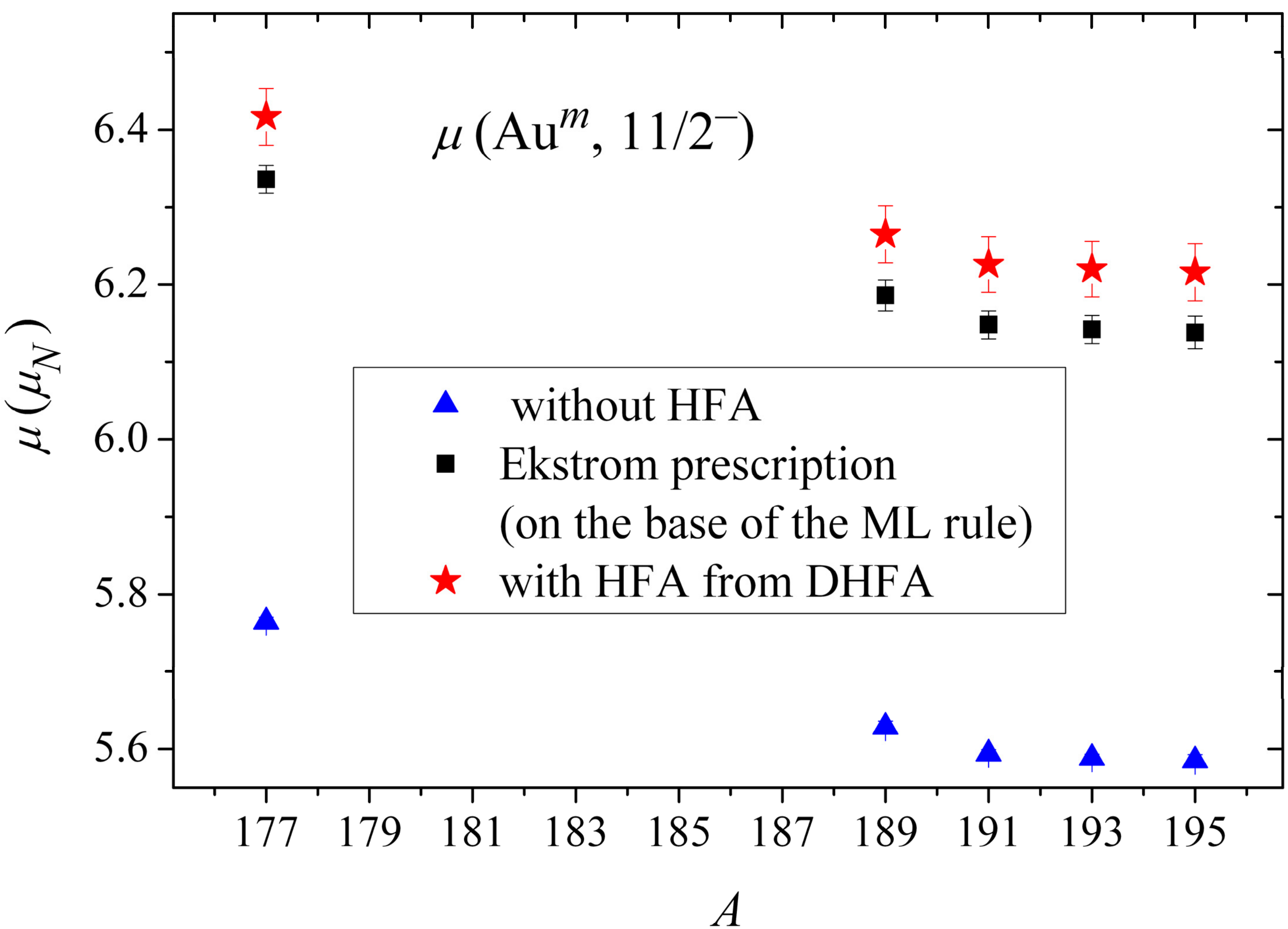


$^{197}\Delta^{191m}(6s;\,\mathrm{ML}) = 0.084$

$\Longrightarrow$ failure of the ML rule

$^{197}\Delta^{(I\,=\,11/2)}(6s) = 0.113(6)$

MAGNETIC MOMENTS of $I = 11/2^-$ Au ISOMERS



NUCLEAR FACTOR

$$\delta = b_N\,(R/\lambda)^{\chi},\ \varepsilon = b_M\,d_{\mathrm{nuc}}\,(R/\lambda)^{\chi},\ \chi = 2\sqrt{1-(Z\alpha)^2} - 1$$
$$a = a_0\,(1-\delta)\,(1-\varepsilon)$$

$^{201,\,199}$Hg II: hfs constants calculated with uncertainties 0.06 and 0.08% and ε = 2.0 and 1.8 (ML)

$\Downarrow$

conservative estimation of a_0 uncertainty ~2.5%

A	I	d_{nuc}, expt	d_{nuc}, sp	
199	3/2	3.2(5)	3.7	
197	3/2	5.1(5)	8.0	$\longrightarrow$ 7% error in a
195m	11/2		0.73	

ATOMIC CALCULATIONS of η FACTOR in Ra^+, HFA and μ EVALUATION

$\eta\ (Ra^+;\ 7s_{1/2},\ 7p_{1/2})_{expt} = 2.15(80)$

theory:

$\eta = 2.90$ (Demidov, 2021)

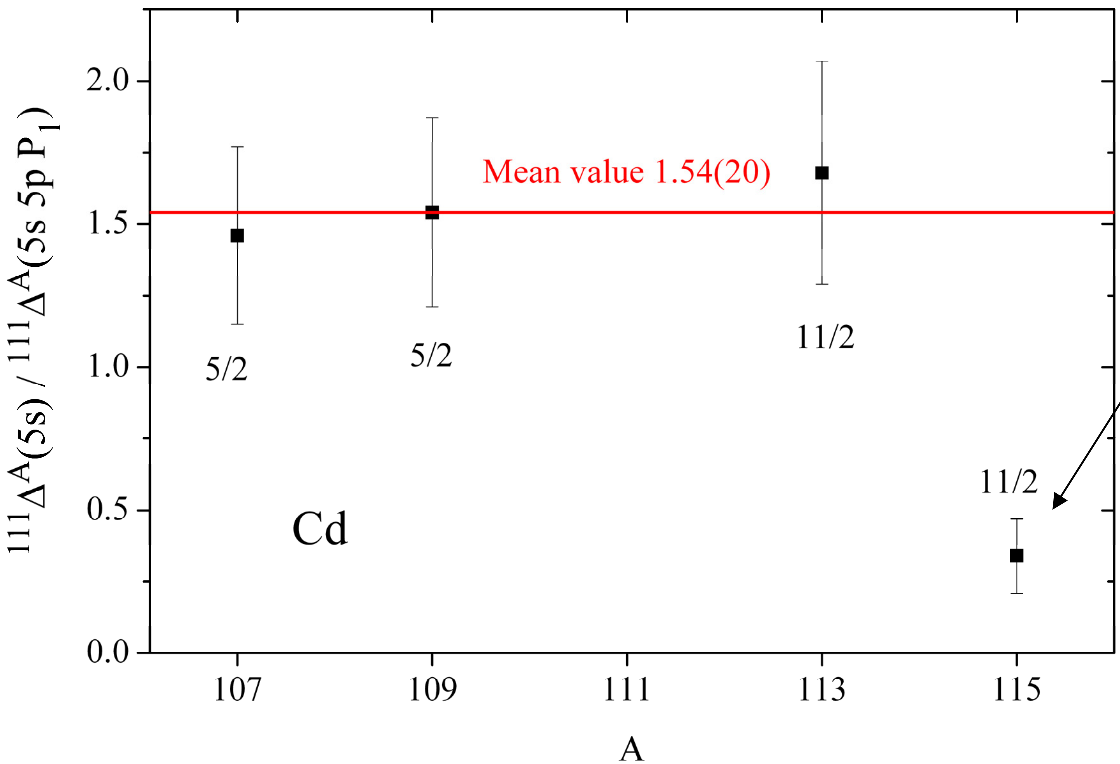
$\eta = 3.09$ (Skripnikov, 2021)

A	$^{213}\Delta^{A}$ (Ra II) %	$^{213}\Delta^{A}$ (Ra I) %	μ (μ_N)
211	-0.96(20)	-0.86(24)	0.8778(31)
213	0.00	0.0000	0.6133(18)[a]
221	0.03(90)	-0.04(109)	-0.1805(17)
223	-1.12(51)	-1.33(59)	0.2702(16)
225	-0.80(27)	-0.66(31)	-0.7338(15)
227	-0.53(40)	-0.75(61)	-0.4043(20)
229	-0.87(31)	-0.86(50)	0.5026(22)

NUCLEAR FACTOR IN Ra

A	N	I	d_{nuc}, expt	d_{nuc}, sp
211	123	5/2	1.39	1.30
213	125	1/2	1.72	1.76
221	133	5/2	1.67	
223	135	3/2	1.25	
225	137	1/2	1.24	
227	139	3/2	1.44	
229	141	5/2	1.30	

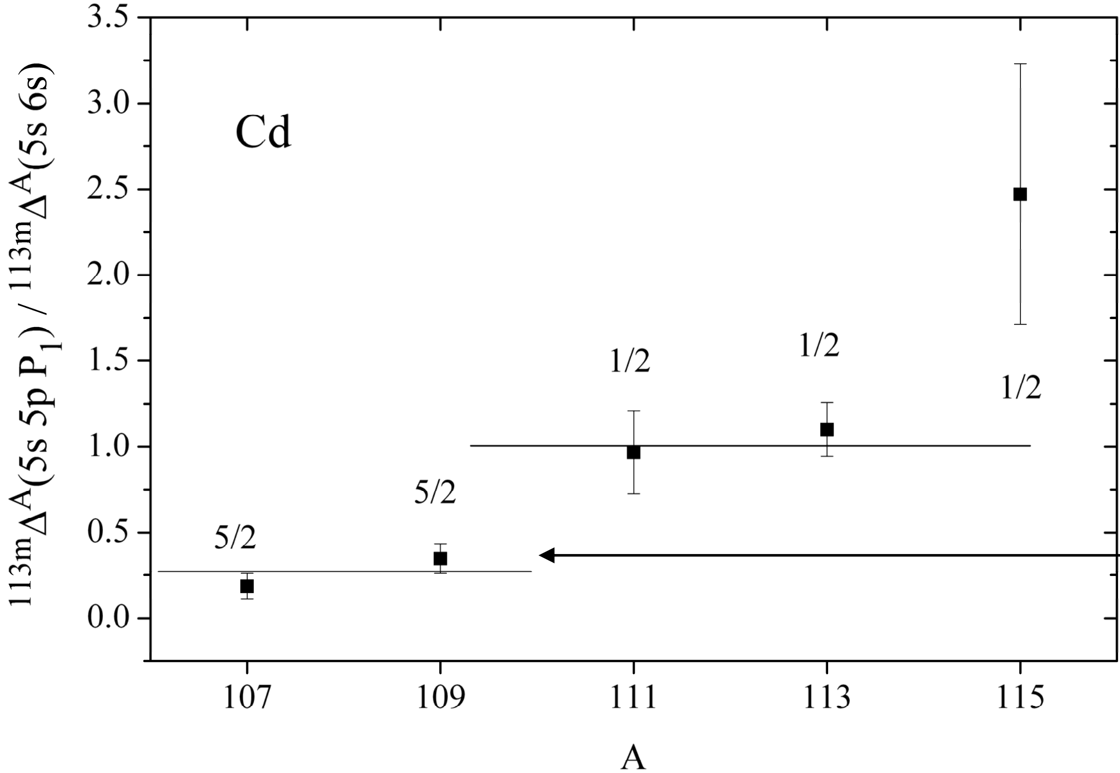
FACTORIZATION in Cd



Experimental error or some nuclear physics? Violation of factorization?

Experimental data are from:
N. Frömmgen et al., Eur. Phys. J. D 69, 164 (2015);
D. T. Yordanov et al., Phys. Rev. Lett. 110, 192501 (2013) and references therein

FACTORIZATION in Cd



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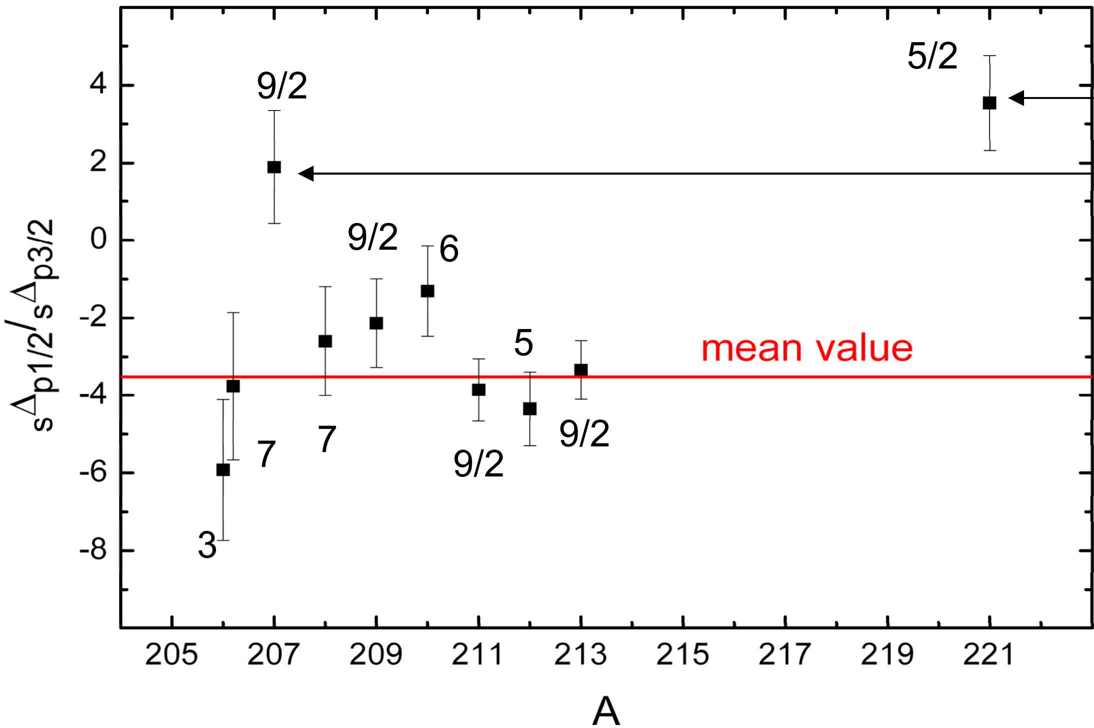
FACTORIZATION in Fr

Experimental data are from:
R. Collister, et al., PR A 90, 052502 (2014); J. Zhang, et al., PRL 115, 042501 (2015);
A. Voss et al., PR C 91, 044307 (2015) and references therein



Experimental error or some nuclear physics? Violation of factorization?

^{207}Fr nuclear physics-wise is identical to $^{209, 211, 213}$Fr. There are no reasons for such a departure

SUMMARY

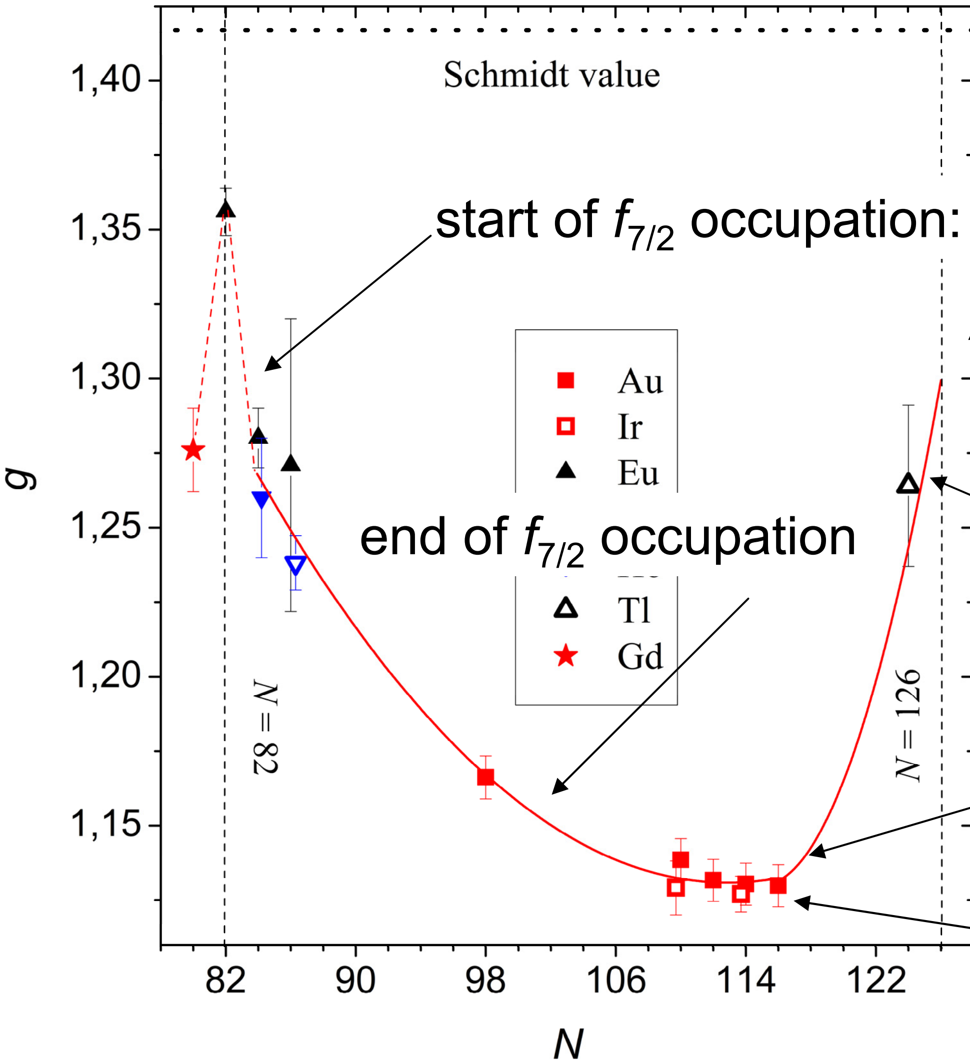
1. Анализ RHFA позволяет уточнить значения μ (для Au — до 15%) и дать надежную оценку его погрешности.
2. ML rule не должно использоваться для оценки RHFA (кроме Hg).
3. «Правило Экштрёма» для вычисления μ в ядрах золота плохо обосновано, заведомо занижает погрешности и должно быть заменено анализом DHFA. Практически все измеренные прежде магнитные моменты ядер золота должны быть пересчитаны с учетом RHFA или же их погрешности должны быть пересмотрены.
4. «Новый» метод извлечения RHFA из экспериментальных данных в сочетании с атомными расчетами позволяет расширить наше знание о распределении намагниченности на ядра удаленные от стабильности.
5. Данные для цепочек ядер Cd и Fr демонстрируют заметное «нарушение» факторизации. Дальнейшие экспериментальные и теоретические исследования должны объяснить эти эффекты или показать их отсутствие.

SUMMARY

1. The analysis of RHFA gives more accurate values for μ and reliable estimation of their uncertainty.
2. ML rule does not work in many cases and should be used with caution. The spectacular success of the ML rule for several Hg, Au, Ir nuclei remains unexplainable.
3. Ekström prescription for μ determination in Au nuclei is based on the dubious grounds and should be replaced by the direct accounting for the RHFA. Nearly all previously derived magnetic moments in gold nuclei should be recalculated or, at least, the corresponding uncertainties should be revised
4. "New" method of the RHFA-values extraction from the experimental data in combination with the advanced atomic calculations enables one to extend our knowledge of RHFA to far from stability nuclei.
5. From the analysis of the available data for Cd and Fr isotopic chains marked "violations" of the factorization approximation were found. Experimental and theoretical works are necessary to confirm (explain) or reject these departures from one of the fundamental premises of the HFA theory.

SYSTEMATICS of $g(\pi h_{11/2})$

$$\langle \psi | \mu | \psi \rangle = \langle 0^+ \otimes h_{11/2} | \mu | 0^+ \otimes h_{11/2} \rangle + \alpha \langle 0^+ \otimes h_{11/2} | \mu | 1^+ \otimes h_{11/2} \rangle$$



start of $f_{7/2}$ occupation: CP1=0

end of $f_{7/2}$ occupation

core excitation: $\nu f_{7/2} \rightarrow \nu f_{5/2}$

$$\Delta\mu_{\mathrm{CM1}}(N) = C \cdot [(2j_1+1)v_{j_1}^2(N)][(2j_2+1)u_{j_2}^2(N)]$$

$j_1 = 7/2$, $j_2 = 5/2$

end of $f_{5/2}$ occupation: CP1=0

start of $f_{5/2}$ occupation

maximal CP1 (minimal μ):
full $f_{7/2}$
empty $f_{5/2}$