Possibility of experimental confirmation of 3 + 1 neutrino model with one sterile neutrino

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Neutrino-4 collaboration

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[2]JSC "SSC Research Institute of Atomic Reactors", Dimitrovgrad, Russia

[3]Dimitrovgrad Engineering and Technological Institute MEPhI, Dimitrovgrad, Russia

NRC KI - Petersburg Nuclear Physics Institute

16 September 2021

1.The direct observation of effect of oscillation in Neutrino-4 experiment on search for sterile neutrino

2. Comparison of results of Neutrino-4 experiment with results of other experiments

a) with the results of reactor, gallium anomaly, KATRIN, DANSS and NEOS

b) MiniBoone and LSND

c) with IceCube

3. Possibility of experimental confirmation of the 3 + 1 neutrino model with one sterile neutrino

a) The structure of 3+1 neutrino model and representation of probabilities of different oscillation

b) Prediction of the effective mass of electron neutrino from Neutrino-4 experiment

c) Comparison with experiments on measuring neutrino mass: KATRIN

d) Comparison with neutrino mass constraints from neutrino less double beta-decay: GERDA

4. Sterile Neutrino Parameters Neutrino model with one sterile neutrino

a) PMNS matrix for 3 + 1 model

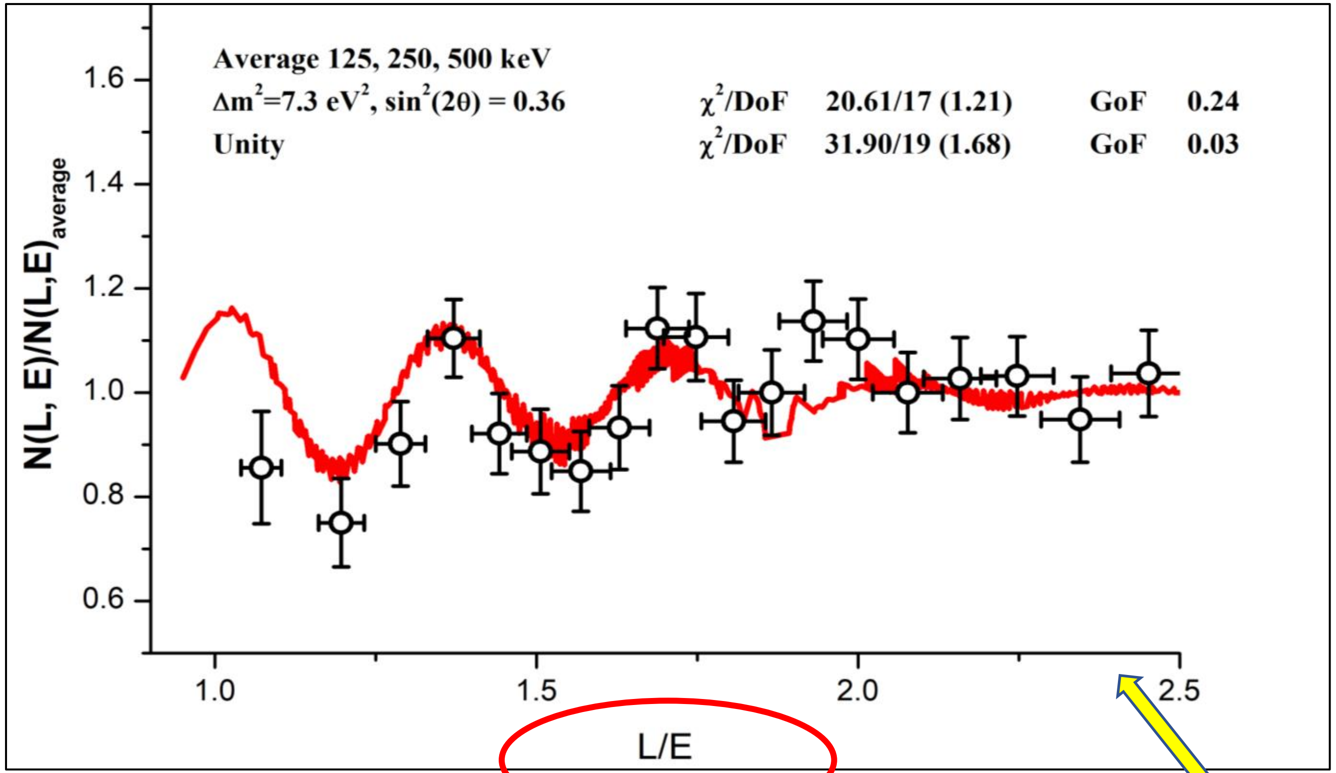
b) Neutrino flavors mixing scheme including sterile neutrino

c) effective mass hierarchy

5. Cosmology – the role of sterile neutrinos during the formation of Universe

The direct observation of effect of oscillation in Neutrino-4 experiment on search for sterile neutrino

A.P. Serebrov, et al. PHYSICAL REVIEW D 104, 032003 (2021)



The period of oscillation for neutrino energy 4 MeV is 1.4 m

$$P(\tilde{\nu}_e \to \tilde{\nu}_e) = 1 - \sin^2 2\theta_{14} \ \sin^2\left(1.27 \frac{\Delta m^2_{14}[eV^2] L[m]}{E_{\tilde{\nu}}[MeV]}\right)$$

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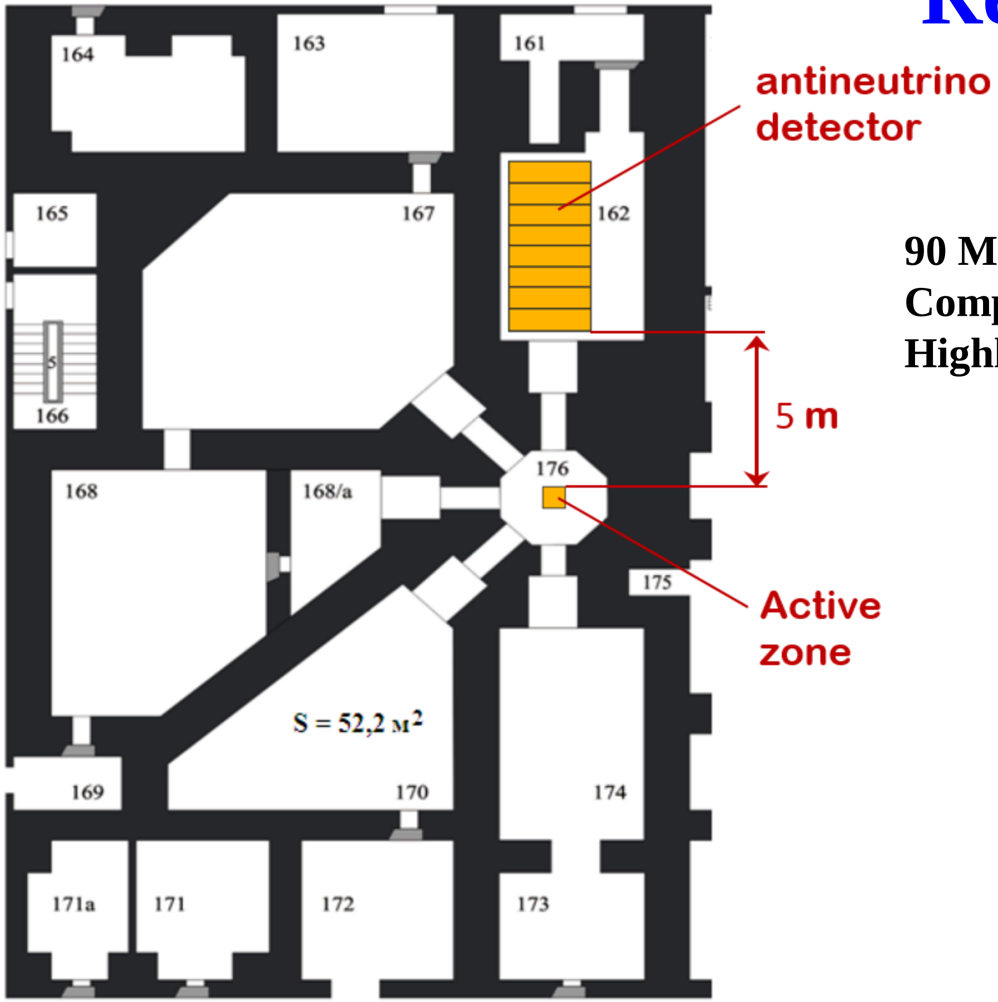
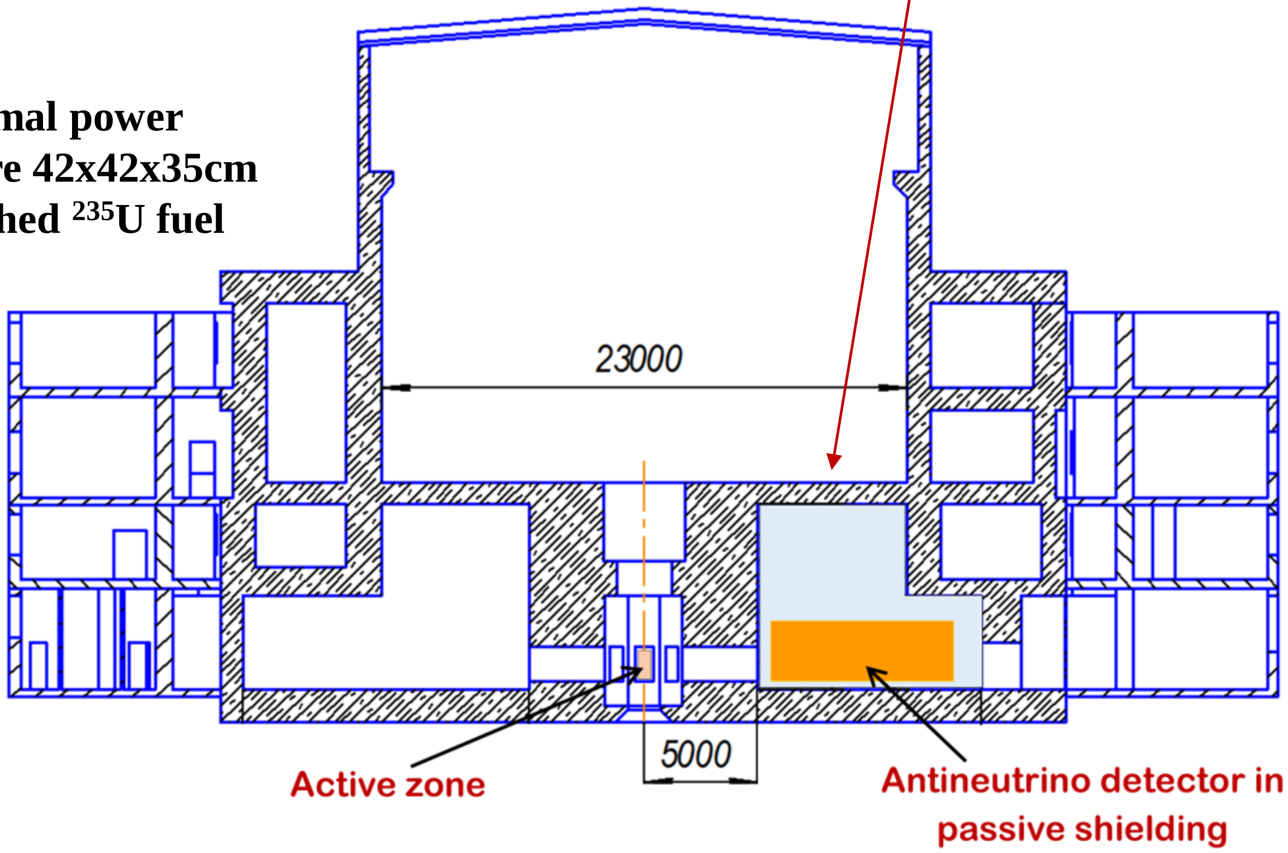
arxiv:2003.03199
arXiv:2005.05301v 8

NEUTRINO 2020

Reactor SM-3

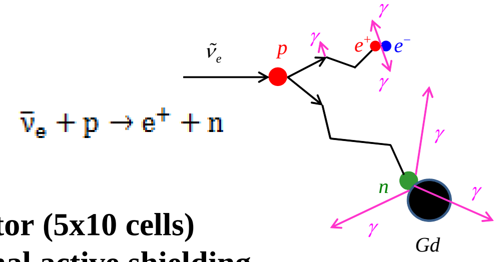


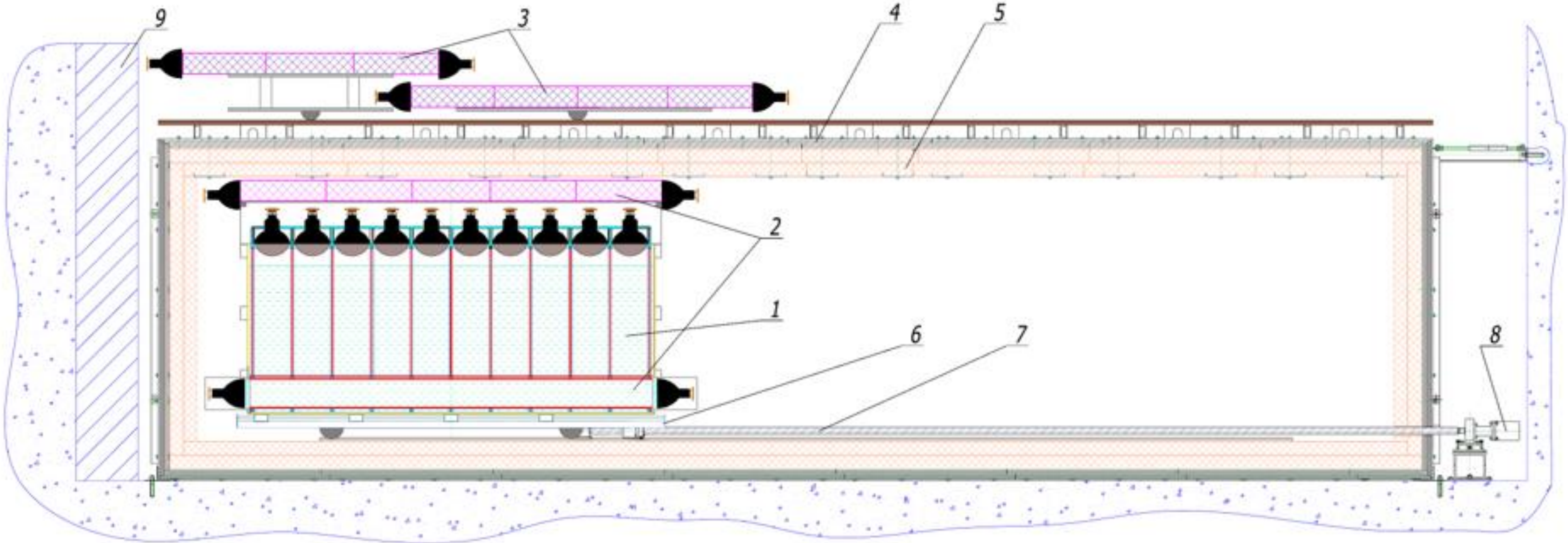
90 MW thermal power
Compact core 42x42x35cm
Highly enriched ^{235}U fuel



Due to some peculiar characteristics of its construction, reactor SM-3 provides the most favorable conditions to search for neutrino oscillations at short distances. However, SM-3 reactor, as well as other research reactors, is located on the Earth's surface, hence, cosmic background is the major difficulty in considered experiment.

Movable and spectrum sensitive antineutrino detector at SM-3 reactor





1. detector (5x10 cells)
2. internal active shielding
3. external active shielding
4. steel and lead
5. borated polyethylene
6. moveable platform
7. feed screw
8. step motor
9. shielding

Passive shielding - 60 tons

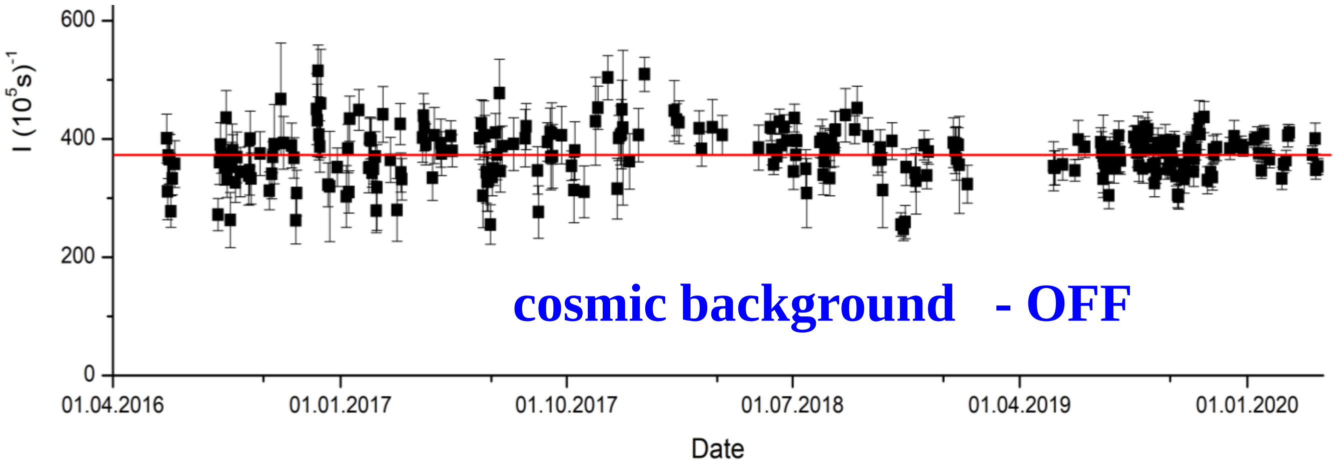
Neutrino channel outside and inside

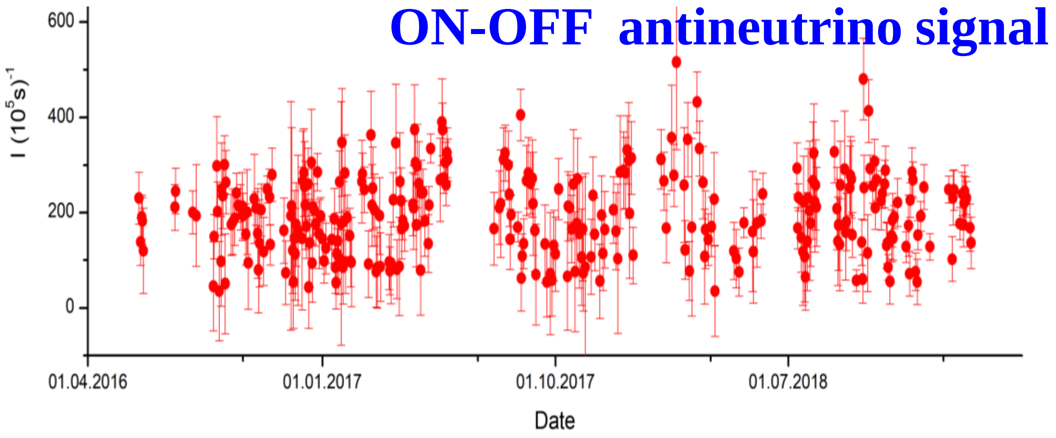
Range of measurements is 6 – 12 meters

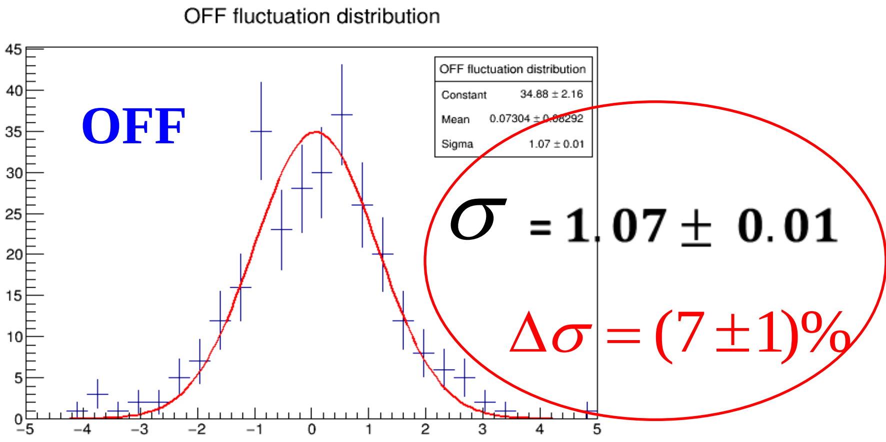
Detector prototype

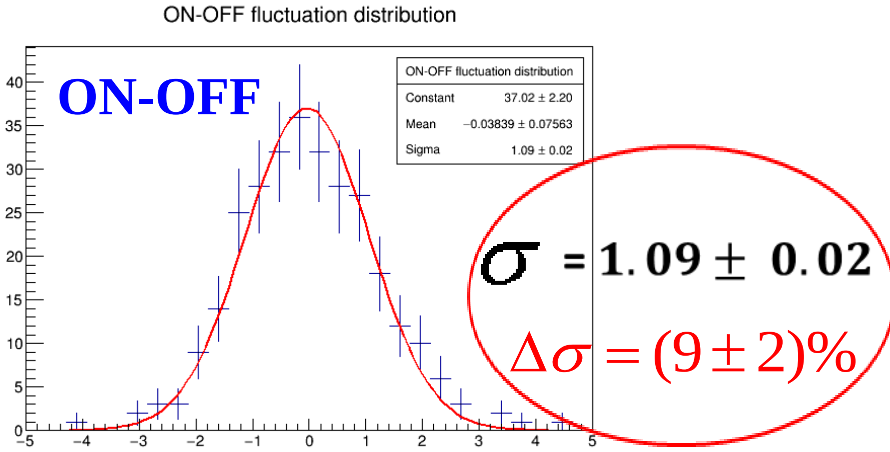
Full-scale detector

Liquid scintillator detector 50 sections 0.235x0.235x0.85м^3





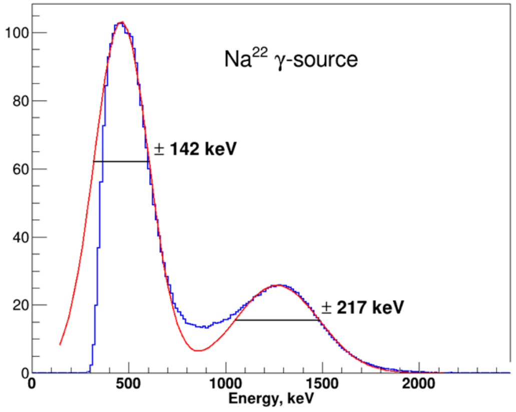


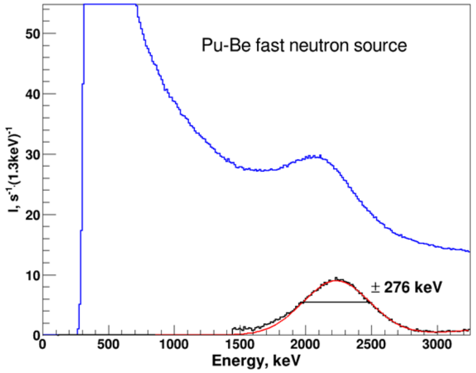


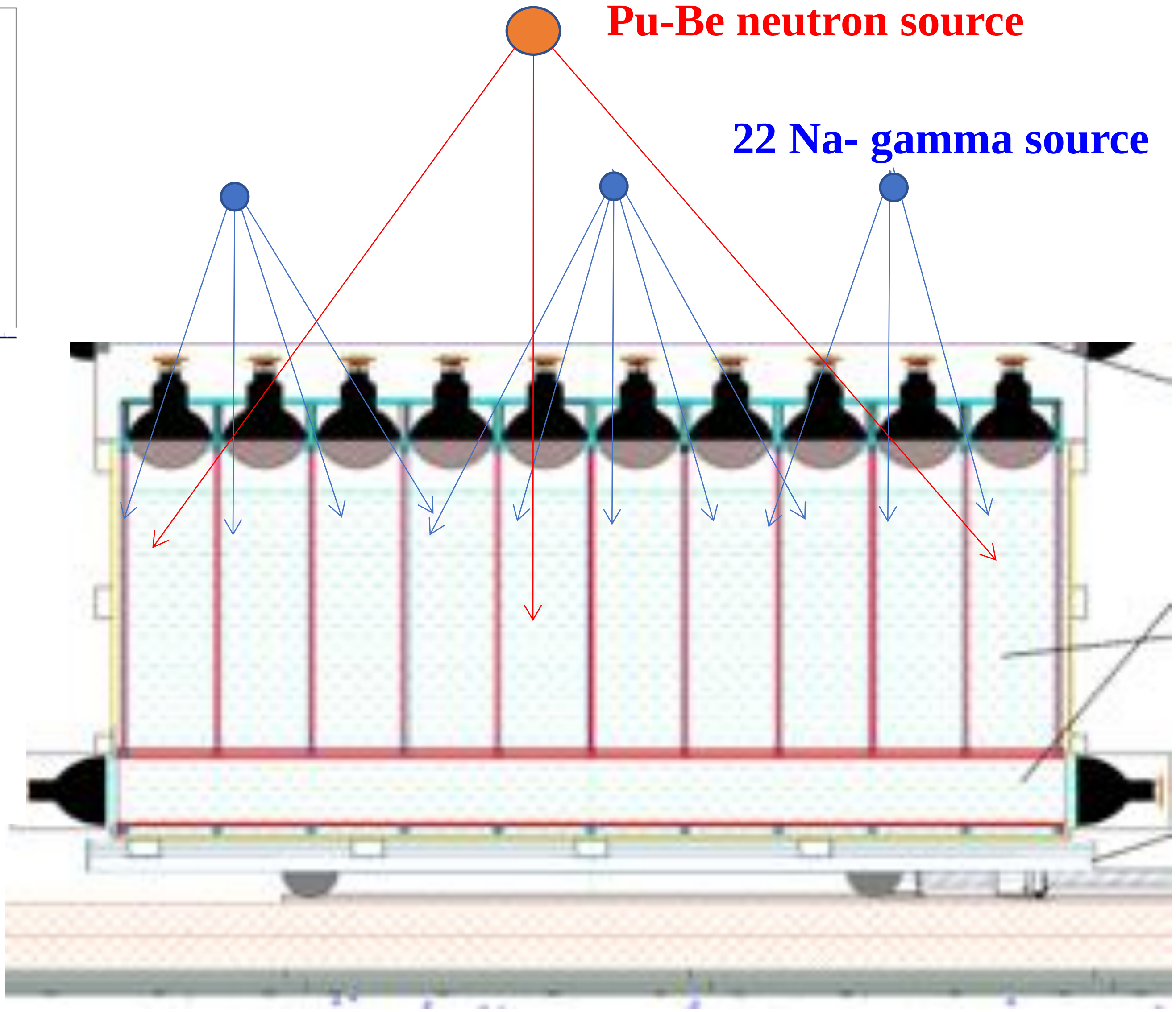
The correlated signal produced by the cosmic background measured over the whole time (up). The correlated ON-OFF signal over the whole time (down).

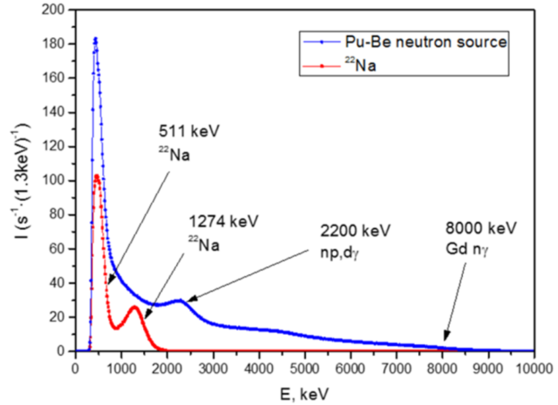
The distribution of deviations from average value of correlated events rates background (OFF) and differences (ON-OFF) normalized on their statistical uncertainties.

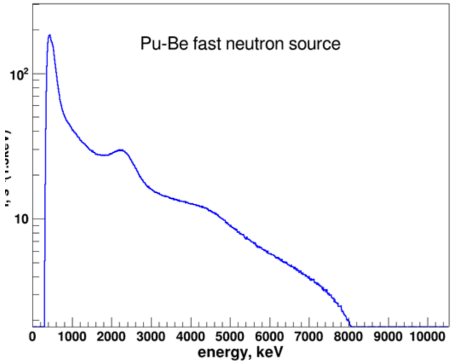
Energy calibration of the full-scale detector

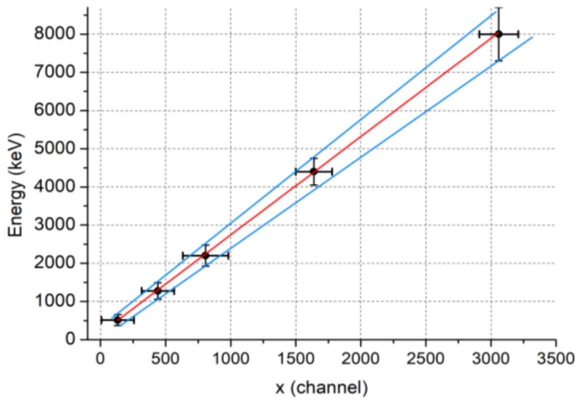




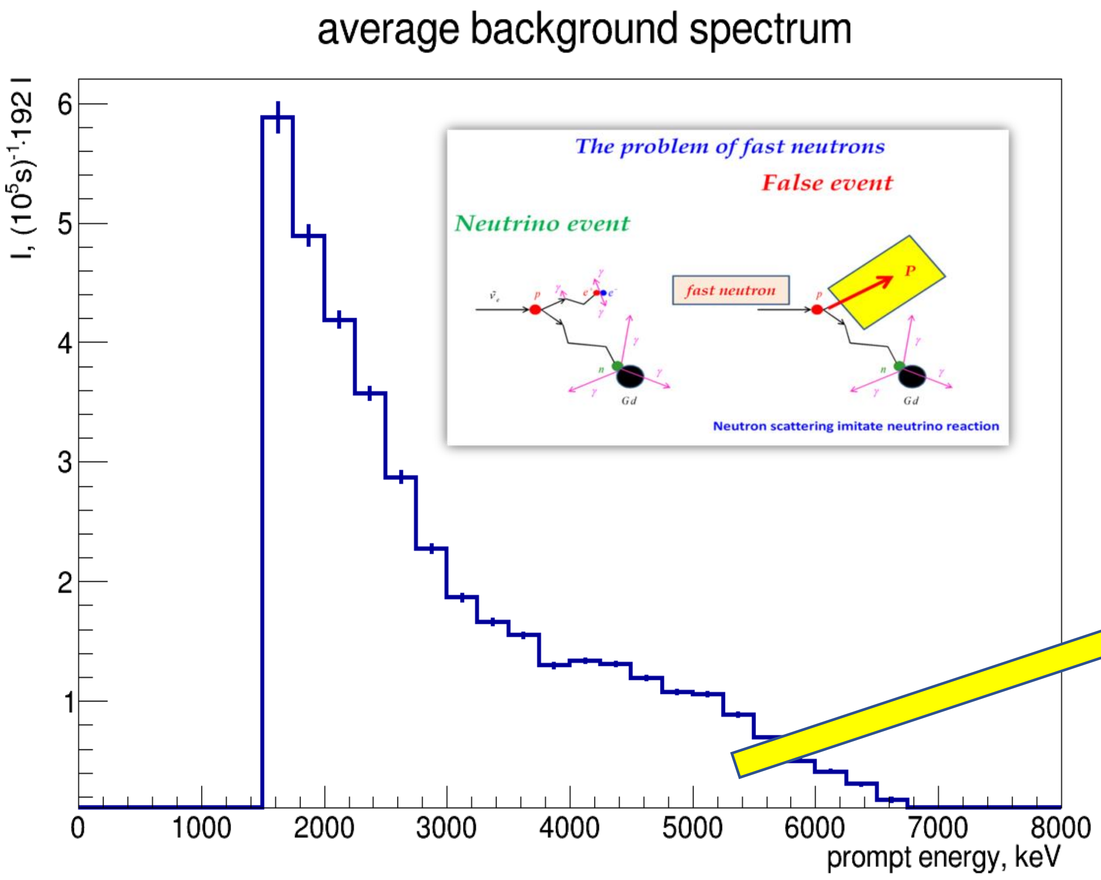


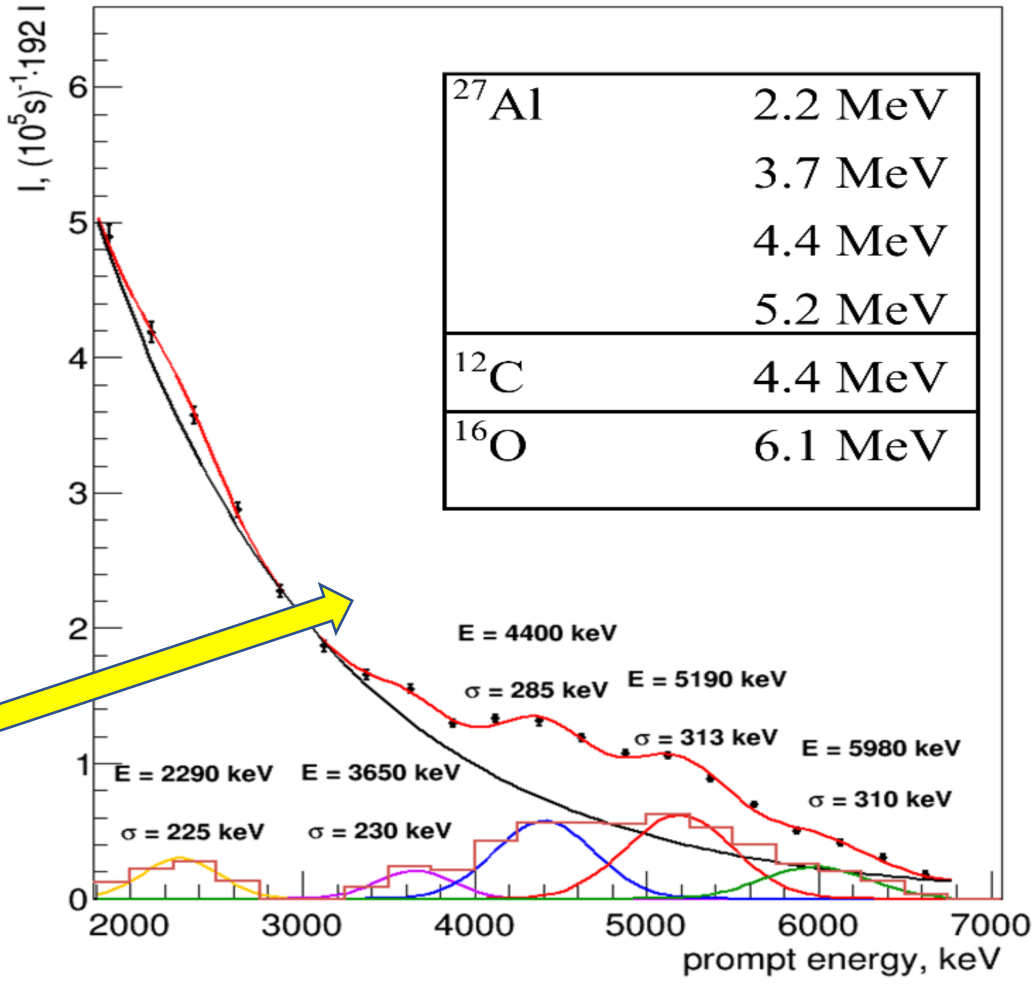






BACKGROUND SPECTRUM

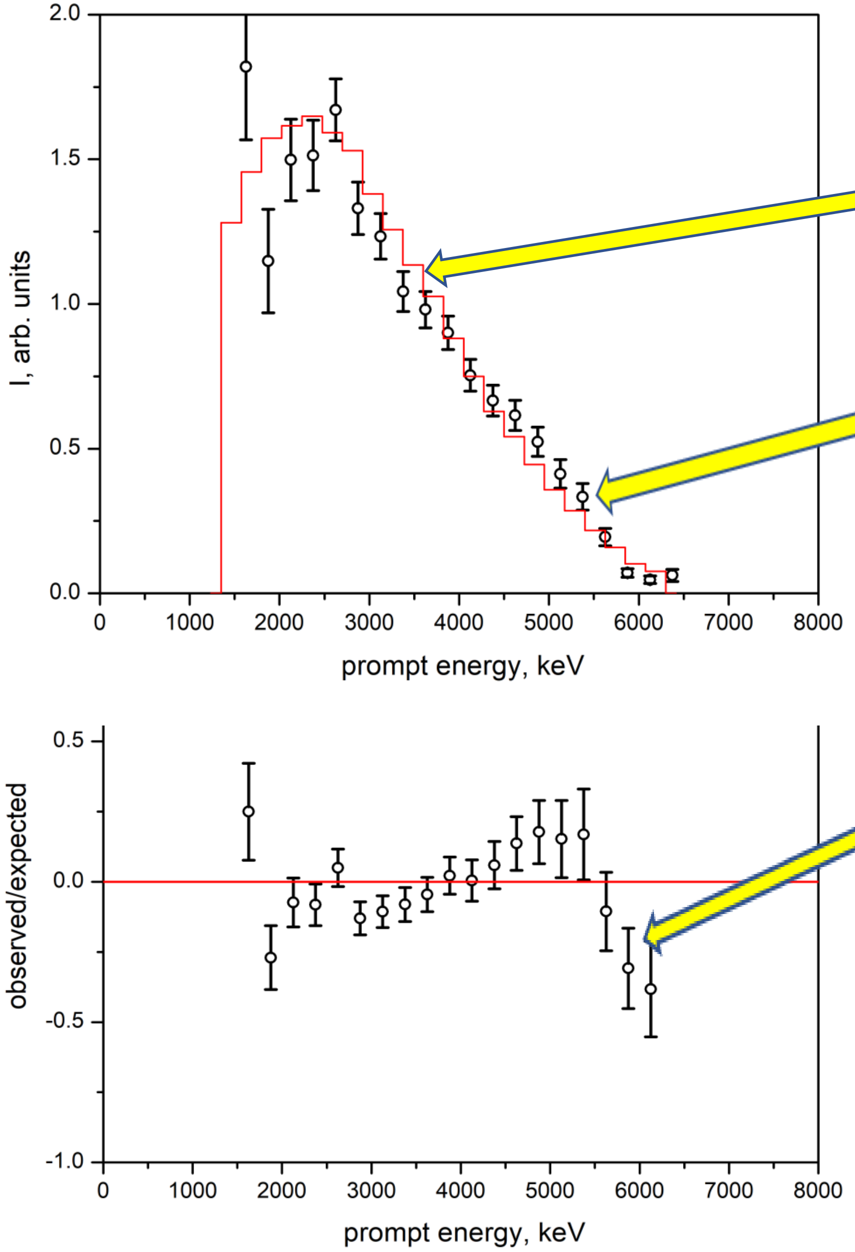




Presence of that structure in the energy spectrum indicates that energy calibration of the detector was the same in all measurements.

Energy resolution of the detector σ=250 keV which does not depend on the energy of a positron.

Comparison of MC spectrum of antineutrino for ^{235}U with the experimental ON-OFF spectrum



calculated MC spectrum of antineutrino for ^{235}U

experimental ON-OFF spectrum.

difference normalized to the calculated spectrum

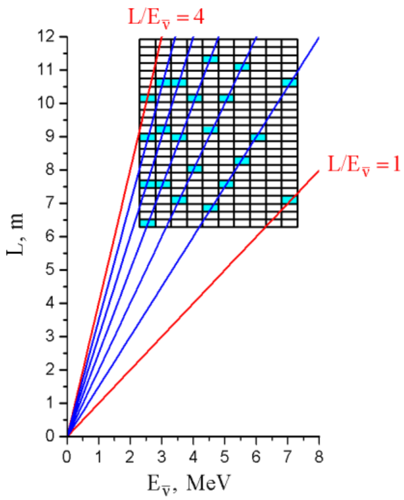
Spectrally independent method for analyzing experimental data is needed

$N(E_i, L_k)$

Number of antineutrino events

Probability of antineutrino disappearance

$$P(\tilde{\nu}_e \rightarrow \tilde{\nu}_e) = 1 - \sin^2 2\theta_{14} \; \sin^2\left(1.27 \frac{\Delta m^2_{14}[\text{eV}^2]L[\text{m}]}{E_{\tilde{\nu}}[\text{MeV}]}\right) \quad (1)$$



The spectrum independent method of experimental data analysis

$$R^{\exp}_{i,k} = \frac{N(E_i, L_k)L_k^2}{K^{-1}\sum_k^K N(E_i, L_k)L_k^2} = \frac{S(E)\mathcal{E}(E)[1-\sin^2 2\theta_{14}\sin^2(1.27\Delta m^2_{14}L_k / E_i)]}{S(E)\mathcal{E}(E)K^{-1}\sum_k^K[1-\sin^2 2\theta_{14}\sin^2(1.27\Delta m^2_{14}L_k / E_i)]} = R^{th}_{i,k} \quad (2)$$

The denominator is significantly simplified with a range of measurement distances significantly greater than the characteristic oscillation period:

$S(E)$ - Spectrum

$\mathcal{E}(E)$ - Detector efficiency

$$R^{\text{th}}_{ik} \approx \frac{1 - \sin^2 2\theta_{14} \sin^2(1.27\Delta m^2_{14}L_k/E_i)}{1 - 1/2\sin^2 2\theta_{14}} \xrightarrow[\theta_{14}=0]{} 1 \quad (3)$$

$$\sum_{i,k}[(R^{\exp}_{i,k} - R^{th}_{i,k})^2 / (\Delta R^{\exp}_{i,k})^2] = \Delta\chi^2(\sin^2 2\theta_{14}, \Delta m^2_{14}) \quad (4)$$

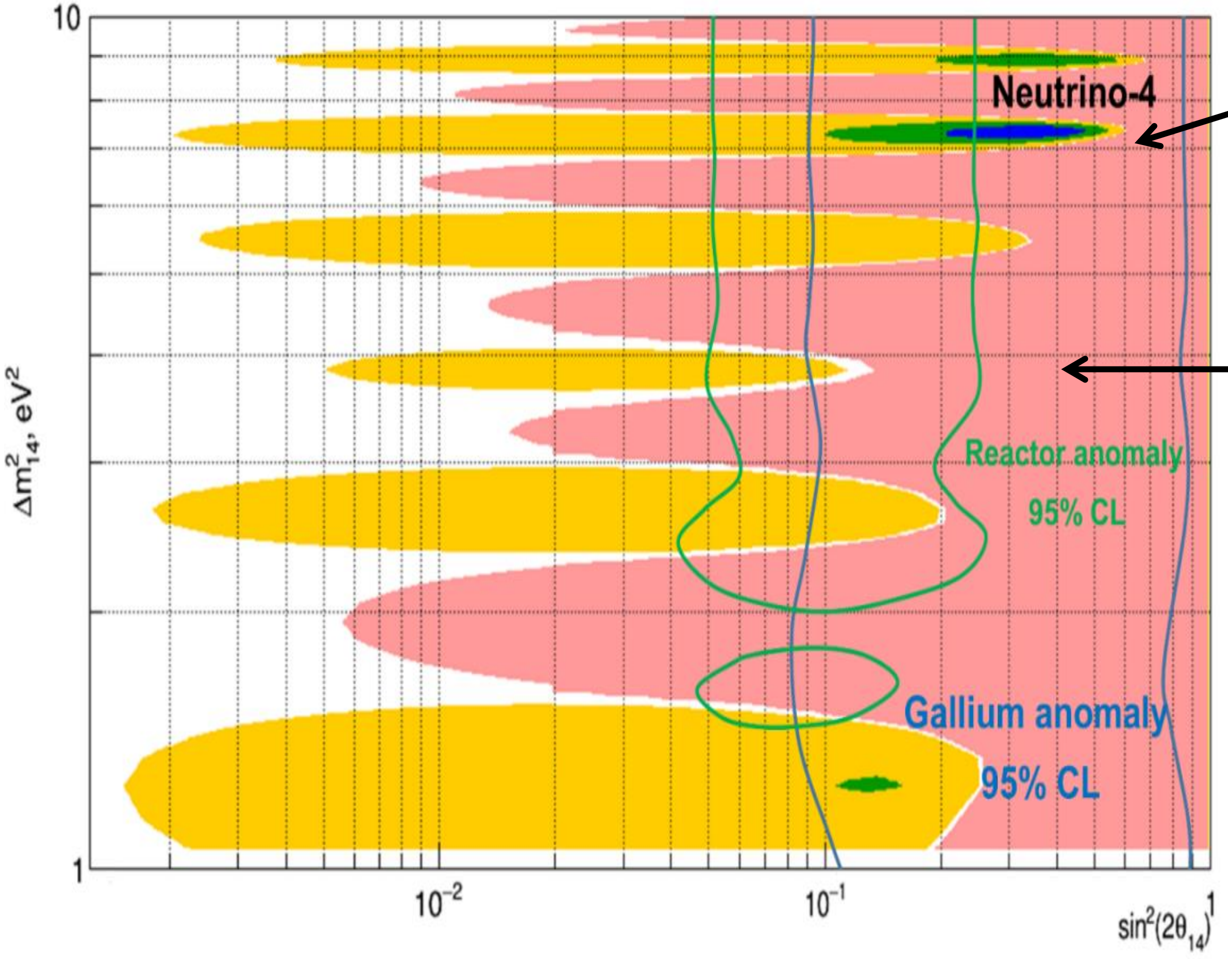
The results of the analysis of optimal parameters Δm^2_{14} and $\sin^2 2\theta_{14}$ using $\Delta\chi^2$ method

$$\sum_{i,k}[(R^{exp}_{i,k} - R^{th}_{i,k})^2 / (\Delta R^{exp}_{i,k})^2] = \chi^2(\sin^2 2\theta_{14}, \Delta m^2_{14})$$

We observed the oscillation effect at C.L. 2.9 σ in vicinity of :

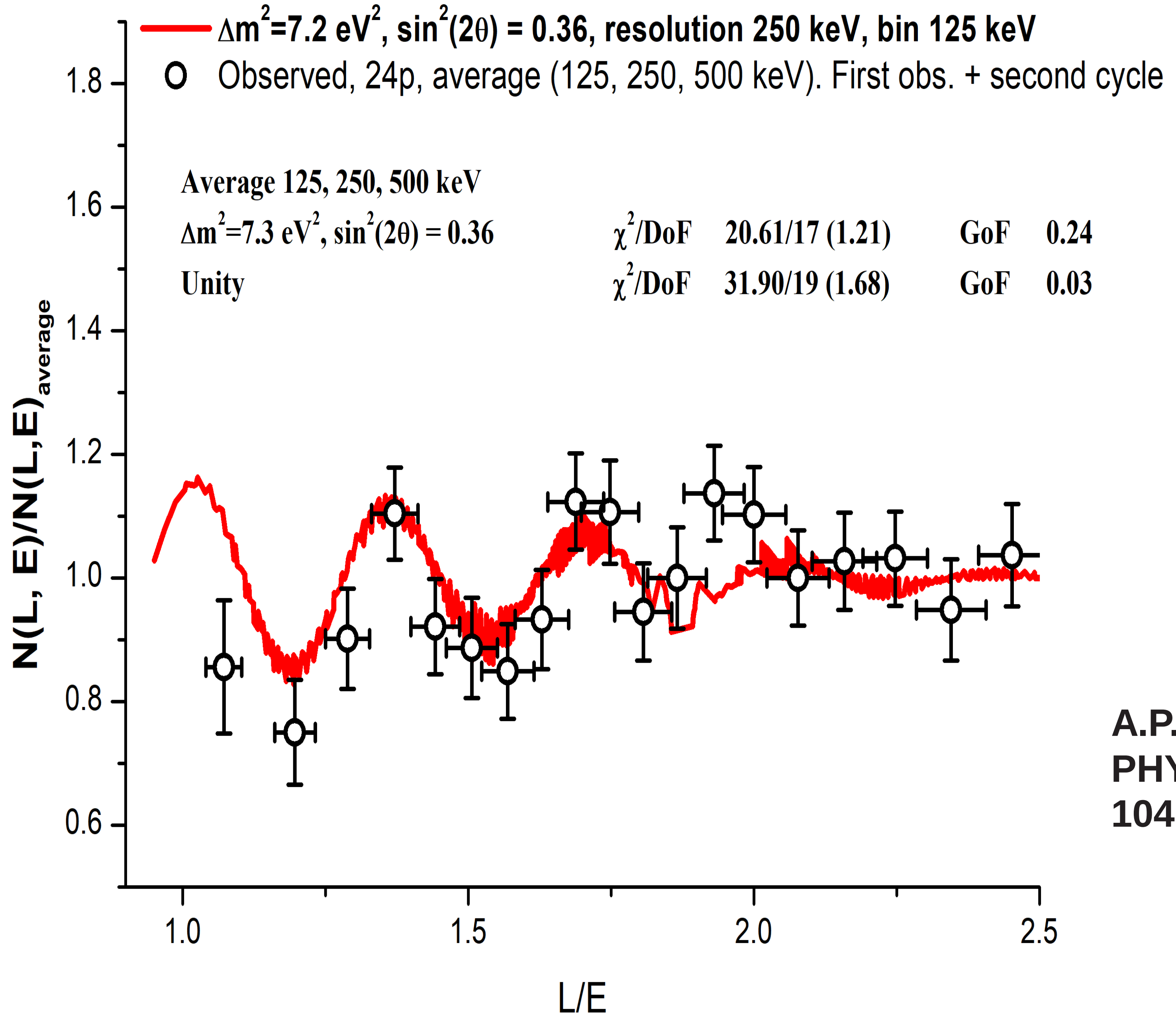
$$\Delta m^2_{14} \approx 7.25 eV^2$$

$$\sin^2 2\theta_{14} \approx 0.36$$



Expected from Neutrino-4 CL 99.7%

Excluded from Neutrino-4 CL>99.9% CL



$$\Delta m^2_{14} = 7.30 \pm 0.13_{st} \pm 1.16_{syst}$$

$$sin^2 2\theta_{14} = 0.36 \pm 0.12$$

The period of oscillation for neutrino energy 4 MeV is 1.4 m

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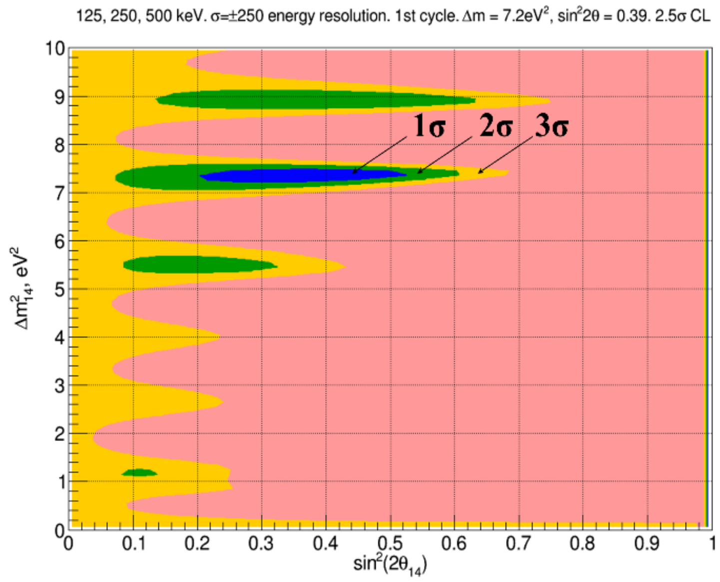

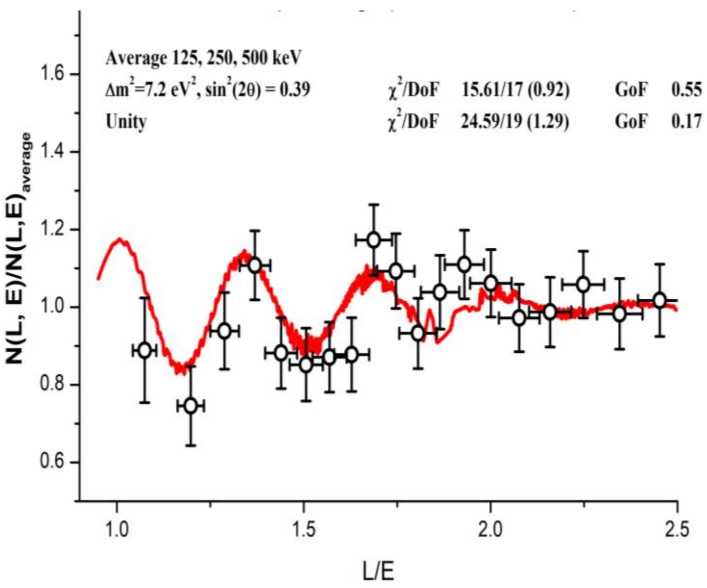

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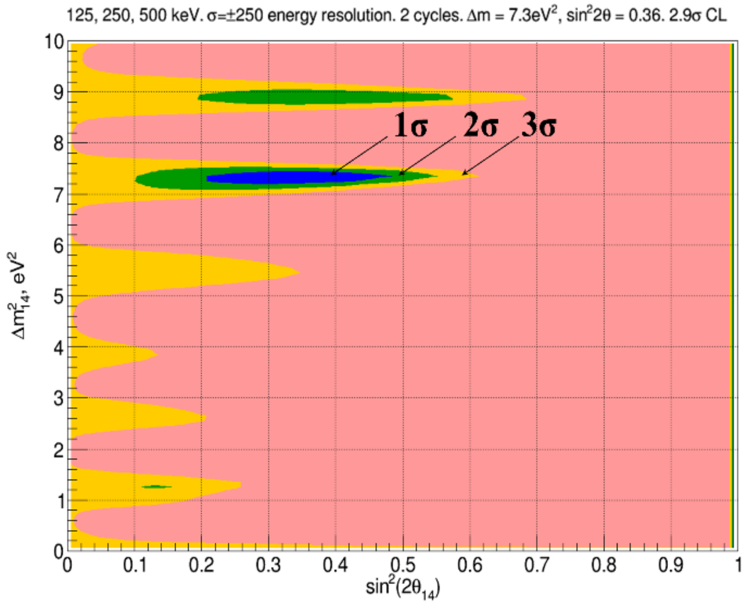
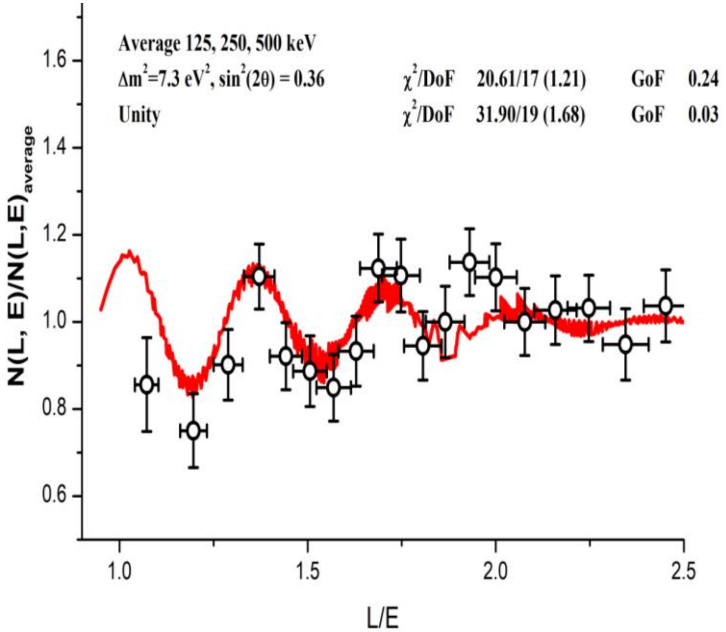




Best fit oscillation parameters Δm^2_{14}, $\sin^2 2\theta_{14}$	χ^2/d.o.f. (Reduced χ^2) fit w/ and w/o oscillation	Goodness of fit w/ and w/o oscillation
7.2 eV^2, 0.39(2.5σ)	**15.61/17(0.92)** 24.59/19 (1.29)	0.55 0.17
7.3 eV^2, 0.36(2.9σ)	**20.61/17(1.21)** 31.90/19 (1.68)	0.24 0.03

There is a maximum at zero, from here the process of oscillations begins

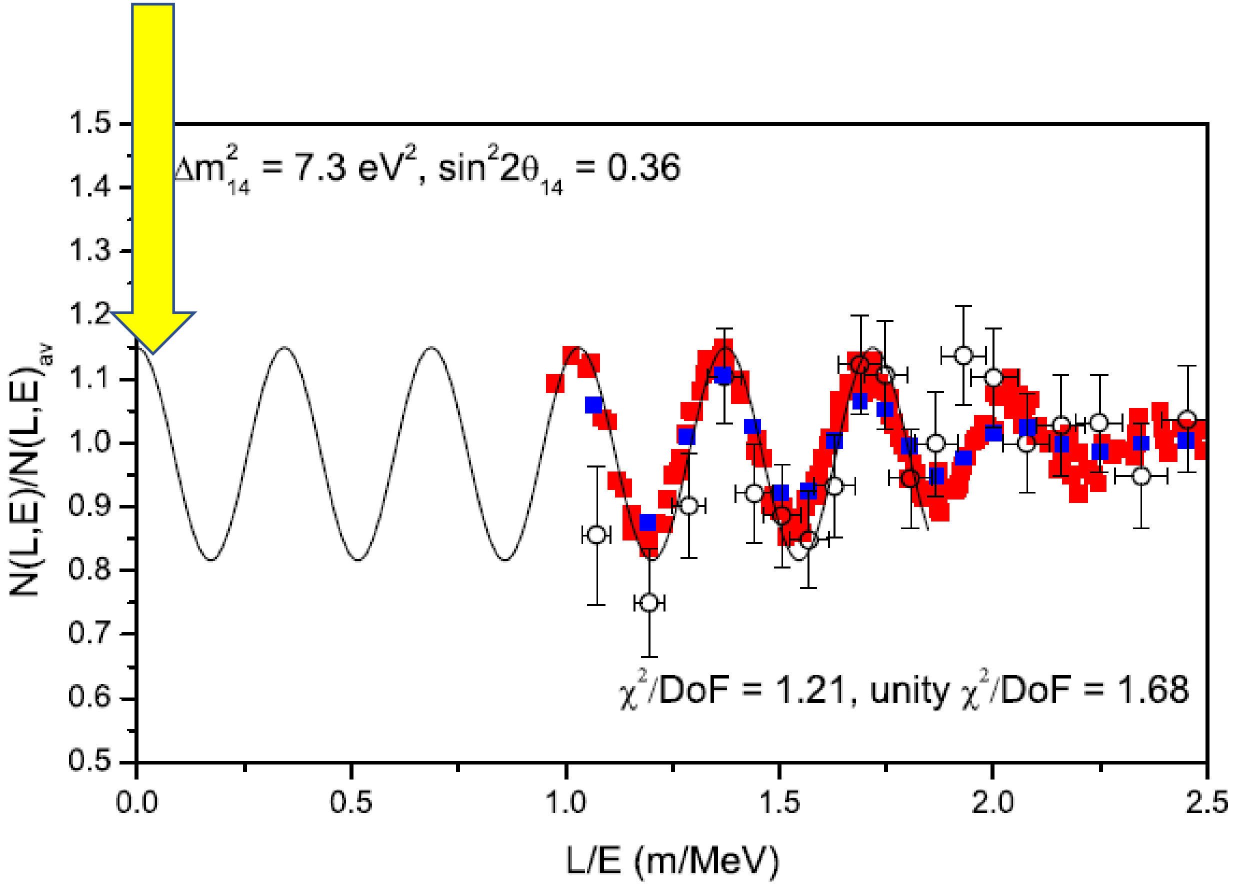
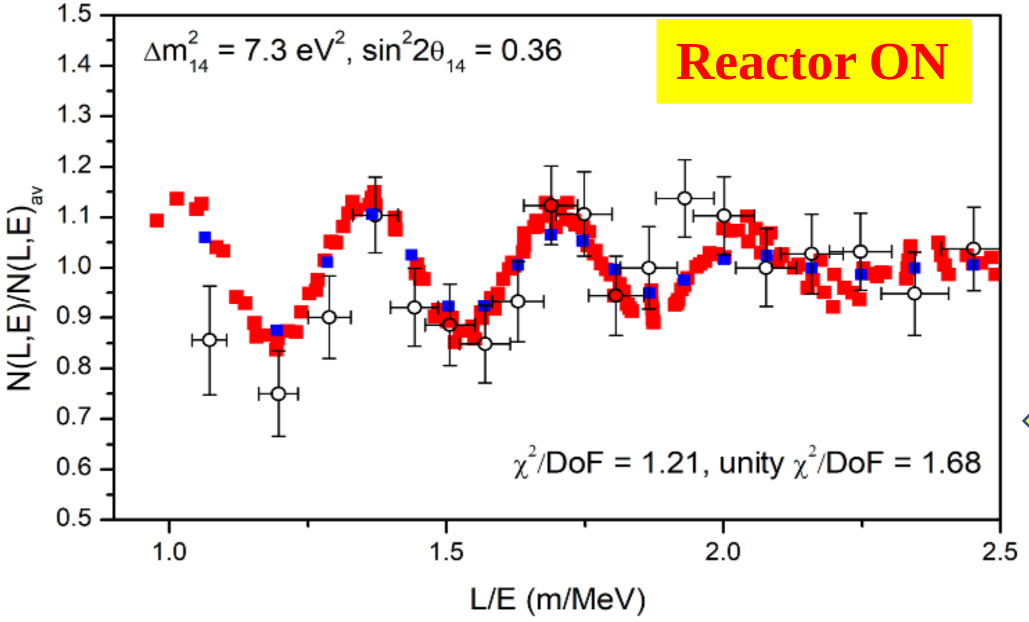
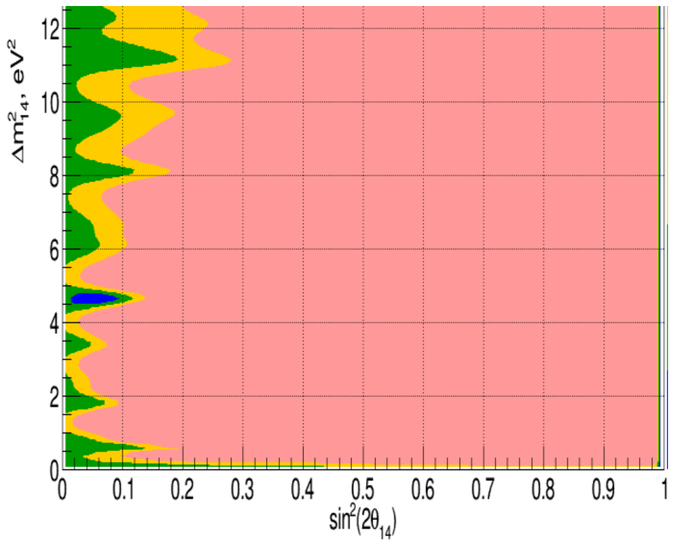


FIG. 47. Complete curve of the oscillation process starting from the reactor core center.

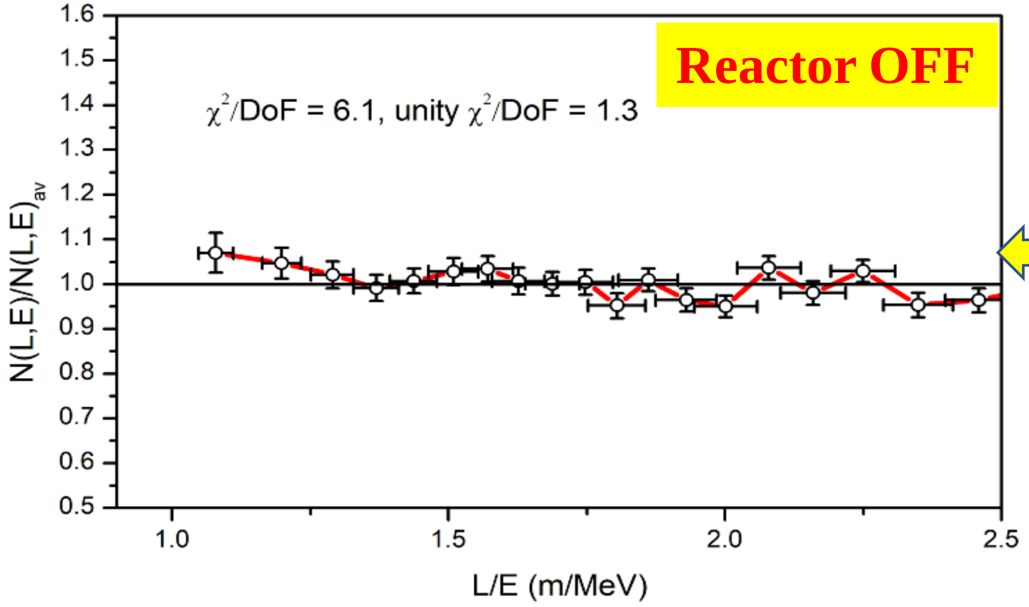
It should be noted that the experimental points should be fitted with such a sinusoidal dependence, which has a maximum at the origin, since the process of oscillations starts from the source. This significantly reduces the set of sinusoids available for fitting. FIG. shows the complete curve of the oscillation process starting from the reactor.

ANALYSIS OF POSSIBLE SYSTEMATIC EFFECTS

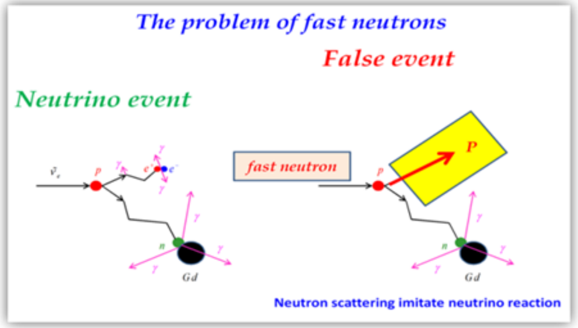




Comparison of the R-ratio versus L/E for the neutrino signal (top)



and the R-ratio versus L/E for the background (bottom).



RESULT OF EXPERIMENT NEUTRINO-4

$$\Delta m^2_{14} = 7.30 \pm 0.13_{st} \pm 1.16_{syst} = 7.30 \pm 1.17$$

$$sin^2 2\theta_{14} = 0.36 \pm 0.12 (2.9\sigma)$$

Monte Carlo based statistical analysis gave an estimation of the confidence level at 2.7σ.

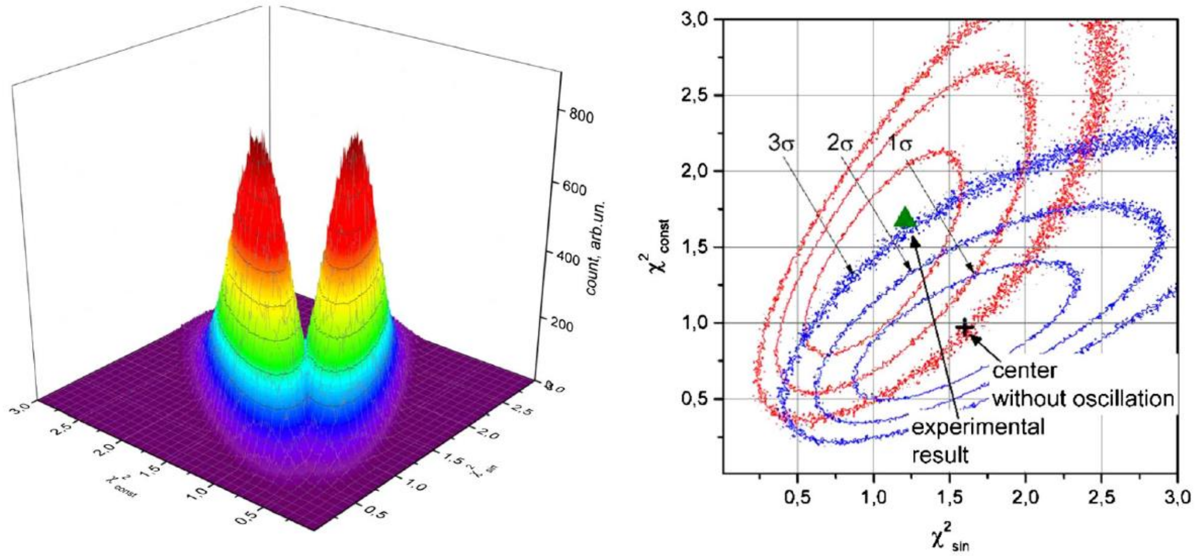


FIG. 53. On the left, the distribution of the reduced χ^2 on the plane $(\chi^2_{\text{const}}, \chi^2_{\text{sin}})$ is shown for modeling with accuracy according to the experimental statistics and with a background level corresponding to the experimental background. Two cases were simulated with the hypothesis of oscillations and with the hypothesis without oscillations. For each case, an analysis was carried out with the hypothesis of oscillations and with the hypothesis without oscillations. On the right is the same picture, where the center of the distribution without oscillations (black cross), and the deviation from this center of the experimental result (green triangle—first and second cycle) are indicated. The result of the analysis of the experimental data when processed with the hypothesis with and without oscillations gives $\chi^2_{\text{sin}} = 1.21$ or 20.6 for 17 degrees of freedom and $\chi^2_{\text{const}} = 1.68$, or 31.9 for 19 degrees of freedom, respectively, and $\Delta\chi^2 = 11.3$. Distribution contours with 1σ, 2σ, and 3σ are marked. The deviation of the experimental result from the center of the distribution without oscillations is approximately 3σ.

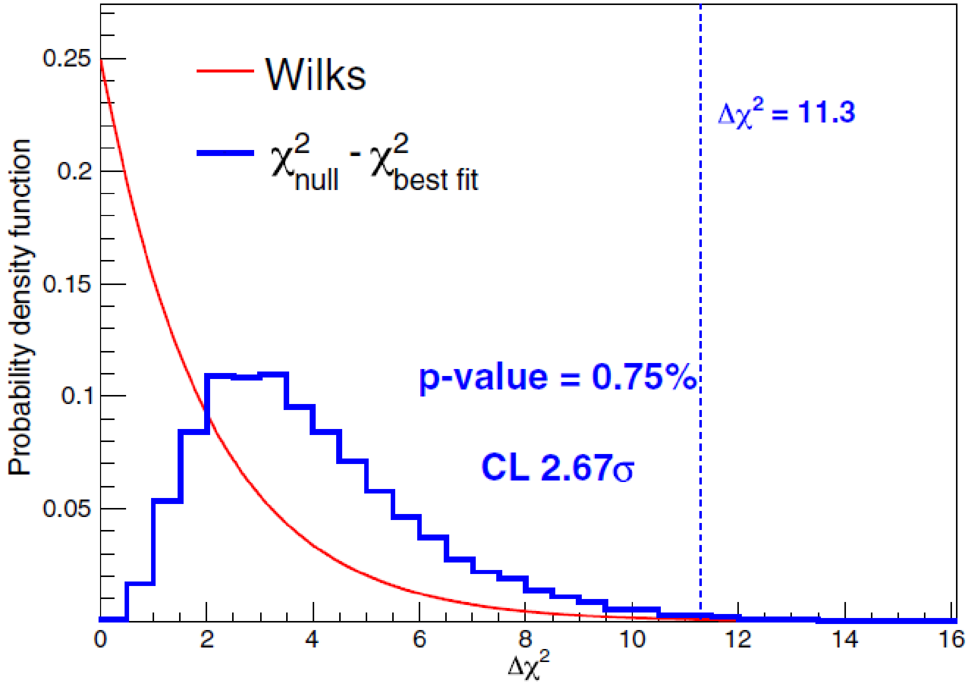
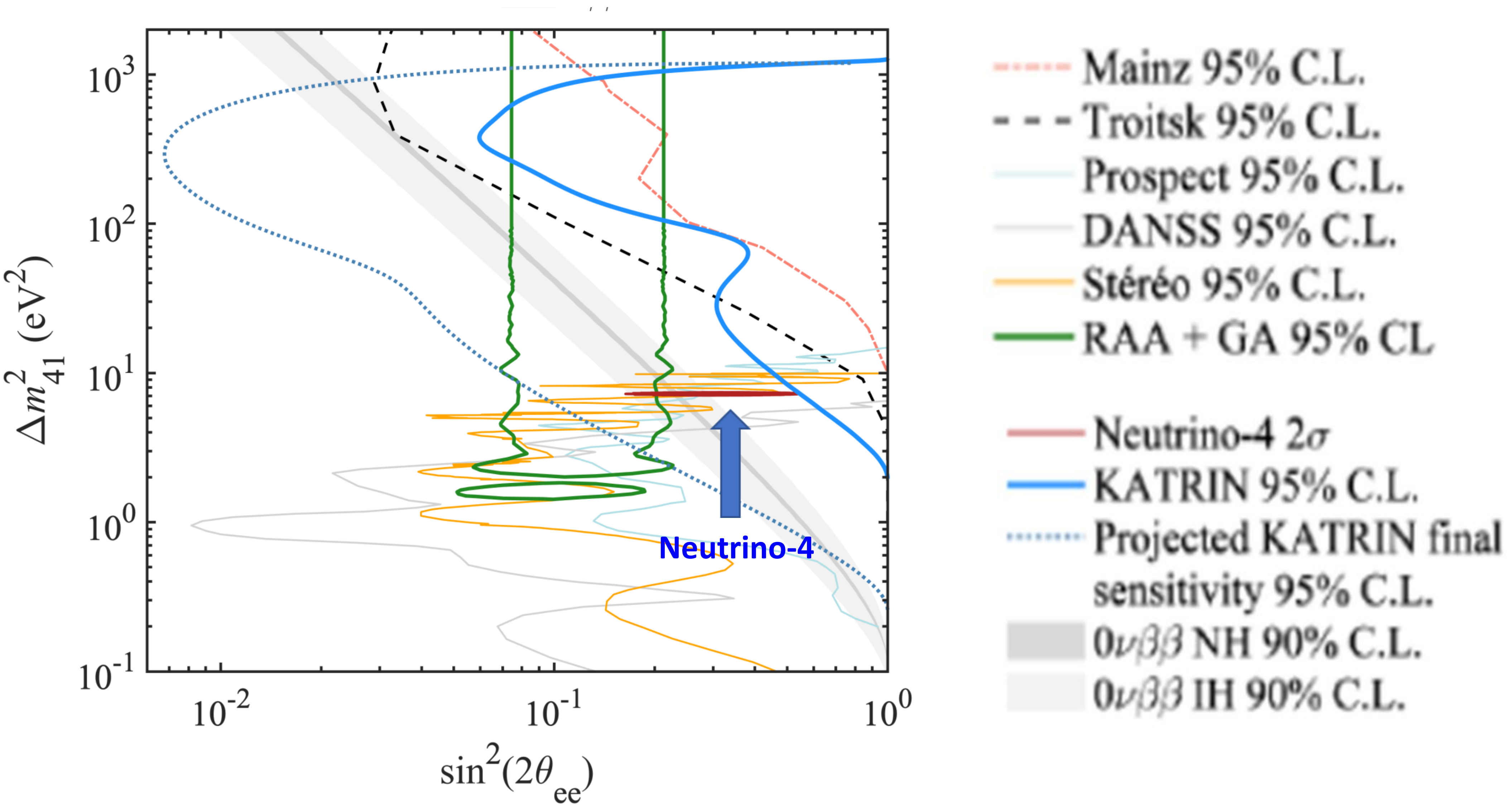


FIG. 54. T distribution for MC based approach to the statistical analysis (blue line) and χ^2 with 2 degrees of freedom function, which is claimed by Wilks's theorem.

Comparison of results of Neutrino-4 experiment with results of other experiments



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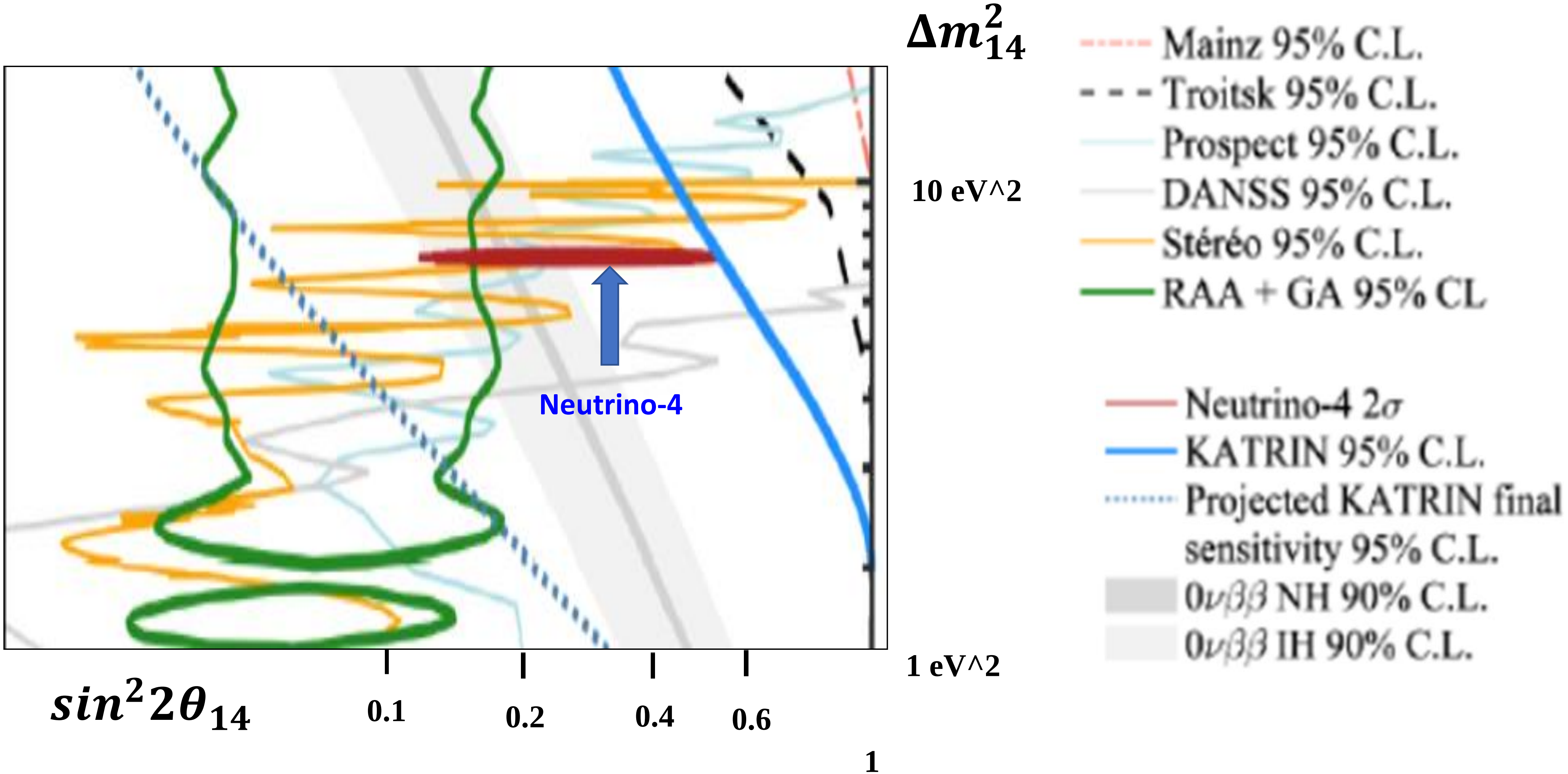

The results of the analysis of optimal parameters Δm_{14}^2 and $\sin^2 2\theta_{14}$ using $\Delta\chi^2$ method

$$\sum_{i,k}[(R_{i,k}^{\exp}-R_{i,k}^{th})^2/(\Delta R_{i,k}^{\exp})^2]=\chi^2(\sin^2 2\theta_{14},\Delta m_{14}^2)$$

We observed the oscillation effect at C.L. 2.9 σ in vicinity of :

$$\Delta m_{14}^2 \approx 7.25 eV^2$$

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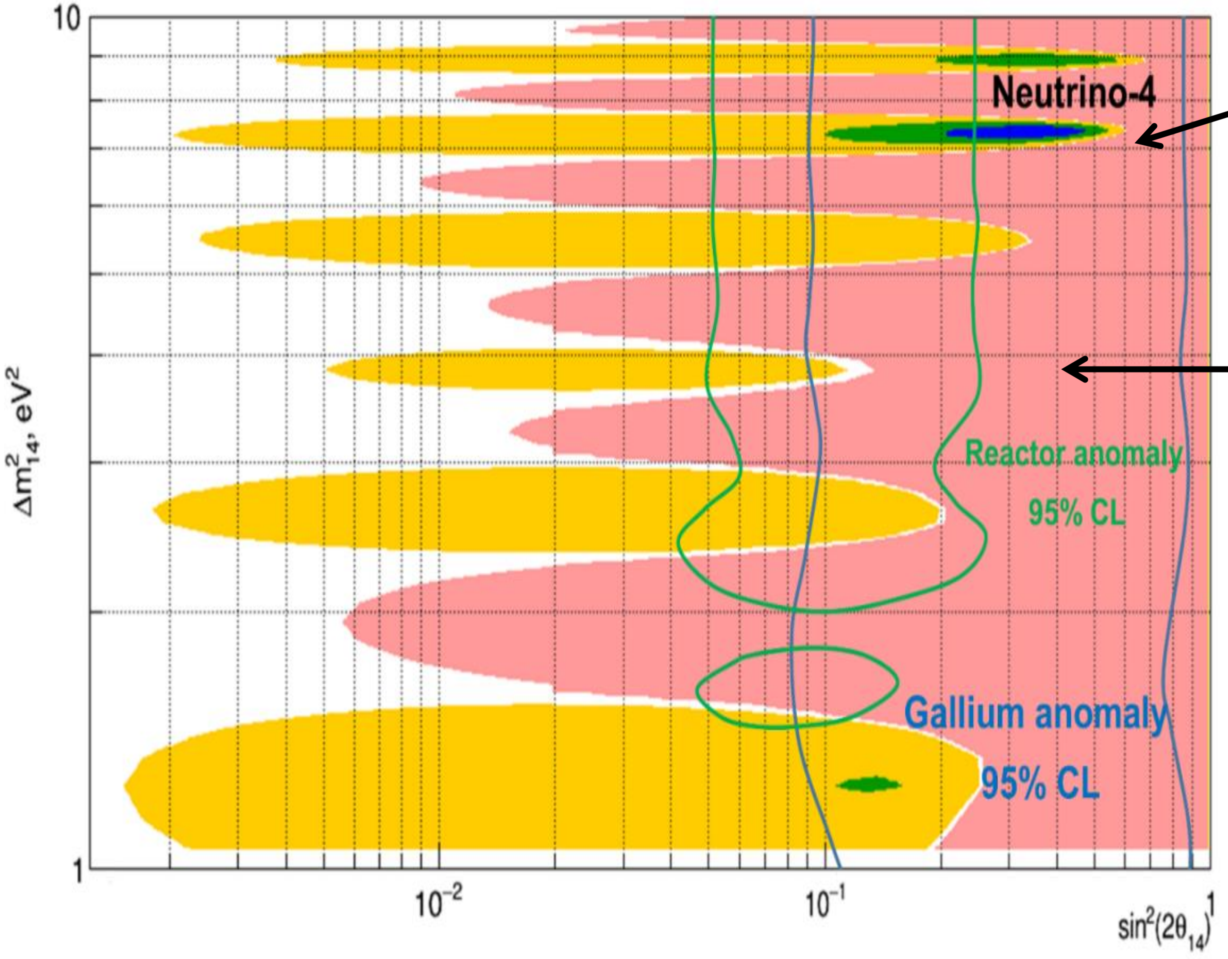
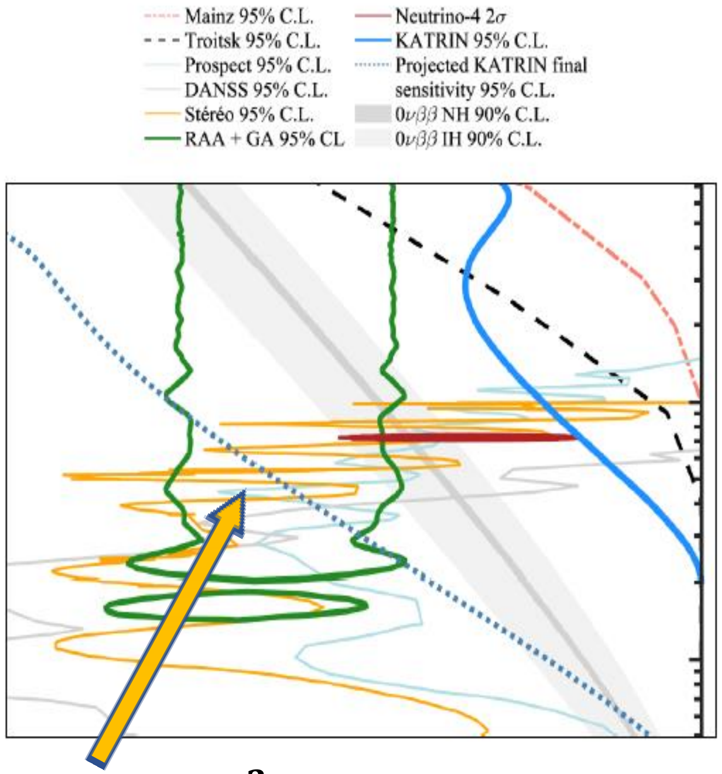
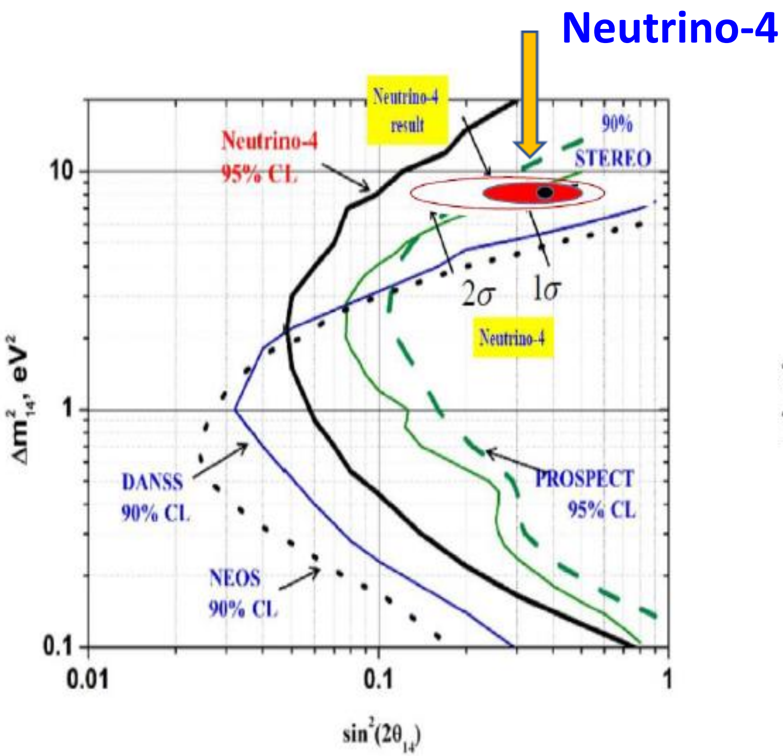


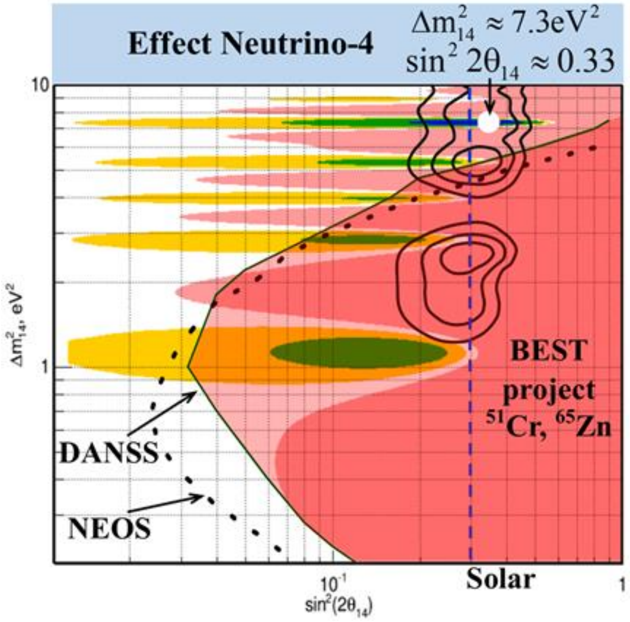
Expected from Neutrino-4 CL 99.7%

Excluded from Neutrino-4 CL>99.9% CL

Comparison of the results of Neutrino-4 experiment with the results of reactor, gallium anomaly and results of the KATRIN and GERDA experiments and results of BEST experiment







RAA $\sin^2 2\theta_{14} \approx 0.13 \pm 0.05(2.6\sigma)$

$\sin^2 2\theta_{14} \approx 0.36 \pm 0.12(2.9\sigma)$

Neutrino-4

$\sin^2 2\theta_{14} \approx 0.32 \pm 0.10(3.2\sigma)$

gallium anomaly

$$\sin^2 2\theta_{14} \approx 0.34 \pm 0.08(4.3\sigma)$$

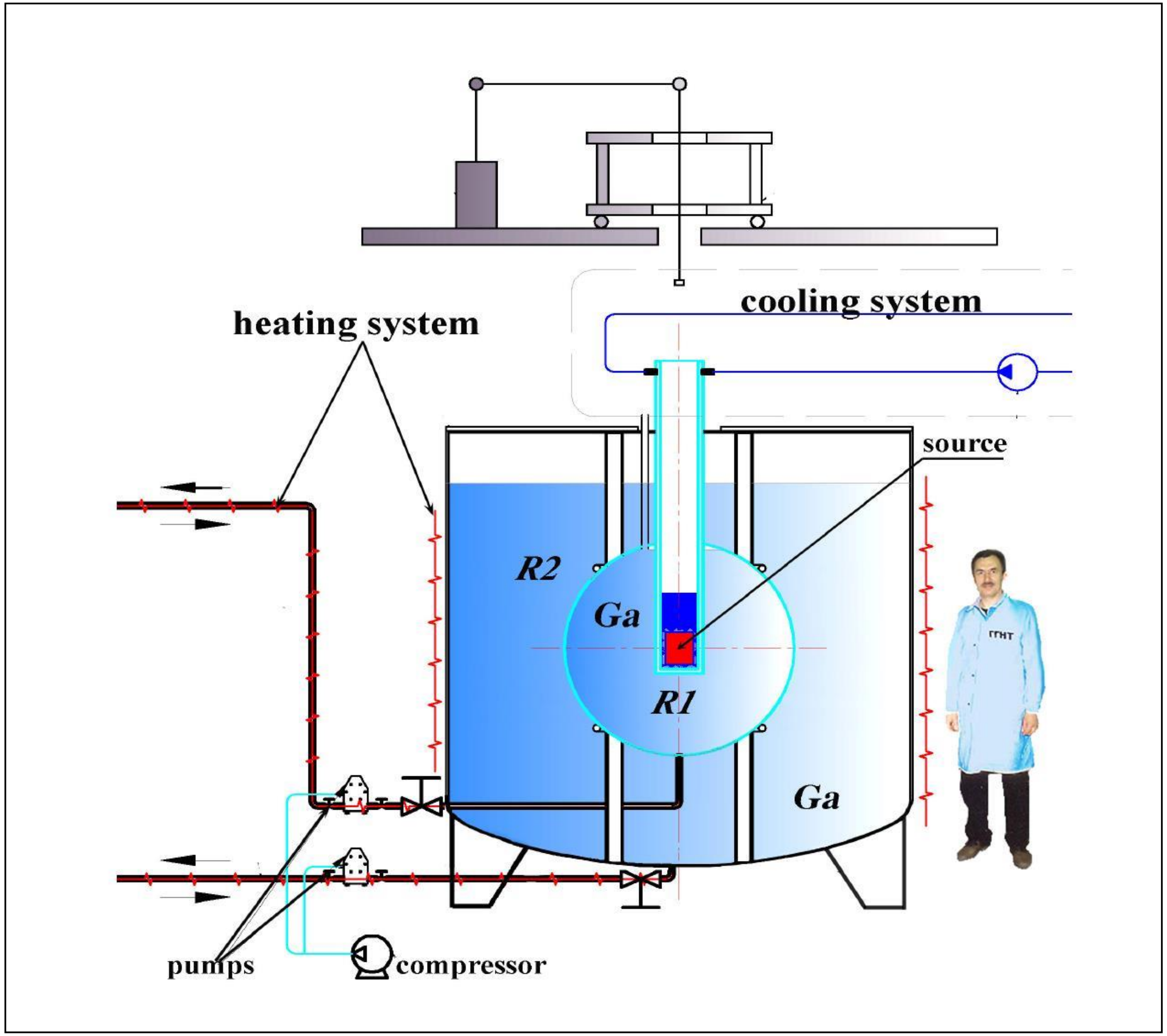
Neutrino-4 +gallium anomaly

We use the sterile neutrino parameters

$sin^2 2\theta_{14} \approx 0.34 \pm 0.08, \quad \Delta m^2_{14} = (7.3 \pm 0.13_{st} \pm 1.16_{syst})\ eV^2$

in our further analysis.

Results from the Baksan Experiment on Sterile Transitions (BEST)

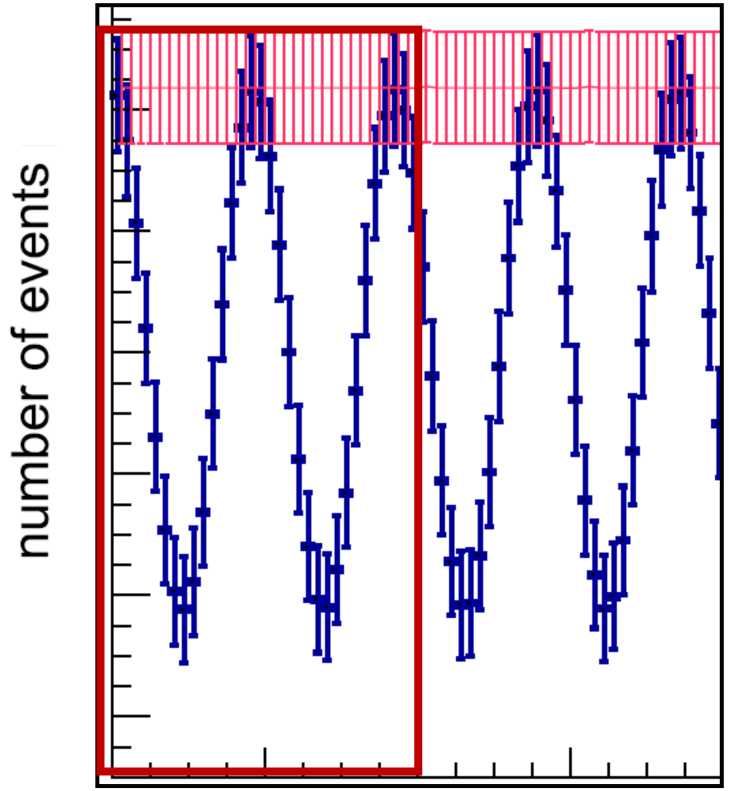


The source was delivered to the Baksan Neutrino Observatory (BNO) on July 5, 2019 and was placed into the two-zone target at 14:02 that same day and this is our chosen reference time for the source strength. The source was manufactured by irradiating 4 kg of 50Cr-enriched metal for 100 d in a reactor at the State Scientic Research Institute of Atomic Center Research Institute of Atomic Reactors, Dimitrovgrad, Russia. The activity (A) at the reference time is 3.4140.008 MCi.

The 51Cr isotope emitse's at 4 energies;
747 keV (81.63%),
427 keV (8.95%),
752 keV (8.49%) and
432 keV (0.93%).

Result for

$\Delta m^2_{14} = 7.3\ \text{eV}^2$

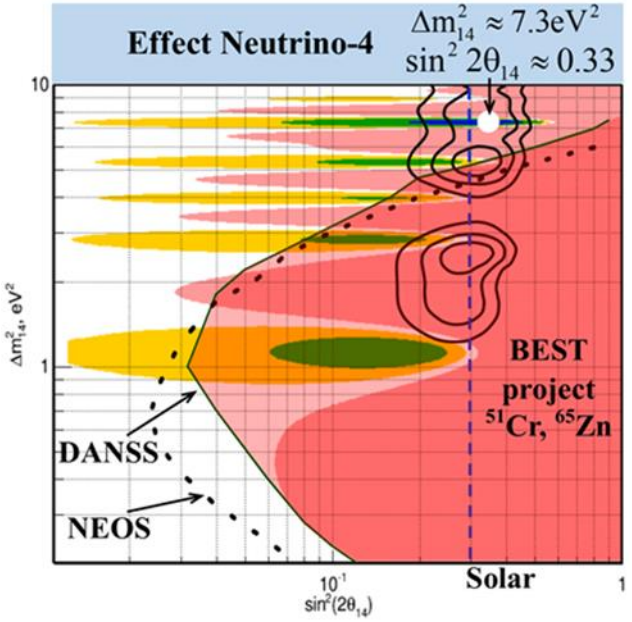


0 0.5 1.0 m

Inner Volume

Outer Volume

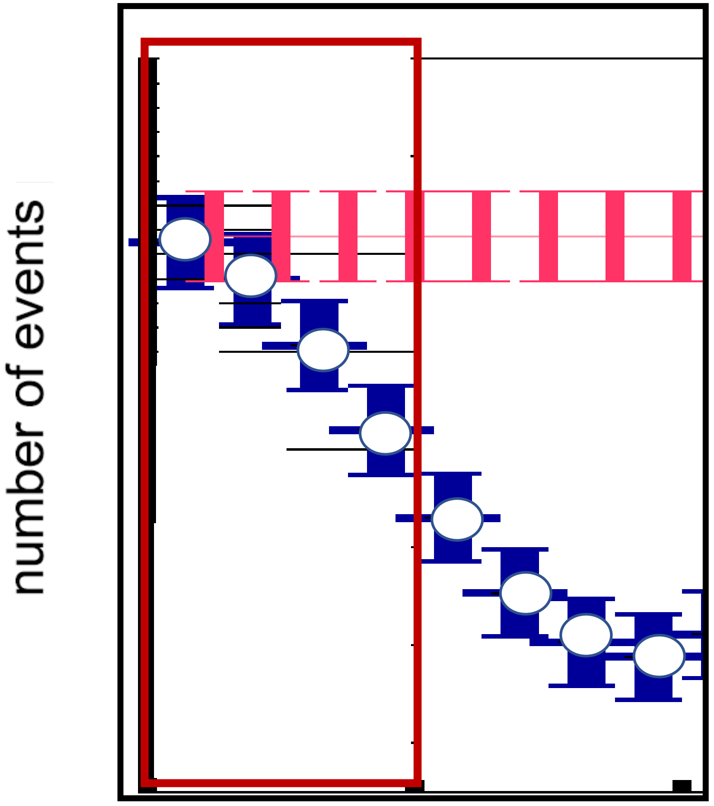
Experiment BEST



$\sin^2 2\theta_{14} \approx 0.32 \pm 0.10 (3.2\sigma)$

Project for

$\Delta m^2_{14} = 1.0\ \text{eV}^2$

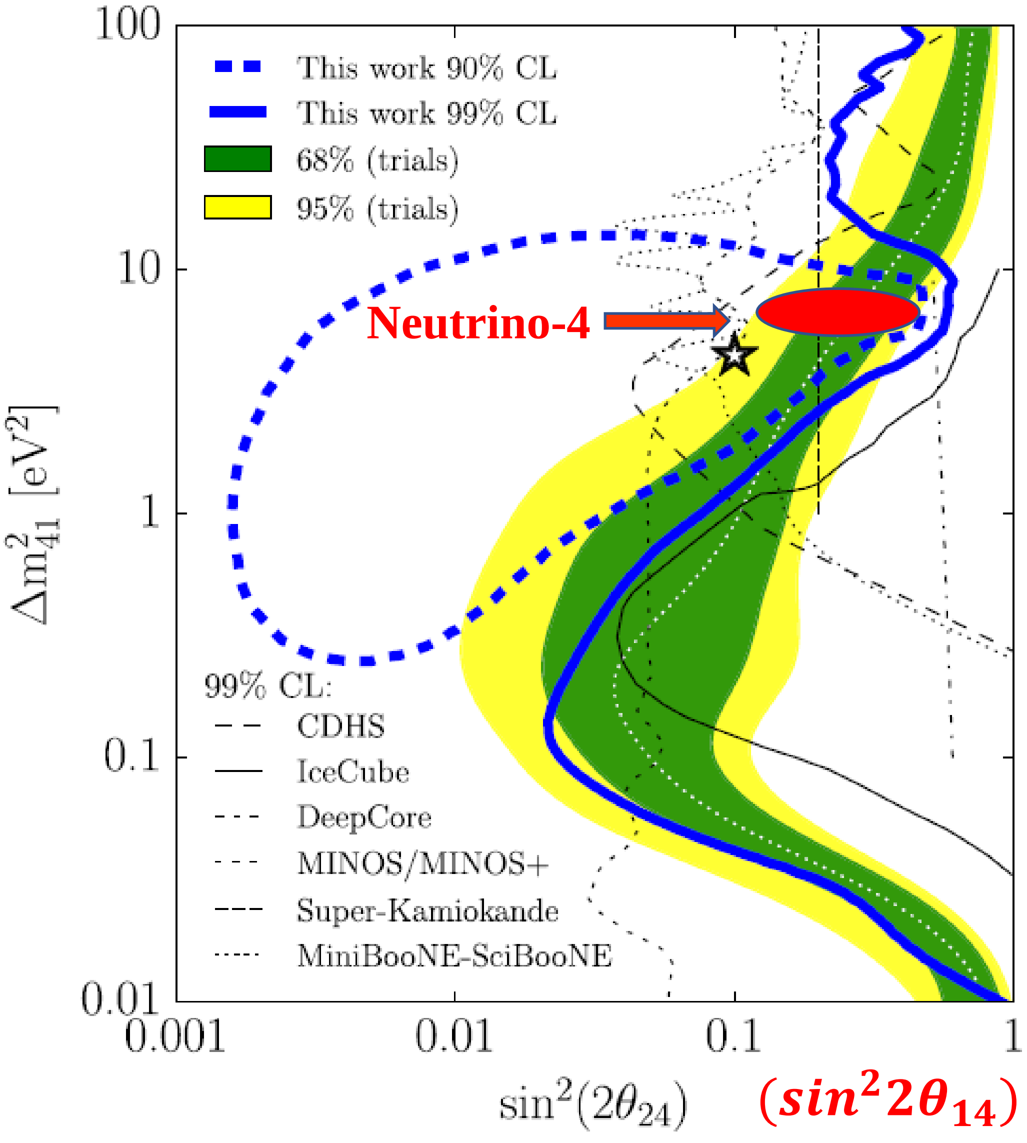


0 0.5 1.0 m

Inner Volume

Outer Volume

Comparison of results of Neutrino-4 experiment with results of IceCube experiment



$$\Delta m^2_{14} = 4.47^{+3.53}_{-2.08}$$

$$sin^2(2\theta_{24}) = 0.10^{+0.102}_{-0.07}$$

FIG. 4. Frequentist analysis result. The 90% and 99% C.L. contours, assuming Wilks's theorem, shown as dashed and solid bold blue lines, respectively. The green and yellow band shows the region where 68% and 95% of the pseudoexperiment 99% C.L. observations lie; the dashed white line corresponds to the median. Other muon-neutrino disappearance measurements at 99% C.L. are shown in black [25–30,123,124]; where results were not available at 99% C.L., methods of Ref. [11] were applied using public data releases. Finally, the star marks the analysis best-fit point location.

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Possibility of experimental confirmation of the 3 + 1 neutrino model with one sterile neutrino

THE STRUCTURE OF 3+1 NEUTRINO MODEL AND REPRESENTATION OF PROBABILITIES OF VARIOUS OSCILLATIONS

$$\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\\ \nu_s\end{bmatrix} = \begin{bmatrix}U_{e1} & U_{e2} & U_{e3} & U_{e4}\\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4}\\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4}\\ U_{s1} & U_{s2} & U_{s3} & U_{s4}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\\ \nu_4\end{bmatrix}$$

$$|U_{e4}|^2 = \sin^2(\theta_{14})$$

$$|U_{\mu4}|^2 = \sin^2(\theta_{24}) \cdot \cos^2(\theta_{14})$$

$$|U_{\tau4}|^2 = \sin^2(\theta_{34}) \cdot \cos^2(\theta_{24}) \cdot \cos^2(\theta_{14})$$

$$P_{\nu_e\nu_e} = 1 - 4|U_{e4}|^2(1 - |U_{e4}|^2)\, sin^2\left(\frac{\Delta m_{14}^2 L}{4E_{\nu_e}}\right) = 1 - sin^2 2\theta_{ee}\, sin^2\left(\frac{\Delta m_{14}^2 L}{4E_{\nu_e}}\right)$$

$$P_{\nu_\mu\nu_\mu} = 1 - 4|U_{\mu4}|^2(1 - |U_{\mu4}|^2)\, sin^2\left(\frac{\Delta m_{14}^2 L}{4E_{\nu_\mu}}\right) = 1 - sin^2 2\theta_{\mu\mu}\, sin^2\left(\frac{\Delta m_{14}^2 L}{4E_{\nu_\mu}}\right)$$

$$P_{\nu_\mu\nu_e} = 4|U_{e4}|^2|U_{\mu4}|^2\, sin^2\left(\frac{\Delta m_{14}^2 L}{4E_{\nu_e}}\right) = sin^2 2\theta_{\mu e}\, sin^2\left(\frac{\Delta m_{14}^2 L}{4E_{\nu_e}}\right)$$

The relations of oscillations parameters required for comparative analysis of experimental results are:

$$\sin^2 2\theta_{ee} \equiv \sin^2 2\theta_{14}$$

$$\sin^2 2\theta_{\mu\mu} = 4\sin^2\theta_{24}\cos^2\theta_{14}(1-\sin^2\theta_{24}\cos^2\theta_{14}) \approx \sin^2 2\theta_{24}$$

$$\sin^2 2\theta_{\mu e} = 4\sin^2\theta_{14}\sin^2\theta_{24}\cos^2\theta_{14} \approx \frac{1}{4}\sin^2 2\theta_{14}\sin^2 2\theta_{24}$$

The first important conclusion of the 3 + 1 model is that the oscillation frequency in all processes should be the same, i.e. it is determined by the value

$$\boldsymbol{\Delta m^2_{14}}$$

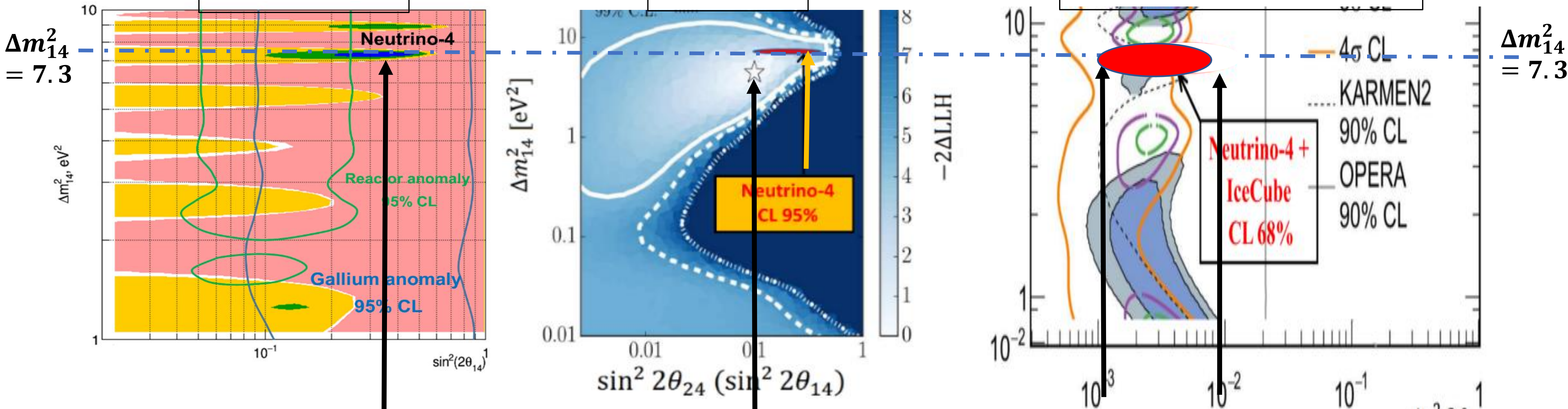
The second important relation for experimental verification of the 3 + 1 model.

$$sin^2 2\theta_{\mu e} \approx \frac{1}{4} sin^2 2\theta_{14} sin^2 2\theta_{24}$$

This relationship can be interpreted in fairly simple way. The appearance of electron neutrinos in muon neutrino beam: this is a second-order process, i.e. transition of muon neutrino to sterile neutrino, and then the transition of sterile neutrino into electron neutrino.

COMPARISON OF NEUTRINO-4 WITH ICECUBE, MINIBOONE AND LSND

NEUTRINO-4 | **ICECUBE** | **MINIBOONE AND LSND**



$\sin^2 2\theta_{14} \approx$ **0.34** $\quad \sin^2 2\theta_{24} \approx$ **0.1** $\quad \sin^2 2\theta_{\mu e} \approx$ **$0.001 \div 0.009$**

$$\sin^2 2\theta_{\mu e} \approx \frac{1}{4} \sin^2 2\theta_{14} \sin^2 2\theta_{24}$$

Test of neutrino model 3+1

A.P. Serebrov *, R.M. Samoilov JETP Letters, Volume 112, 2020 Issue 4, pp 211–225
arxiv:2003.03199

$$\Delta m^2_{14} = 4.47^{+3.53}_{-2.08} \quad \sin^2(2\theta_{24}) = 0.10^{+0.102}_{-0.07}$$

ICECUBE

Prediction of the effective mass of electron neutrino from Neutrino-4 experiment and comparison with experiments on measuring neutrino mass: KATRIN and GERDA

$$m_{4\nu_e}^{eff} = \sqrt{\sum m_i^2 |U_{ei}|^2}\,; \quad sin^2 2\theta_{14} \approx 4|U_{14}|^2;$$

$$\sum m_\nu = m_1 + m_2 + m_3 \approx 0.54 \div 0.11\text{eV}$$

$$\Delta m_{14}^2 \approx m_4^2 \approx 7.25\ eV^2, \quad m_1^2, m_2^2, m_3^2 \ll m_4^2$$

$$m_4 = (2.70 \pm 0.22)\text{eV}$$

$$sin^2 2\theta_{14} \approx 0.34 \pm 0.08\ (4.3\sigma)$$

$$m_{4\nu_e}^{\text{eff}} \approx \sqrt{\text{m}_4^2 |U_{e4}|^2} \approx \frac{1}{2}\sqrt{m_4^2 \sin^2 2\theta_{14}}.$$

$$m_{4\nu_e}^{eff} = (0.82 \pm 0.18)\text{eV}$$

$$m_{4\nu_e}^2 = 0.68 \pm 0.29$$

The majorana mass is determined in the double β-decay experiments by the ratio:

$$m(0\nu\beta\beta) = /\sum U_{ei}^2 m_i\ /$$

$$m(0\nu\beta\beta) \approx m_4 U_{14}^2$$

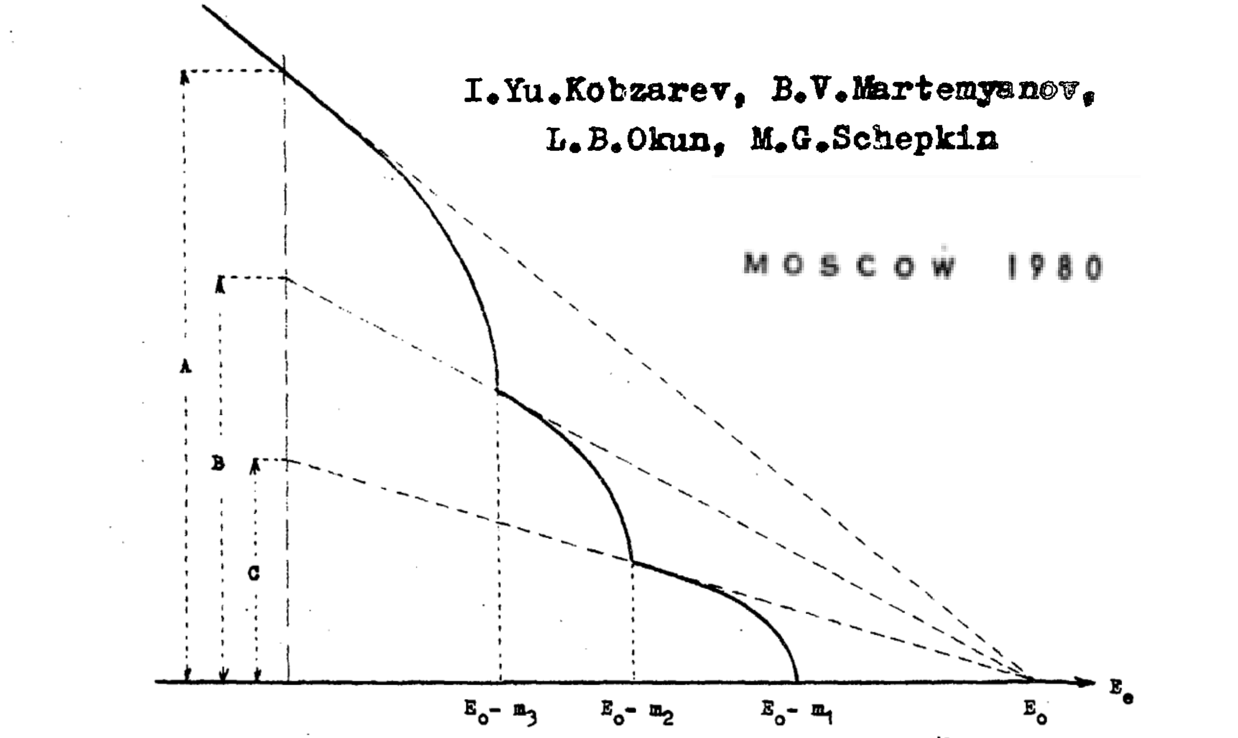
$$m(0\nu\beta\beta) = (0.25 \pm 0.09)\text{eV}$$

Karlsruhe Tritium Neutrino (KATRIN) experiment

First direct neutrino-mass measurement with sub-eV sensitivity

arXiv:2105.08533v1 [hep-ex] 18 May 2021

THE PHENOMENOLOGY OF NEUTRINO OSCILLATIONS



The edge of β -spectrum (Fermi spectrum). $A : B : C = (|a_1|^2 + |a_2|^2)^{1/2} : |a_1|$ (See Eqs.33,34)

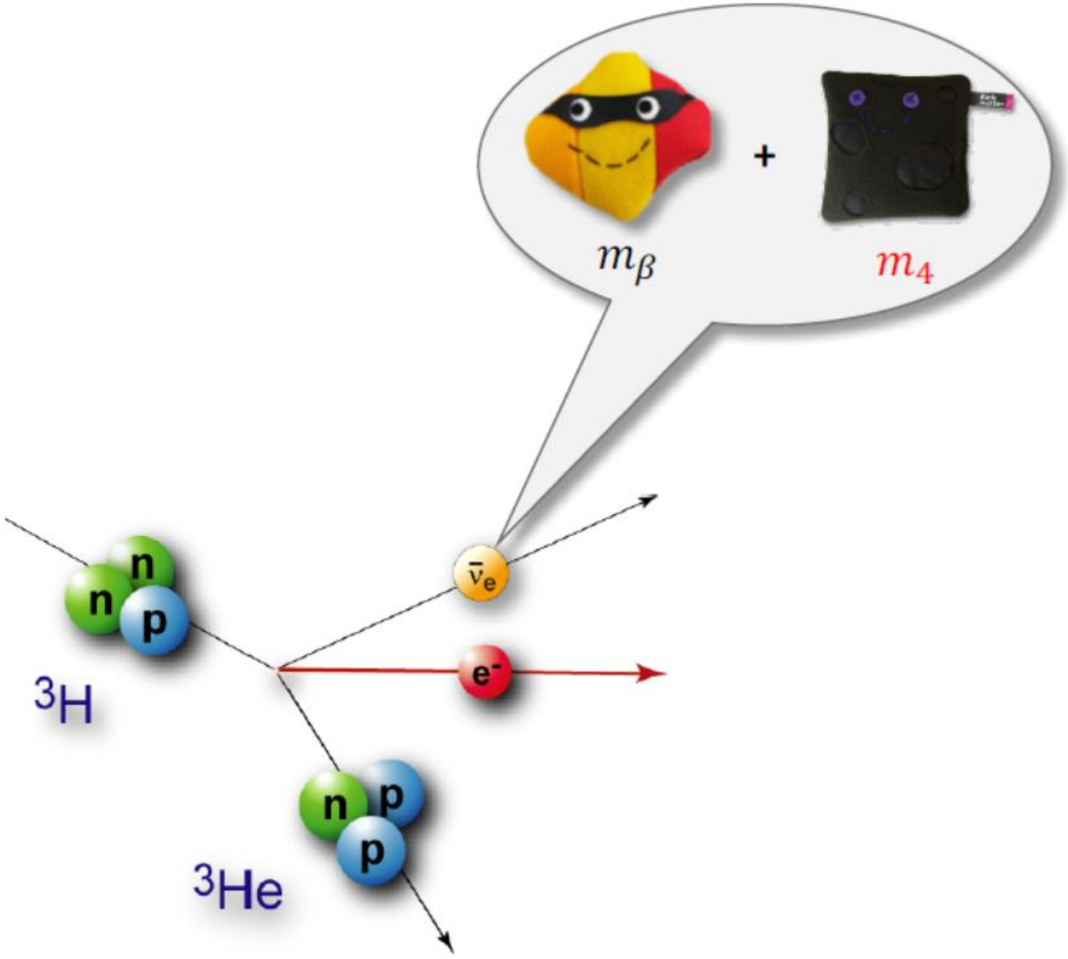
The best fit to the spectral data yields $m^2_{\nu_e} = 0.26 \pm 0.34$, resulting in an upper limit of $m_{\nu_e} < 0.9$ eV (90%)

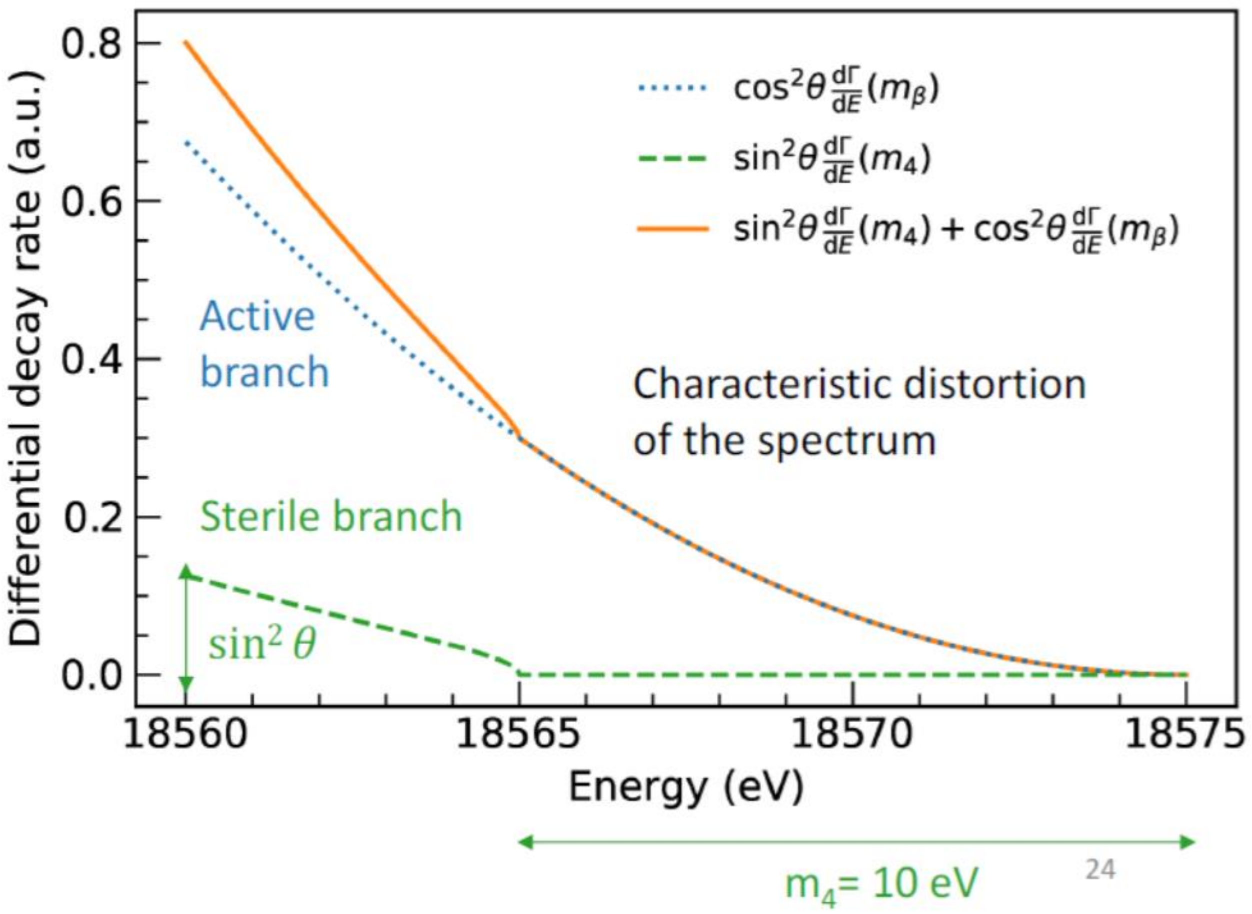
By combining this result with the first neutrino mass campaign, we find an upper limit of $m_{\nu_e} < 0.8$ eV (90%)

What does KATRIN measure?

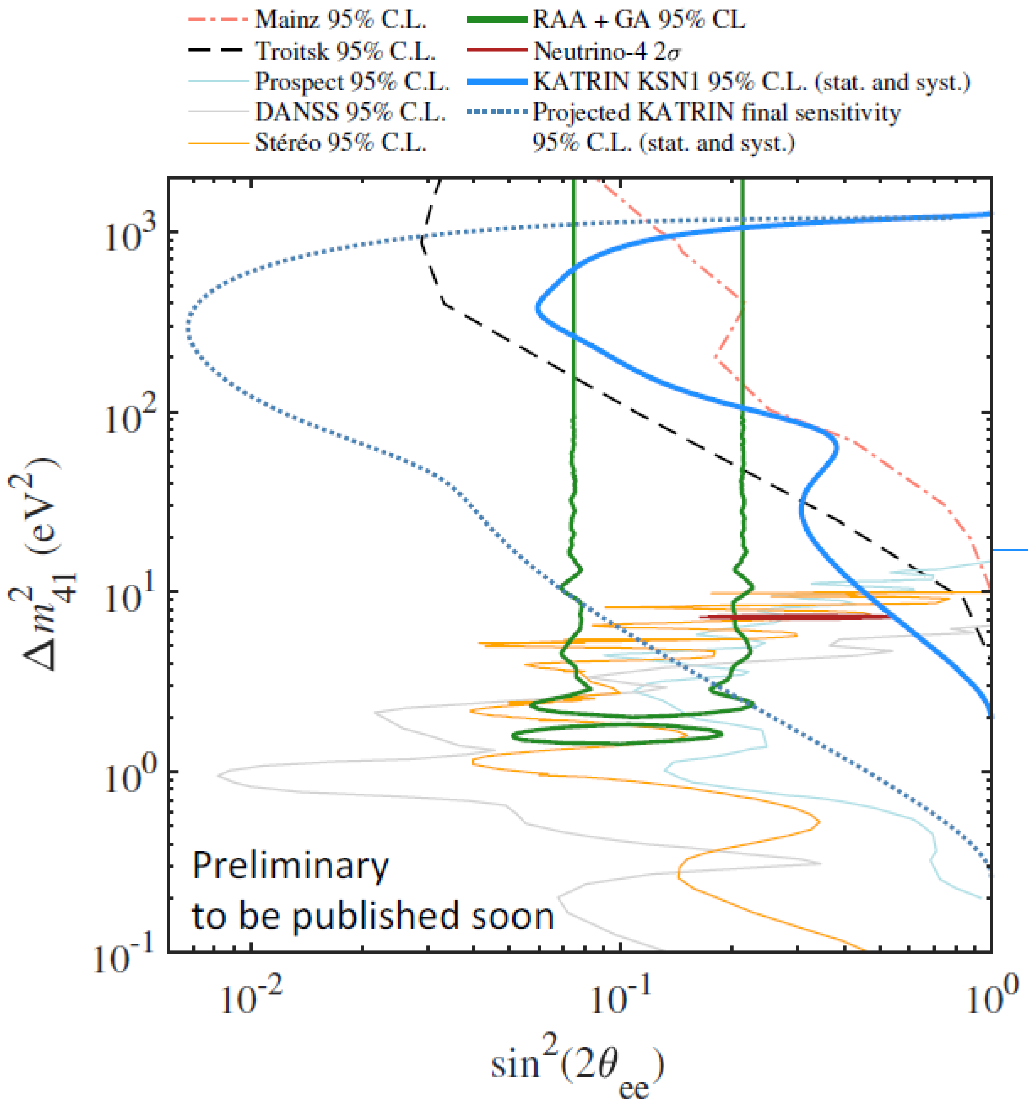
Signature of light sterile neutrino

→ See **Reactor and Geo ν** session on **25 June**
→ See **sterile ν** session on **2 July**





eV-scale sterile neutrino search



High Δm_{41} region:

- ✓ Improve exclusion with respect to DANSS, PROSPECT, and STEREO
- ✓ Exclude parameter space of Reactor Anomaly (RAA)

Low Δm_{41} region:

- ✓ Improve MAINZ and TROITSK limit
- ✓ The NEUTRINO-4 hint at the edge of exclusion limit

KATRIN

$m^{\text{eff}}_{3\nu_e} < 0.8$ eV (90%)

$\left(m^{eff}_{3\nu_e}\right)^2 = \mathbf{0.26 \pm 0.34}$

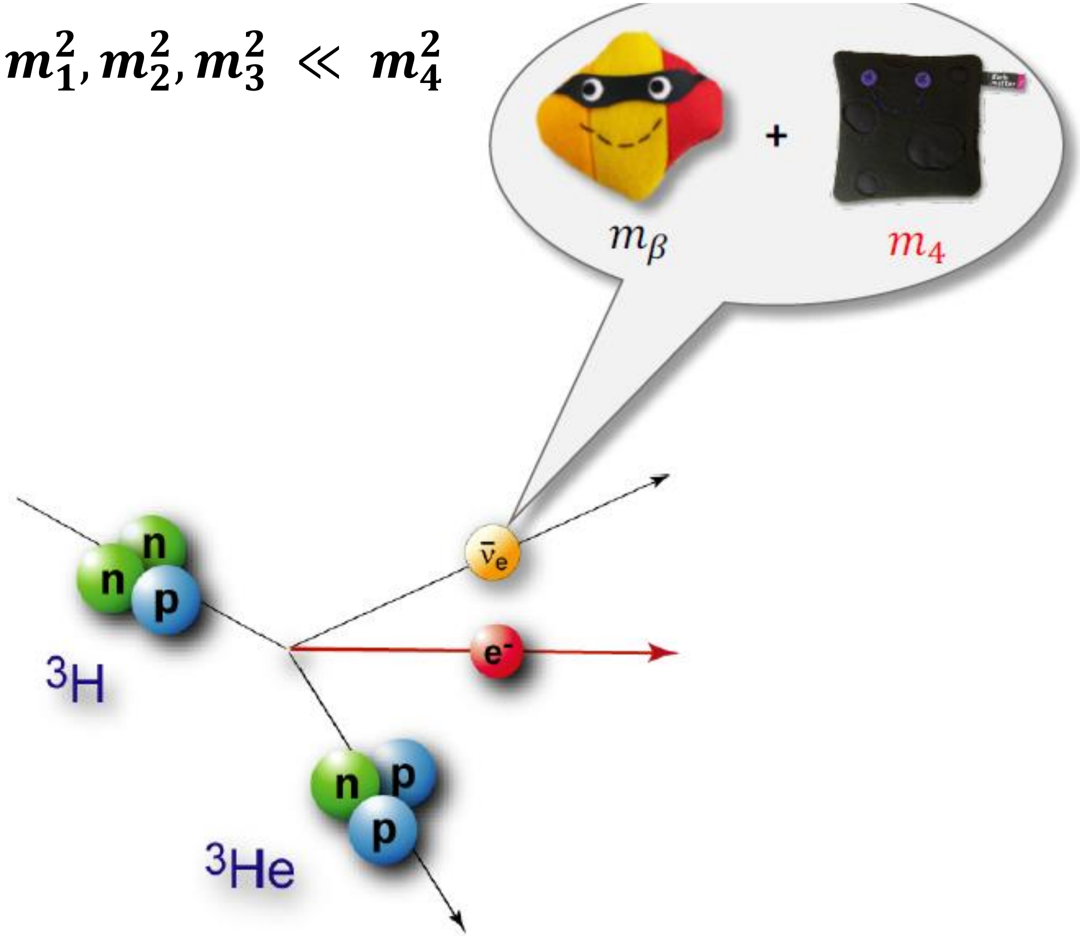
$m^{eff\,2}_{4\nu_e} = ?$

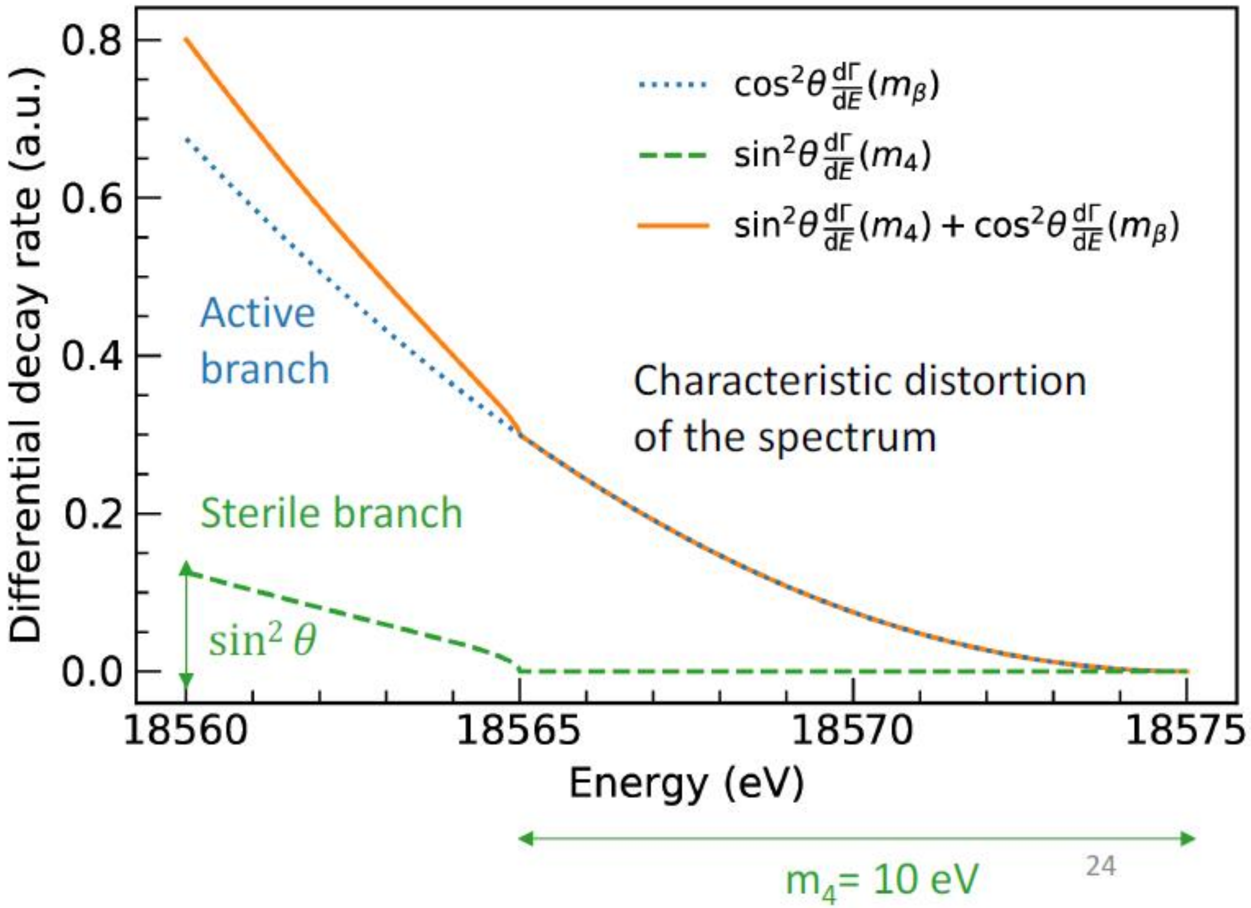
$\mathbf{m^{eff}_{4\nu_e} = (0.82 \pm 0.18)eV}$

Neutrino4

$\left(m^{eff}_{4\nu_e}\right)^2 = \mathbf{0.68 \pm 0.29}$

$m_1^2, m_2^2, m_3^2 \ll m_4^2$





Susanne Mertens

$$R_\beta(E, m_\nu) = \frac{G_F^2 \cos^2\theta_C}{2\pi^3} |\mathcal{M}|^2 F(E, Z+1) \times (E + m_e)\sqrt{(E+m_e)^2 - m_e^2} \times \sum_i \varsigma_f \sqrt{(E_0 - E - V_i)^2 - m_\nu^2}\, \Theta(E_0 - E - V_i - m_\nu)$$

$$R_\beta^s(E, m'_\nu, m_4) = (1 - |U_{e4}|^2) R_\beta(E, m'_\nu) + |U_{e4}|^2 R_\beta(E, m_4)$$

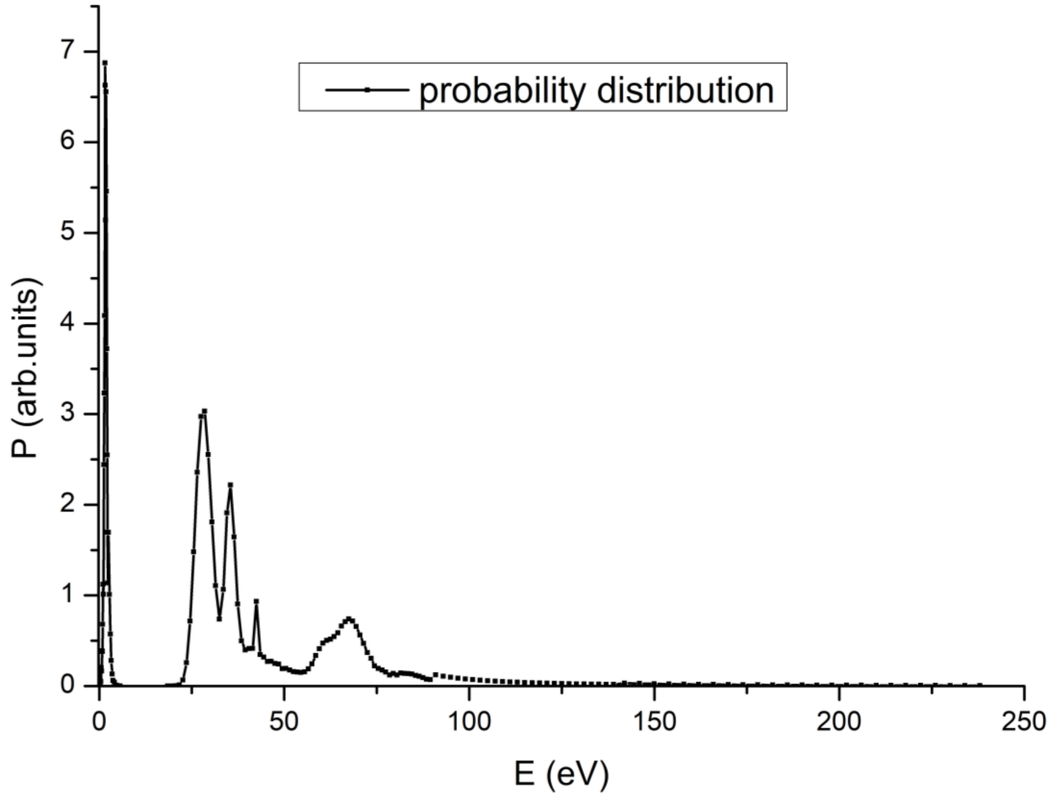


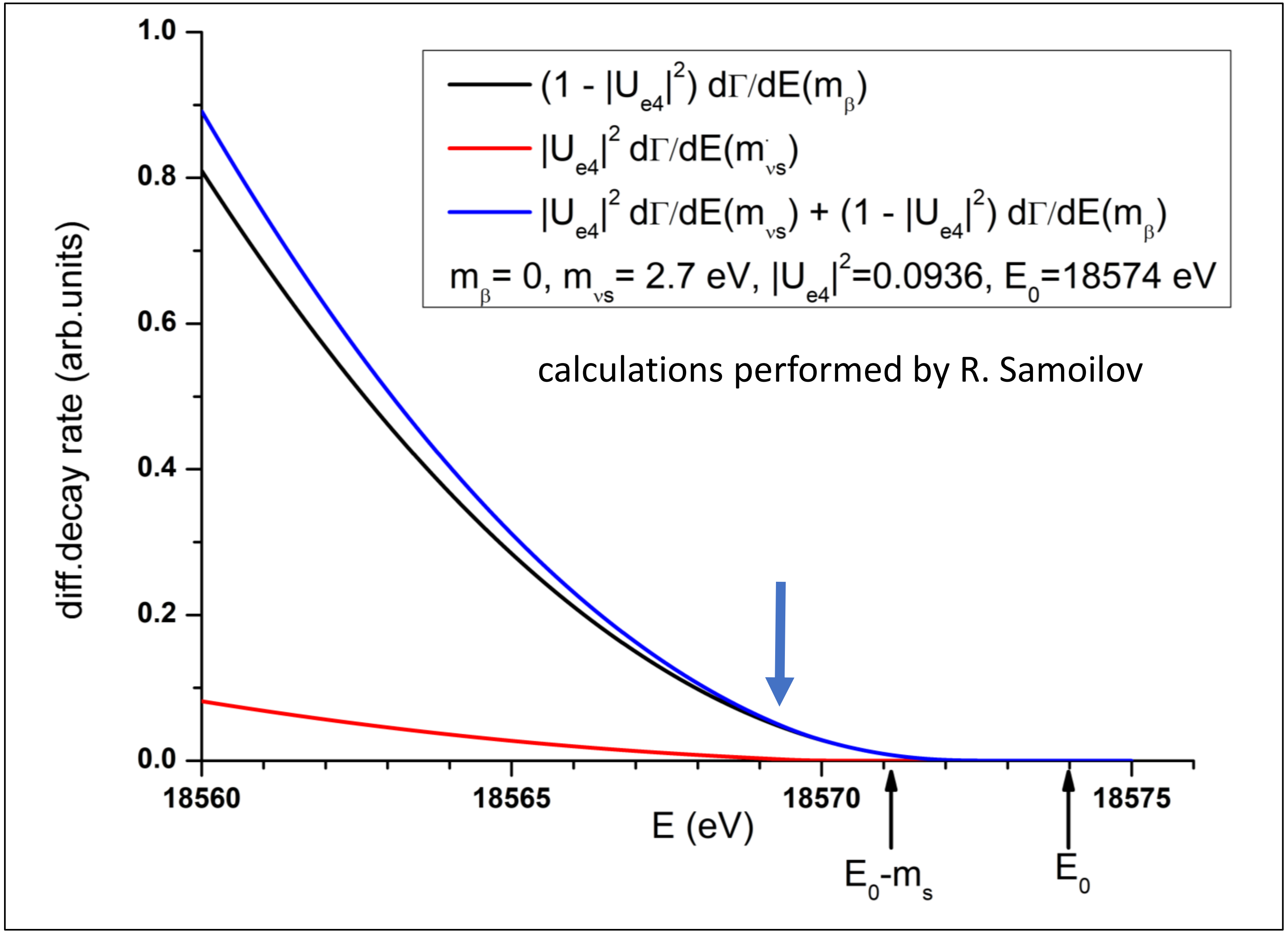
TABLE I. Discretized final-state probability distribution $P(E_i)$ for $^3HeT^+$ ($^3HeH^+$) following the β decay of a free T_2 (HT) molecule within the sudden approximation. The mean excitation energies E_i for both isotopes are given relative to the rovibronic ground state of $^3HeT^+$ and the recoil energy for $^3HeT^+$. The spectrum extends to 240 eV after which an atomic distribution may be used (see text for details).

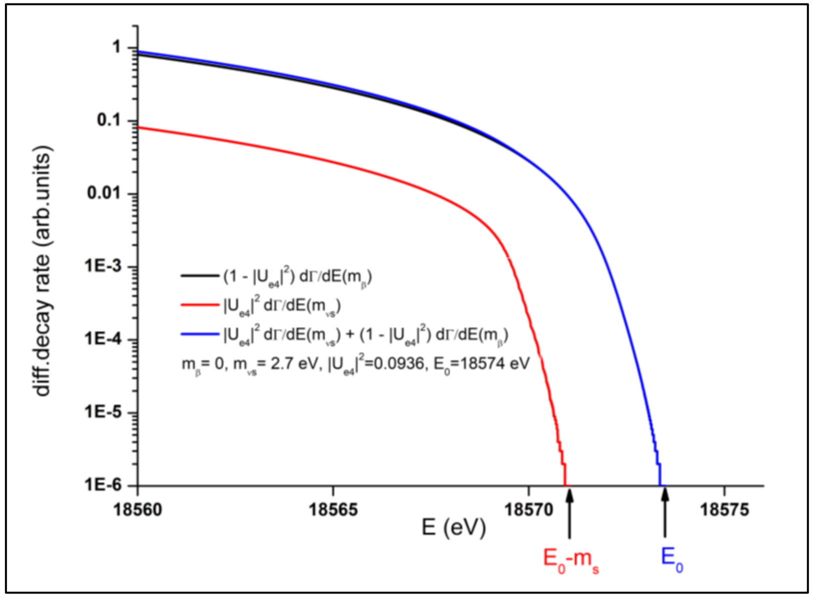
E_i(eV)	$P(E_i)$(%)	E_i(eV)	$P(E_i)$(%)	E_i(eV)	$P(E_i)$(%)
0.053(0.049)	0.0069(0.0000)	36.459(36.467)	1.6434(1.5166)	88.498(88.493)	0.0757(0.0806)
0.124(0.147)	0.0046(0.0000)	37.445(37.457)	0.9037(0.9546)	89.495(89.498)	0.0696(0.0715)
0.247(0.247)	0.0233(0.0000)	38.463(38.467)	0.4989(0.5951)	90.967(90.962)	0.1236(0.1276)
0.351(0.347)	0.0553(0.0000)	39.499(39.485)	0.3978(0.4492)	92.983(92.979)	0.1080(0.1102)
0.442(0.447)	0.0457(0.0000)	40.506(40.504)	0.4124(0.4280)	94.982(94.981)	0.0966(0.0973)
0.556(0.547)	0.2033(0.0000)	41.498(41.502)	0.4152(0.3905)	96.982(96.981)	0.0864(0.0867)
0.665(0.647)	0.1649(0.0000)	42.498(42.494)	0.3925(0.3795)	98.981(98.979)	0.0776(0.0779)
0.759(0.747)	0.3877(0.0000)	43.504(43.486)	0.3457(0.3308)	100.981(100.980)	0.0703(0.0707)
0.850(0.876)	0.3808(0.2252)	44.511(44.474)	0.3186(0.3122)	102.983(102.982)	0.0636(0.0638)
0.937(0.955)	0.6809(0.5814)	45.494(45.495)	0.2701(0.2906)	104.984(104.984)	0.0578(0.0580)
1.048(1.087)	1.1214(1.2129)	46.481(46.494)	0.2713(0.2538)	106.985(106.985)	0.0528(0.0529)
1.143(1.162)	1.0112(0.6216)	47.483(47.472)	0.2481(0.2438)	108.986(108.986)	0.0483(0.0484)
1.249(1.260)	2.4406(0.6313)	48.513(48.452)	0.2412(0.2236)	110.986(110.986)	0.0444(0.0445)
1.359(1.344)	3.2337(3.0441)	49.498(49.445)	0.1907(0.2201)	112.987(112.986)	0.0409(0.0410)
1.451(1.452)	4.0864(9.5173)	50.462(50.472)	0.1938(0.1989)	114.987(114.987)	0.0378(0.0378)
1.552(1.585)	6.8745(5.7372)	51.501(51.513)	0.1760(0.1808)	116.987(116.987)	0.0350(0.0350)
1.657(1.663)	6.6279(6.9898)	52.496(52.491)	0.1575(0.1609)	118.988(118.988)	0.0325(0.0324)
1.745(1.739)	5.1412(5.7803)	53.491(53.494)	0.1541(0.1627)	120.988(120.988)	0.0302(0.0302)
1.834(1.847)	6.5561(7.1064)	54.502(54.507)	0.1485(0.1632)	122.988(122.988)	0.0282(0.0281)
1.940(1.942)	5.4588(4.1076)	55.508(55.512)	0.1557(0.1963)	124.989(124.989)	0.0263(0.0262)
2.044(2.044)	3.7231(3.3984)	56.526(56.531)	0.1895(0.2533)	126.989(126.989)	0.0246(0.0245)
2.144(2.150)	2.5473(2.7572)	57.518(57.520)	0.2427(0.3266)	128.989(128.989)	0.0230(0.0229)
2.244(2.270)	1.6959(1.2274)	58.529(58.535)	0.3357(0.3956)	130.989(130.989)	0.0216(0.0215)
2.344(2.346)	1.1369(1.0046)	59.518(59.507)	0.4095(0.4490)	132.990(132.990)	0.0203(0.0202)
2.510(2.509)	1.6947(1.4798)	60.505(60.501)	0.4714(0.4895)	134.990(134.990)	0.0191(0.0190)
2.762(2.756)	1.0094(0.8381)	61.505(61.506)	0.5034(0.5065)	136.990(136.990)	0.0180(0.0179)
3.009(3.009)	0.5732(0.4292)	62.501(62.504)	0.5152(0.5320)	138.990(138.990)	0.0169(0.0168)
3.257(3.257)	0.2806(0.2117)	63.503(63.500)	0.5442(0.5690)	141.962(141.962)	0.0311(0.0309)
3.507(3.506)	0.1316(0.0985)	64.512(64.496)	0.5859(0.6180)	145.964(145.964)	0.0278(0.0277)
3.757(3.756)	0.0623(0.0446)	65.504(65.496)	0.6617(0.7003)	149.965(149.965)	0.0250(0.0249)
4.083(4.082)	0.0420(0.0288)	66.510(66.507)	0.7094(0.6969)	153.966(153.967)	0.0225(0.0225)
4.579(4.579)	0.0080(0.0054)	67.501(67.497)	0.7404(0.6898)	157.967(157.968)	0.0204(0.0203)
5.132(5.076)	0.0015(0.0009)	68.492(68.488)	0.7164(0.6457)	161.969(161.969)	0.0185(0.0185)
5.647(5.658)	0.0000(0.0002)	69.489(69.488)	0.6563(0.5835)	165.970(165.970)	0.0169(0.0169)
18.500(18.773)	0.0000(0.0001)	70.486(70.487)	0.5620(0.5037)	169.971(169.971)	0.0154(0.0154)
19.500(19.680)	0.0000(0.0016)	71.473(71.475)	0.4691(0.4318)	173.972(173.972)	0.0141(0.0142)
20.696(20.645)	0.0012(0.0125)	72.478(72.485)	0.3680(0.3547)	177.973(177.973)	0.0130(0.0130)
21.658(21.615)	0.0113(0.0618)	73.486(73.487)	0.3049(0.3003)	181.973(181.973)	0.0120(0.0120)
22.627(22.590)	0.0656(0.2137)	74.443(74.447)	0.2210(0.2437)	185.974(185.974)	0.0111(0.0111)
23.598(23.568)	0.2567(0.5507)	75.446(75.469)	0.1928(0.2029)	189.975(189.975)	0.0103(0.0103)
24.573(24.549)	0.7149(1.1046)	76.465(76.465)	0.1761(0.1862)	193.976(193.975)	0.0095(0.0095)
25.550(25.532)	1.4804(1.7838)	77.430(77.428)	0.1530(0.1723)	197.976(197.976)	0.0089(0.0089)
26.529(26.517)	2.3583(2.3800)	78.512(78.518)	0.1215(0.1236)	201.977(201.977)	0.0083(0.0083)
27.510(27.503)	2.9715(2.6793)	79.464(79.474)	0.1390(0.1597)	205.977(205.977)	0.0077(0.0077)
28.493(28.491)	3.0307(2.5894)	80.504(80.522)	0.1216(0.1238)	209.978(209.978)	0.0072(0.0072)
29.478(29.480)	2.5527(2.1803)	81.510(81.515)	0.1422(0.1459)	213.979(213.978)	0.0068(0.0067)
30.464(30.471)	1.8080(1.6225)	82.523(82.504)	0.1384(0.1356)	217.979(217.979)	0.0064(0.0063)
31.455(31.470)	1.1070(1.1150)	83.505(83.503)	0.1368(0.1325)	221.979(221.979)	0.0060(0.0059)
32.490(32.507)	0.7377(0.9290)	84.499(84.497)	0.1316(0.1266)	225.980(225.980)	0.0056(0.0056)
33.557(33.539)	1.0637(1.2964)	85.490(85.493)	0.1153(0.1082)	229.980(229.980)	0.0053(0.0053)
34.534(34.519)	1.9095(1.8714)	86.491(86.499)	0.1076(0.1036)	233.981(233.980)	0.0050(0.0050)
35.492(35.490)	2.2178(1.9702)	87.498(87.480)	0.0921(0.0912)	237.981(237.981)	0.0047(0.0047)

What should be seen ***KATRIN*** taking into account sterile neutrino with parameters:

$m_4 = (2.70 \pm 0.22)\text{eV}$

$\sin^2 2\theta_{14} \approx 0.34 \pm 0.08\ (4.3\sigma)$





COMPARISON OF NEUTRINO-4 MASS PREDICTION WITH MEASUREMENT OF NEUTRINO MASS

	Neutrino-4	KATRIN	GERDA $m(0\nu\beta\beta)$
$m_{\nu_e}^{eff} = \sqrt{\sum m_i^2 \lvert U_{ei}\rvert^2}$ $\Delta m_{14}^2 \approx m_4^2$ Effective mass and mass squared: $m_{\nu_e}^{eff}$, m_ν^2	$m_{4\nu_e}^{eff} = 0.82 \pm 0.18$ $\left(m_{4\nu_e}^{eff}\right)^2 = 0.68 \pm 0.29$ $m_1^2, m_2^2, m_3^2 \ll m_4^2$	$m_{3\nu_e}^{eff} < 0.8$ eV (90%) $m_{3\nu_e}^{eff\,2} = 0.26 \pm 0.34$ $m_{4\nu_e}^{eff\,2} =$?	
Maiorana mass $m(0\nu\beta\beta)$	$m(0\nu\beta\beta) = (0.25 \pm 0.09)$eV		$m_{\beta\beta} < [0.080 - 0.182]$eV

Comparison with neutrino mass constraints from experiments for neutrino less double beta-decay search

This expression for the model 3 + 1 and with $m1, m2, m3 \ll m4$ assumption can be simplified:

$$m_{ee} = \left|\sum_i m_i U_{ei}^2\right| = \begin{cases} \left|m_0 c_{12}^2 c_{13}^2 + \sqrt{\Delta m_{21}^2 + m_0^2}\, s_{12}^2 c_{13}^2 e^{2i(\eta_2-\eta_1)} + \sqrt{\Delta m_{32}^2 + \Delta m_{21}^2 + m_0^2}\, s_{13}^2 e^{-2i(\delta_{CP}+\eta_1)}\right| & \text{in NO}, \\ \left|m_0 s_{13}^2 + \sqrt{m_0^2 - \Delta m_{32}^2}\, s_{12}^2 c_{13}^2 e^{2i(\eta_2+\delta_{CP})} + \sqrt{m_0^2 - \Delta m_{32}^2 - \Delta m_{21}^2}\, c_{12}^2 c_{13}^2 e^{2i(\eta_1+\delta_{CP})}\right| & \text{in IO}, \end{cases}$$

The numerical for this with Neutrino-4 and other experiments average result is shown below.

$\boldsymbol{m(0\nu\beta\beta) = (0.25 \pm 0.09)}$**eV**
our estimation

$$m(0\nu\beta\beta) \approx m_4 U_{14}^2$$

$\boldsymbol{m(0\nu\beta\beta)}$ **<[0.080−0.182]eV**
experiments

The best restrictions on Majorana mass were obtained in GERDA experiment . In these experiments, the half-life of the isotope is measured, which depends on Majorana mass as follows:

$$1/T_{1/2}^{0\nu} = g_A^4 G^{0\nu} |M^{0\nu}|^2 \frac{\langle m_{\beta\beta}\rangle^2}{m_e^2}$$

The upper limit for the lower limit - the upper limit for the Majorana mass:
Lower limit for $T_{1/2}/0\nu > 1.8\times10^{26}$ years (90% CL)
Upper limit for $m\beta\beta <$[80−182]meV

Neutrino model with one sterile neutrino

Sterile Neutrino Parameters

RESUTS

$$\Delta m_{14}^2 = 7.30 \pm 0.13_{st} \pm 1.16_{syst} = 7.30 \pm 1.17$$
$$sin^2 2\theta = 0.36 \pm 0.12_{stat} (2.9\sigma)$$

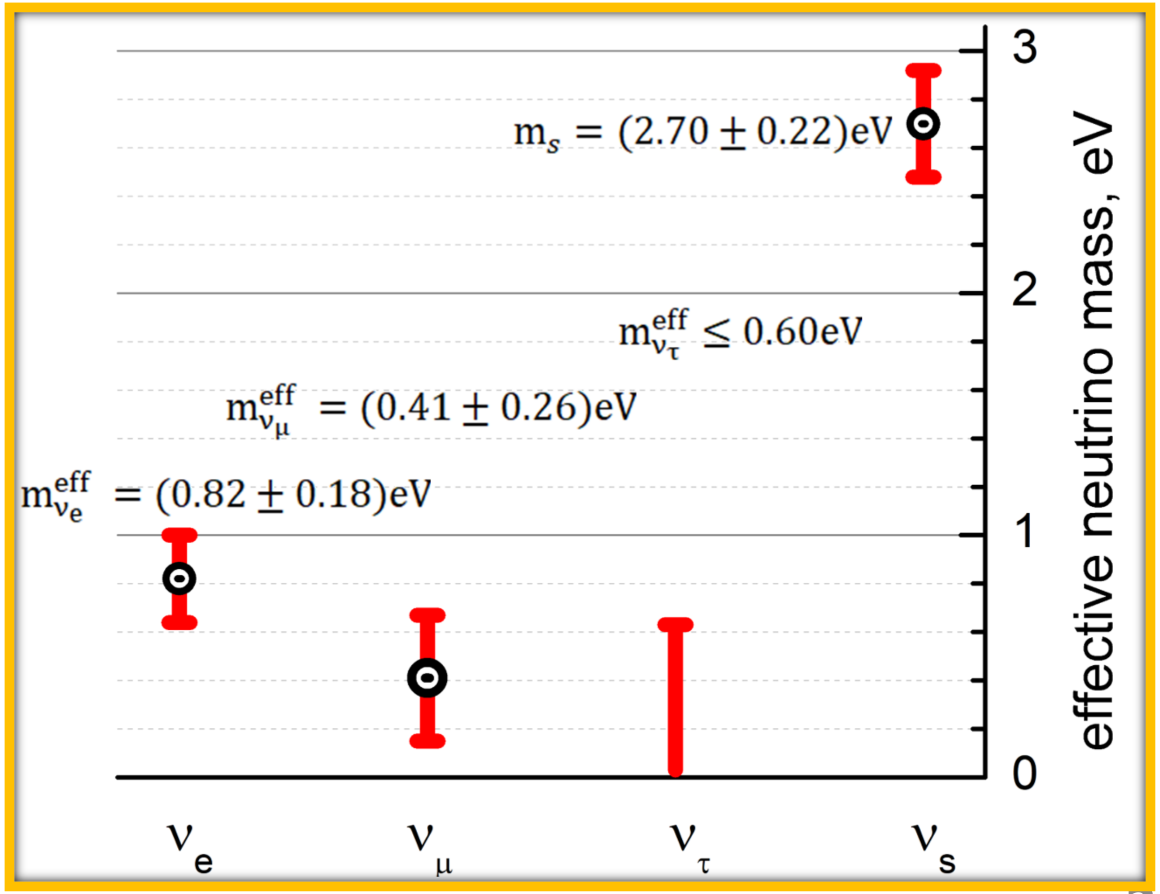
$$m_4 = (2.70 \pm 0.22)\text{eV}$$

$$sin^2 2\theta_{14} \approx 0.34 \pm 0.08 \ (4.3\sigma)$$

$$\mathrm{m}_{4\nu_e}^{\mathrm{eff}} = (0.82 \pm 0.18)\text{eV}$$

$$\mathrm{m}_{4\nu_\mu}^{\mathrm{eff}} = (0.41 \pm 0.26)\text{eV}$$

$$\mathrm{m}_{4\nu_\tau}^{\mathrm{eff}} \leq 0.60\text{eV}$$

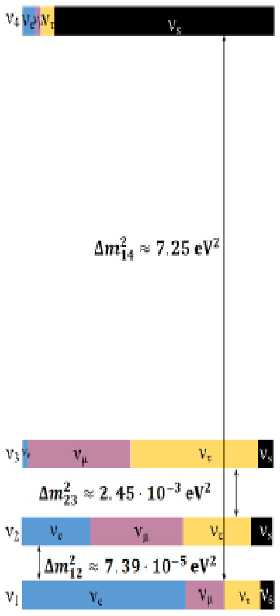


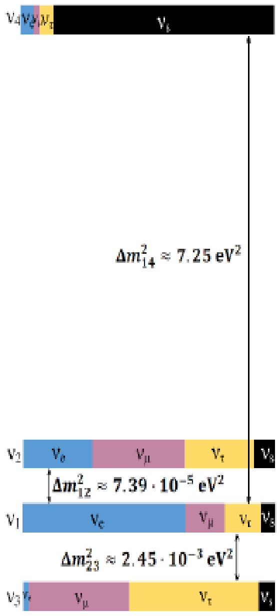
Neutrino flavors mixing scheme including sterile neutrino and effective mass hierarchy

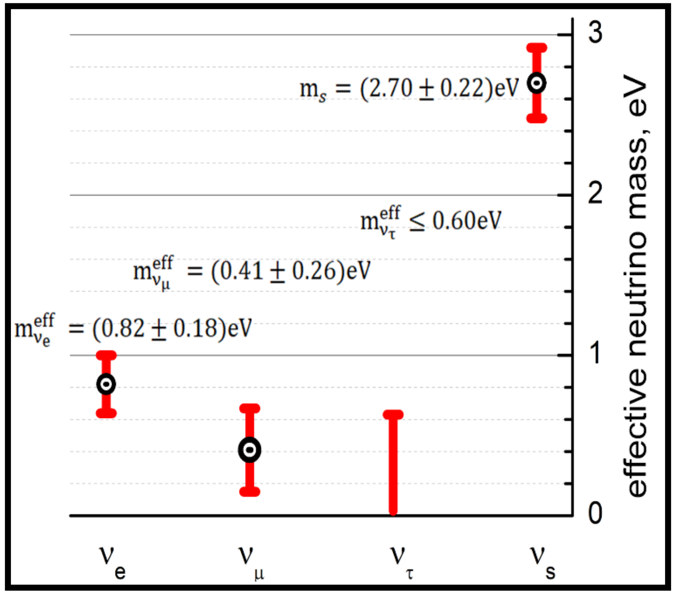
A.P. Serebrov *, R.M. Samoilov JETP Letters, Volume 112, 2020 Issue 4, pp 211–225 arxiv:2003.03199

PMNS matrix for 3 + 1 model

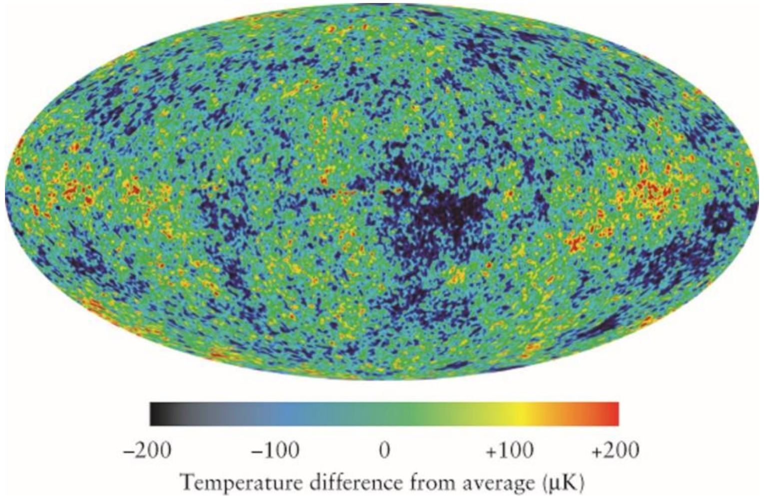
$$U_{PMNS}^{(3+1)} = \begin{pmatrix} 0.824^{+0.007}_{-0.008} & 0.547^{+0.011}_{-0.011} & 0.147^{+0.003}_{-0.003} & 0.306^{+0.041}_{-0.041} \\ 0.409^{+0.036}_{-0.060} & 0.634^{+0.022}_{-0.065} & 0.657^{+0.044}_{-0.014} & 0.15^{+0.08}_{-0.06} \\ 0.392^{+0.025}_{-0.048} & 0.547^{+0.056}_{-0.028} & 0.740^{+0.012}_{-0.048} & < 0.222 \\ < 0.24 & < 0.30 & < 0.26 & > 0.91 \end{pmatrix}$$

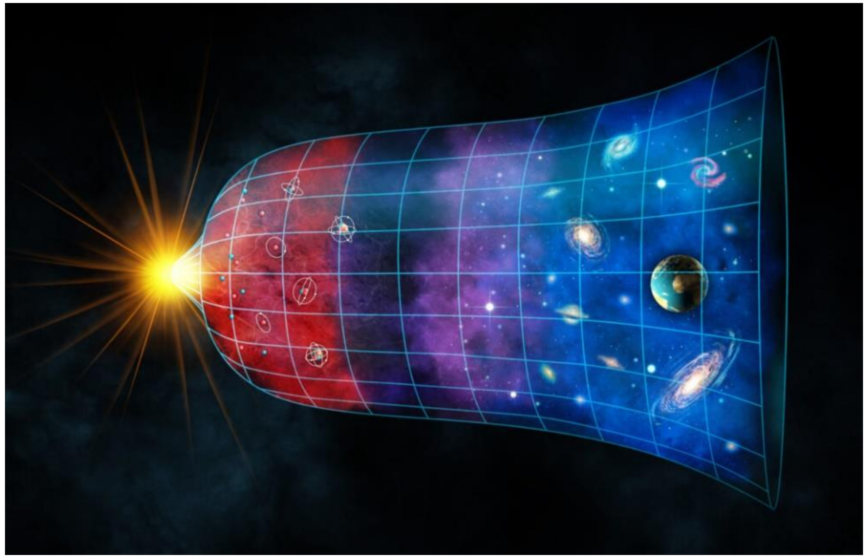






Cosmology – the role of sterile neutrinos during the formation of Universe and the pace of its expansion

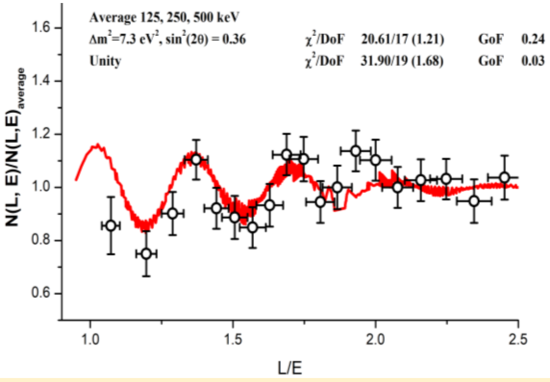




Neutrinos of different masses can affect the early Universe or be responsible for the baryon asymmetry of the Universe and Dark Matter. However, a sterile neutrino with a low mass and a small mixing angle can exist without any significant influence on the formation of the structure of the Universe. **Due to the superweak interaction, such sterile neutrinos do not thermalize in the primary plasma and leave it at an early stage.** Moreover, leaving the primary plasma at an early stage, they reduce its gravitational mass, which accelerates the expansion of the Universe.

Conclusion

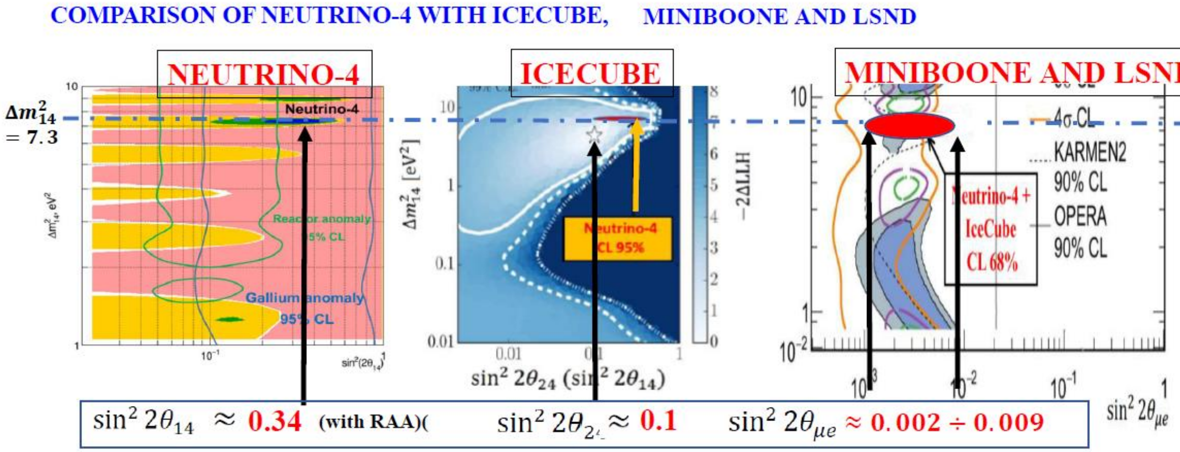
1. The direct observation of sterile neutrino oscillation in Neutrino-4 experiment



$$\Delta m_{14}^2 = 7.30 \pm 0.13_{st} \pm 1.16_{syst}$$

$$\sin^2 2\theta = 0.36 \pm 0.12_{stat} (2.9\sigma)$$

2. Possibility of experimental confirmation of the 3 + 1 neutrino model with one sterile neutrino



$$\sin^2 2\theta_{\mu e} \approx \frac{1}{4} \sin^2 2\theta_{14} \sin^2 2\theta_{24}$$

Test of neutrino model 3+1

3. COMPARISON OF NEUTRINO-4 MASS PREDICTION WITH MEASUREMENT OF NEUTRINO MASS

COMPARISON OF NEUTRINO-4 MASS PREDICTION WITH MEASUREMENT OF NEUTRINO MASS

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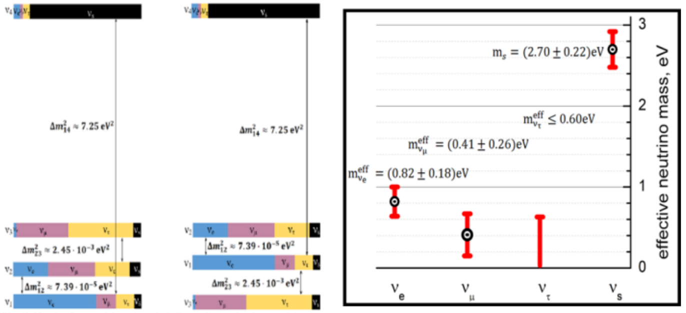
	Neutrino-4	KATRIN	GERDA $m(0\nu\beta\beta)$
$m_{\nu_e}^{eff} = \sqrt{\sum m_i^2 \lvert U_{ei}\rvert^2}$ $\Delta m_{14}^2 \approx m_4^2$ Effective mass and mass squared: $m_{\nu_e}^{eff}$, m_ν^2	$m_{4\nu_e}^{eff} = 0.82 \pm 0.18$ $\left(m_{4\nu_e}^{eff}\right)^2 = 0.68 \pm 0.29$	$m_{3\nu_e}^{eff} < 0.8$ eV (90%) $m_{3\nu_e}^{eff\,2} = 0.26 \pm 0.34$ $m_{4\nu_e}^{eff\,2} = ?$	
Maiorana mass $m(0\nu\beta\beta)$	$m(0\nu\beta\beta) = (0.25 \pm 0.09)$eV .		$m_{\beta\beta} < [0.080 - 0.182]$eV

4. Neutrino flavors mixing scheme including sterile neutrino and effective mass hierarchy



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$$U_{PMNS}^{(3+1)} = \begin{pmatrix} 0.824^{+0.007}_{-0.008} & 0.547^{+0.011}_{-0.011} & 0.147^{+0.003}_{-0.003} & 0.306^{+0.041}_{-0.041} \\ 0.409^{+0.036}_{-0.060} & 0.634^{+0.022}_{-0.065} & 0.657^{+0.044}_{-0.014} & 0.15^{+0.08}_{-0.06} \\ 0.392^{+0.025}_{-0.048} & 0.547^{+0.056}_{-0.028} & 0.740^{+0.012}_{-0.048} & < 0.222 \\ < 0.24 & < 0.30 & < 0.26 & > 0.91 \end{pmatrix}$$

Thus, the analysis performed provides quite interesting generalizations and indication of the possibility of the validity of the 3 + 1 neutrino model with one sterile neutrino.