Baryon and Meson Excited States

L. David Roper[+)] *& Igor Strakovsky*[*)]

[+)] *Prof. Emeritus of Physics, Virginia Polytechnic Institute and State University*
[*)] *The George Washington University*

We would like to dedicate our talk to memory of our friend ***Dick Arndt****.*
We lost ***Dick*** *several years ago…*

L. David Roper & IIS,
arXiv: 2410.11196 [hep-ph]

Supported by DE-SC0016583

Outline

- *A bit of History for Roper*
- *Noble Eightfold Path of Buddhism*
- *PDG & Missing States*
- *Gell-Mann-Okubo Formula*
- *Chew-Frautschi Plot*
- *'t Hooft Model for Mesons*
- *Roper Formula*
- *Potential Energy Approximation*
- *LHCb Pentaquarks*
- *Kaon-Pion Spectroscopy*
- *Where are We Now & …*
- *Tribute to Dick Arndt*

A Bit of History for Roper

$\mathcal{N}(1440)$ $\mathcal{P}_{11}$ Discovery

- **60** yrs ago *the first excited state* of *proton/neutron* was discovered by Dave Roper of his *PhD* work @ MIT & LLNL .

- N(1440) was born in 1963 (M = 1485 MeV)

  B.T. Feld and L.D. Roper, *Proc of the Siena Intern Conf on Elem Part* (Italian Phys Soc, Bologna, Italy, **1963**), p. 400

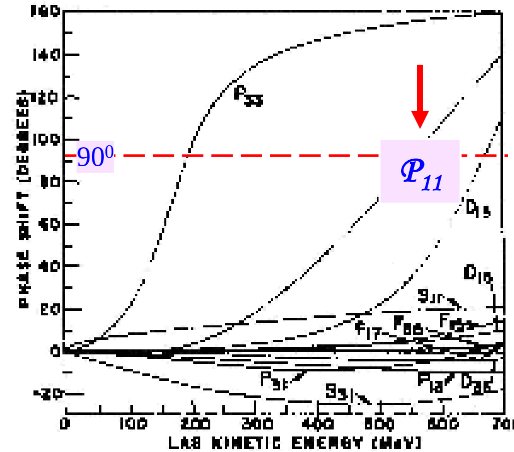
- First official report is

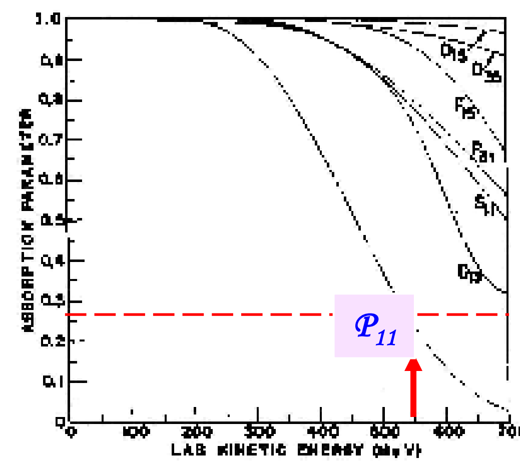
VOLUME 12, NUMBER 12 — PHYSICAL REVIEW LETTERS — 23 MARCH 1964

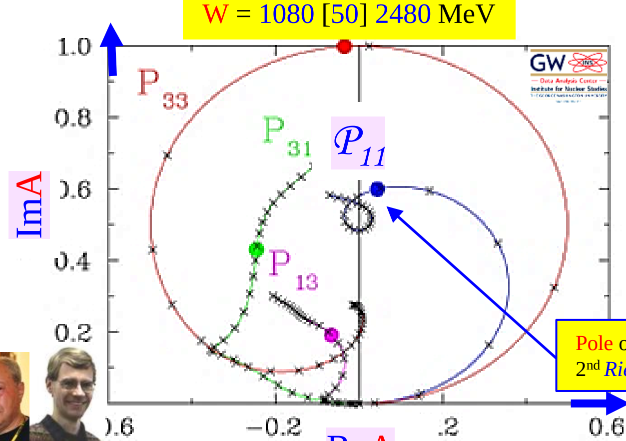
EVIDENCE FOR A P_{11} PION-NUCLEON RESONANCE AT 556 MeV

L. David Roper

- More bio details for Roper are in http://roperld.com/science/roperres.htm







R.A. Arndt *et al*, Phys Rev C **74**, 045205 (**2006**)

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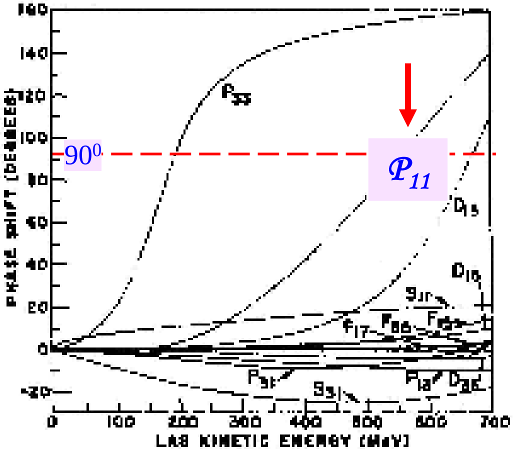
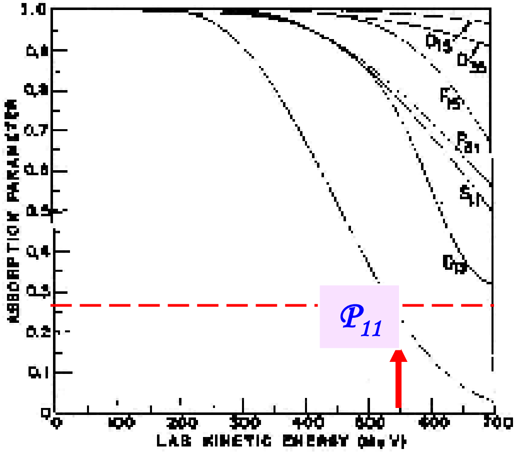
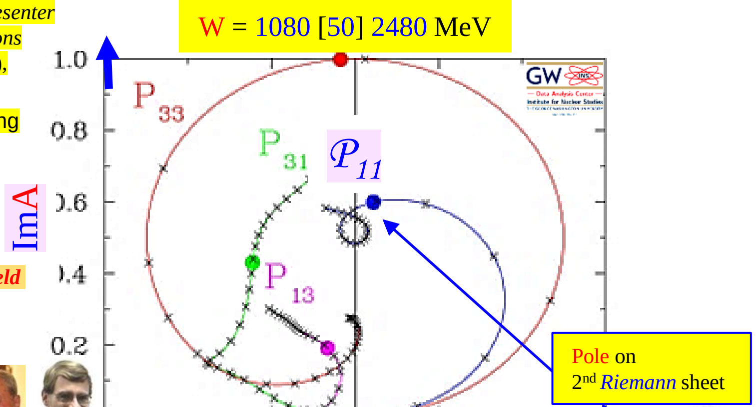




J.-R. Argand, *Essai Sur Une Maniere de Representer les Quantites Imaginaires Dans les Constructions Geometriques* (Sans nom d'auteur, Paris, **1806**), Vol. **I,** 78.

Caspar Wessel invented geometric way of representing complex numbers which pre-dated Argand

Dave Roper: *@ VTech seminar, MIT Prof.* ***Bernard T. Feld*** *talking about it in the early* ***1960s***



R.A. Arndt *et al*, Phys Rev C **74**, 045205 (**2006**)

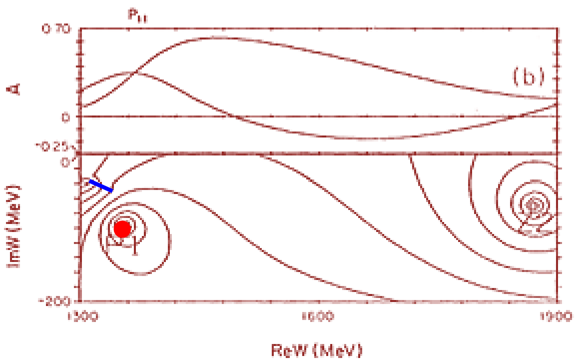
Two Pole Observation for N(1440) P_{11}

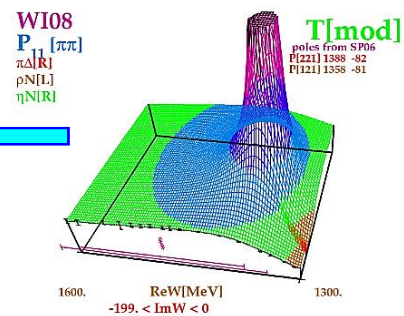
PHYSICAL REVIEW D VOLUME 32, NUMBER 5 1 SEPTEMBER 1985

Pion-nucleon partial-wave analysis to 1100 MeV

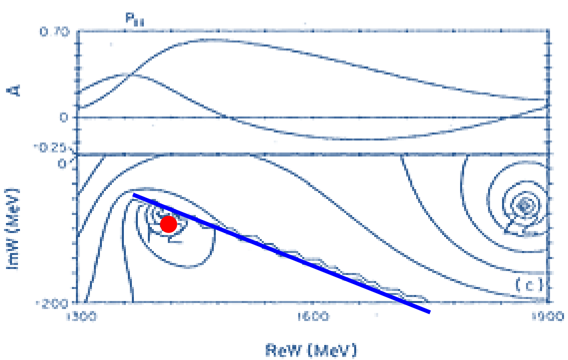
Richard A. Arndt, John M. Ford,* and L. David Roper
Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia

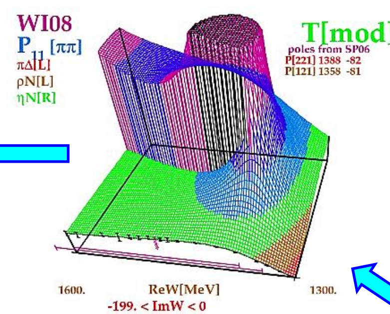
(Received 24 January 1985)





Pole 1: W = 1359–i100 MeV
Pole 2: W = 1410– i80 MeV





- Juelich Model:

M. Döring *et al* Nucl Phys A **829**, 170 (2009)

Pole 1: W = 1387–i73 MeV
Pole 2: W = 1387–i71 MeV

- EBAC Model:

N. Suzuki *et al Phys Rev Lett* **104**, 042302 (2010)

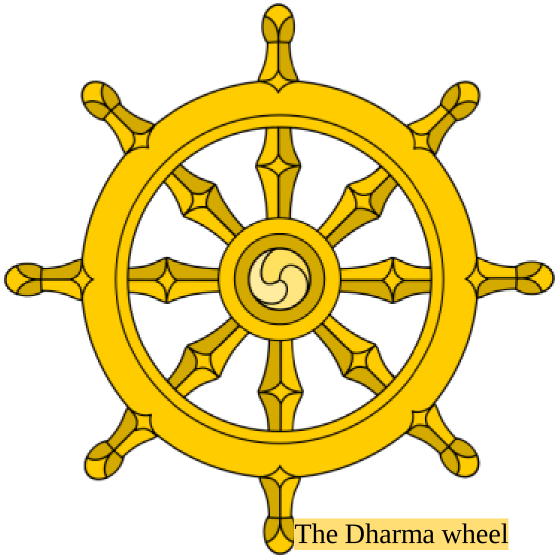
Pole 1: W = 1357– i76 MeV
Pole 2: W = 1364–i105 MeV

Sheet 1 is sheet reached most directly real axis
Sheet 2 is behind *πΔ* Branch Cut

R.E. Cutkosky & S. Wang, Phys Rev D **42**, 235 (1990)

- BW mass is *Real part* of Pole position on *second Riemann* sheet.

Noble Eightfold Path of Buddhism



The Dharma wheel

- QCD gives rise to *hadron spectrum*.

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS

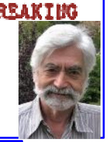
M. GELL-MANN

If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way"

Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc.

CERN-TH-412 (Feb 21, 1964)

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING



G. Zweig *)

CERN – Geneva

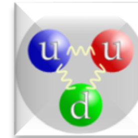
Both mesons and baryons are constructed from a set of three fundamental particles called aces. The aces break up into an isospin doublet and singlet. Each ace carries baryon number $\frac{1}{3}$ and is consequently fractionally charged. SU_3 (but not the Eightfold Way) is adopted as a higher symmetry for the strong interactions. The breaking of this symmetry is assumed to be universal, being due to mass differences among the aces. Extensive space-time and group theoretic structure is then predicted for both mesons and baryons, in agreement with existing experimental information. An experimental search for the aces is suggested.

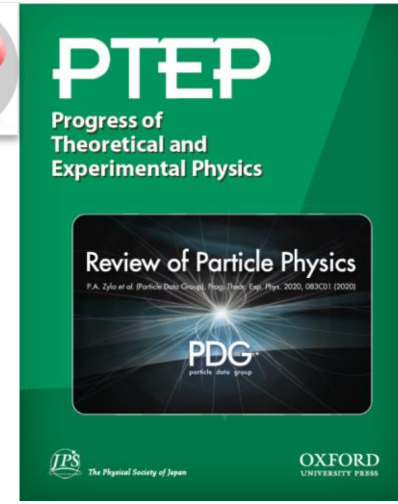
- Many $\bar{q}q$ & *qqq* states have been *observed*.

PDG 220 & 100.

Particle Data Group &
Missing States

Baryon Sector @ PDG2025





GW — Data Analysis Center — Institute for Nuclear Studies, The George Washington University

GW Contribution

S. Navas *et al*, Phys Rev D **110**, 030001 (**2024**)

N	J^P	Status
p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1685)$		*
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	***
$N(17$[illegible]$)$	$3/2^+$	[illegible]
$N(18$[illegible]$)$	$5/2^+$	[illegible]
$N(18$[illegible]$)$	$3/2^-$	**
$N(1$[illegible]$0)$	[illegible]	**
$N($[illegible]$5)$	[illegible]	**
$N($[illegible]$00)$	[illegible]	***
$N($[illegible]$90)$	[illegible]	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	**
$N(2100)$	$1/2^+$	*
$N(2120)$	$3/2^-$	**
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

Δ	J^P	Status
$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	***
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	**
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	[illegible]
$\Delta(1920)$	[illegible]	***
$\Delta(1930)$	[illegible]	[illegible]
$\Delta(1940)$	$3/2^-$	[illegible]
$\Delta(1950)$	$7/2^+$	[illegible]
$\Delta(20$[illegible]$0)$	$5/2^+$	[illegible]
[illegible]	[illegible]	[illegible]
$\Delta(2200)$	$7/2^-$	*
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**

Λ	J^P	Status
Λ	$1/2^+$	****
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	***
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	[illegible]	***
$\Lambda(1820)$	[illegible]	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(200$[illegible]$)$		
[illegible]	$7/2^+$	
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		**

Σ	J^P	Status
Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	**
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1690)$		**
$\Sigma(1750)$	[illegible]	[illegible]
$\Sigma(1770)$	[illegible]	*
$\Sigma(1775)$	[illegible]	****
$\Sigma(1840)$	[illegible]	*
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(191$[illegible]$)$	$5/2^+$	[illegible]
[illegible]	[illegible]	***
$\Sigma(2000)$	$1/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	**
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ, Ω	J^P	Status
Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(153$[illegible]$)$	[illegible]	[illegible]
$\Xi(162$[illegible]$)$		*
$\Xi(1690)$		***
$\Xi(182$[illegible]$)$	[illegible]	***
$\Xi(19$[illegible]$)$		***
$\Xi(2030)$	$\geq$ [illegible]	***
[illegible]		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	[illegible]
$\Omega(2$[illegible]$)^-$		[illegible]
$\Omega($[illegible]$0)^-$		[illegible]
$\Omega($[illegible]$0)^-$		[illegible]

Charm / Bottom	J^P	Status
Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$		***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	***
$\Xi_c'^+$	$1/2^+$	***
$\Xi_c'^0$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2930)$		*
$\Xi_c(2980)$		***
$\Xi_c(3055)$		**
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
Ξ_{cc}^+		*
Λ_b^0	$1/2^+$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
Ξ_b^0, Ξ_b^-	$1/2^+$	***
Ω_b^-	$1/2^+$	***

- First *hyperon* was discovered in **1950**.

THE UNIVERSITY OF MELBOURNE

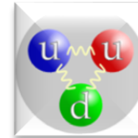
V.D. Hopper & S. Biswas, Phys Rev **80**, 1099 (**1950**)

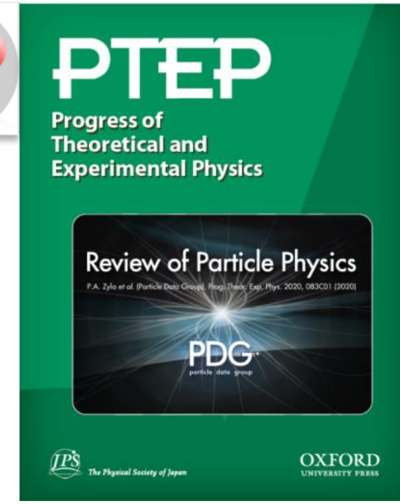
- **Pole** position in complex energy plane for *hyperons* has made *only* in **2010**.

Jefferson Lab

Y. Qung *et al*, Phys Lett B **694**, 123 (**2010**)

Baryon Sector @ PDG2025





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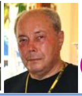

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$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1685)$		*
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	***
$N(172$[illegible])	$3/2^+$	[illegible]
[illegible]	$5/2^+$	[illegible]
[illegible]	$3/2^-$	[illegible]
[illegible]	[illegible]	**
[illegible]	[illegible]	**
[illegible]	[illegible]	***
[illegible]	[illegible]	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	**
$N(2100)$	$1/2^+$	*
$N(2120)$	$3/2^-$	**
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	***
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	**
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	[illegible]	***
$\Delta(1930)$	[illegible]	***
$\Delta(1940)$	$3/2^-$	[illegible]
$\Delta(1950)$	$7/2^+$	****
[illegible]	$5/2^+$	[illegible]
[illegible]	[illegible]	*
$\Delta(2200)$	$7/2^-$	*
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	***
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	[illegible]	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
[illegible]	[illegible]	[illegible]
[illegible]	$7/2^+$	[illegible]
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		**

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	**
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1690)$		**
$\Sigma(1750)$	[illegible]	[illegible]
$\Sigma(1770)$	[illegible]	*
$\Sigma(1775)$	[illegible]	****
$\Sigma(1840)$	[illegible]	*
$\Sigma(1880)$	$1/2^+$	**
[illegible]	$5/2^+$	[illegible]
[illegible]	[illegible]	***
$\Sigma(2000)$	$1/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	**
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
[illegible]	[illegible]	[illegible]
[illegible]		*
$\Xi(1690)$		***
[illegible]	[illegible]	***
[illegible]		***
$\Xi(2030)$	[illegible]	***
[illegible]		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	[illegible]
[illegible]		[illegible]
[illegible]		[illegible]
[illegible]		[illegible]

Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$		***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	***
$\Xi_c^{\prime +}$	$1/2^+$	***
$\Xi_c^{\prime 0}$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2930)$		*
$\Xi_c(2980)$		***
$\Xi_c(3055)$		**
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
Ξ_{cc}^+		*
Λ_b^0	$1/2^+$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
Ξ_b^0, Ξ_b^-	$1/2^+$	***
Ω_b^-	$1/2^+$	***

- First *hyperon* was discovered in **1950**.

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V.D. Hopper & S. Biswas, Phys Rev **80**, 1099 (**1950**)

- Pole position in complex energy plane for *hyperons* has made *only* in **2010**.

Jefferson Lab

Y. Qung *et al*, Phys Lett B **694**, 123 (**2010**)

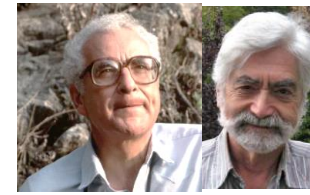
- PDG2024 has 133 Baryon Resonances (69 of them are 4* & 3*).
- In case of SU(6) x O(3), 434 states would be present if all revealed multiplets were fleshed out (*three* **70** & *four* **56**).



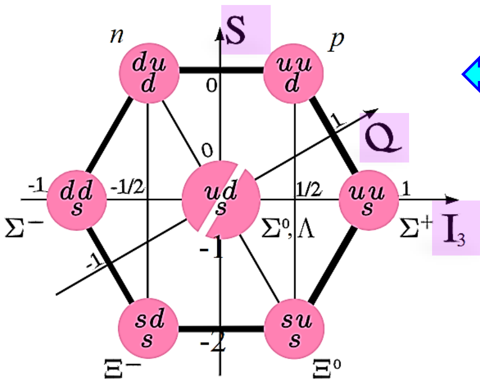
- LQCD results are similar.

R. Koniuk & N. Isgur, Phys Rev Lett **44**, 845 (**1980**)

Baryon Multiplets of Eight-fold Way

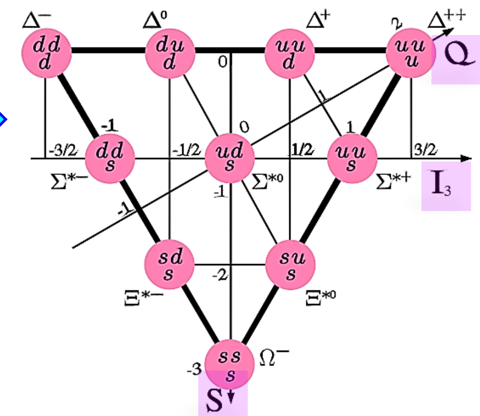


- Three light quarks can be arranged in 6 baryonic families, N*, Δ*, Λ*, Σ*, Ξ*, & Ω*.
- Number of members in family that can exist is not arbitrary.
- If $SU(3)_F$ symmetry of *QCD* is controlling, then:



Spin 1/2* baryon *octet*: *N**, Λ*, Σ*, Ξ

Spin 3/2* baryon *decuplet*: Δ*, Σ*, Ξ*, Ω



had spec

PDG

Resonance	LQCD	Observed
N*	62	36
Δ*	38	29
Λ*	71	23
Σ*	66	28
Ξ*	73	12
Ω*	36	5

R. G. Edwards *et al*, Phys Rev D **87**, 054506 (**2013**)

R. Koniuk & N. Isgur, Phys Rev Lett **44**, 845 (**1980**)

- Seriousness of "*missing-states*" problem is obvious from these numbers.
- One needs to *complete* $SU(3)_F$ multiplets.

B.M.K. Nefkens, πN Newsletter, **14**, 150 (**1997**)

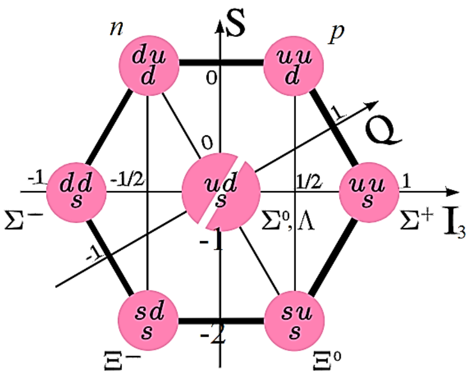
Gell-Mann-Okubo Formula

Gell-Mann–Okubo Mass Formula

M. Gell-Mann, Report CTSL-20, **1961**
S. Okubo, Prog Theor Phys **27**, 949 (**1962**); **28**, 24 (**1962**)


- **GMO** *mass formula* provides sum rule for masses of hadrons within specific *multiplet*, determined by their *isospin* (*I*) & *strangeness* or *hypercharge* (*Y*) generated by SU(3).

$$M = a_0 + a_1 Y + a_2 \left[I(I+1) - \frac{1}{4} Y^2 \right]$$



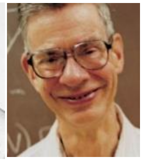
J.J. de Swart, Rev Mod Phys **35**, 916 (**1963**)

- "Equal-spacing" rule for mass shift for baryon **8** members is ~**147** MeV, here *sigma Σ term* playing role.
- **GMO** formula reproduces mass of **8** baryons within ~**0.5**% of determined values.
- Mixing be able to shift some masses for **GMO** mass formula.

Chew-Frautschi Plot

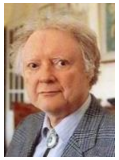
Chew-Frautschi Plot

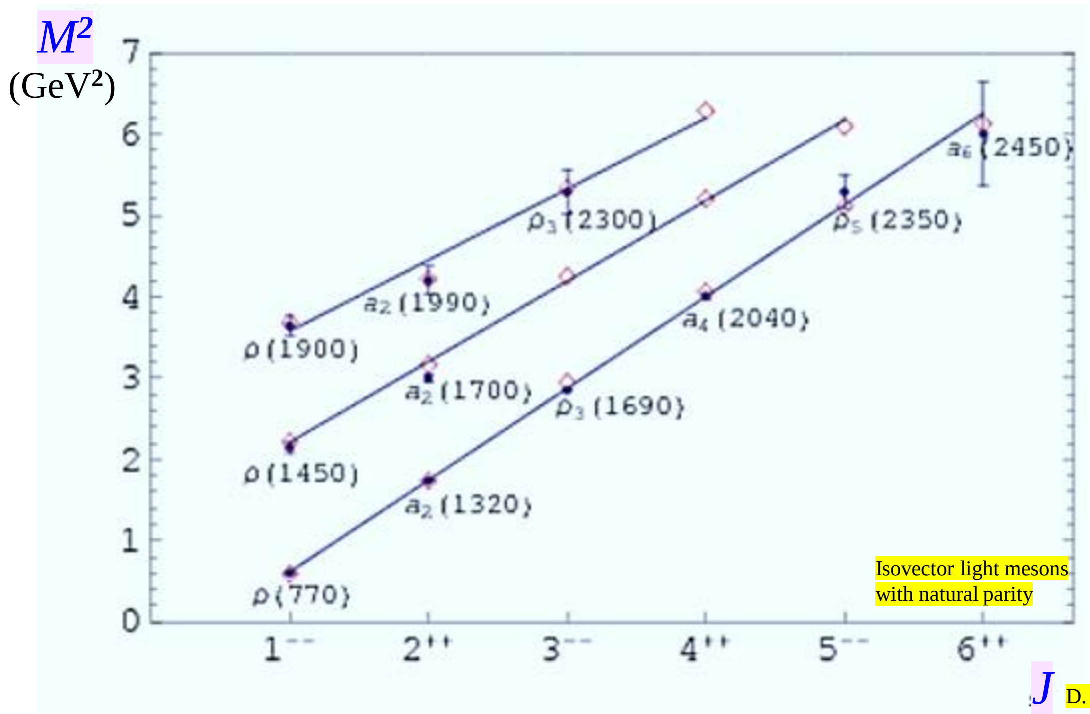
G.F. Chew & S.C. Frautschi, Phys Rev Lett, **8**, 41 (**1962**)

- Geoffrey Chew & Steven Frautschi, in 1961, proposed that mesons, when plotted with them *angular momentum*, against *squared of their masses*, will fall into *straight line trajectories*. These are called Regge trajectories.

T. Regge, Nuov Cim, **14**, 951 (**1959**), **18**, 947 (**1960**)

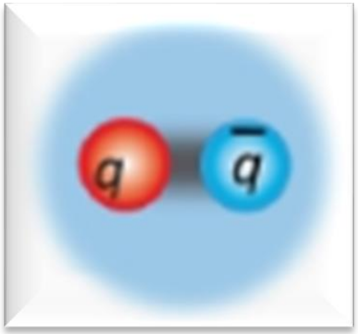




D. Ebert *et al*, Phys Rev D **79**, 114029 (**2009**)

- There are just several samples which prove *straight line* Regge *trajectories* but there is no guarantee that will work when ***J*** goes to *infinity*.

't Hooft's Model for Mesons



Two-Dimensional Model for Mesons

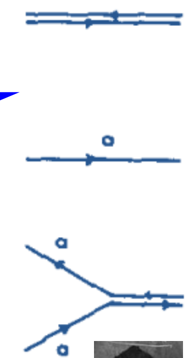
G. 't Hooft, Nucl Phys B **75** 461 (**1974**)

Nuclear Physics B75 (1974) 461–470.

A TWO-DIMENSIONAL MODEL FOR MESONS

G. 't HOOFT

CERN, Geneva



A recently proposed gauge theory for strong interactions, in which the set of planar diagrams play a dominant role, is considered in one space and one time dimension. In this case, the planar diagrams can be reduced to self-energy and ladder diagrams, and they can be summed. The gauge field interactions resemble those of the quantized dual string, and the physical mass spectrum consists of a nearly straight "Regge trajectory".

$$\mu^2_{(n)} \xrightarrow[n\to\infty]{} \pi^2 n + (\alpha_1 + \alpha_2)\log n + C^{st}(\alpha_1, \alpha_2), \qquad n = 0, 1, \ldots$$

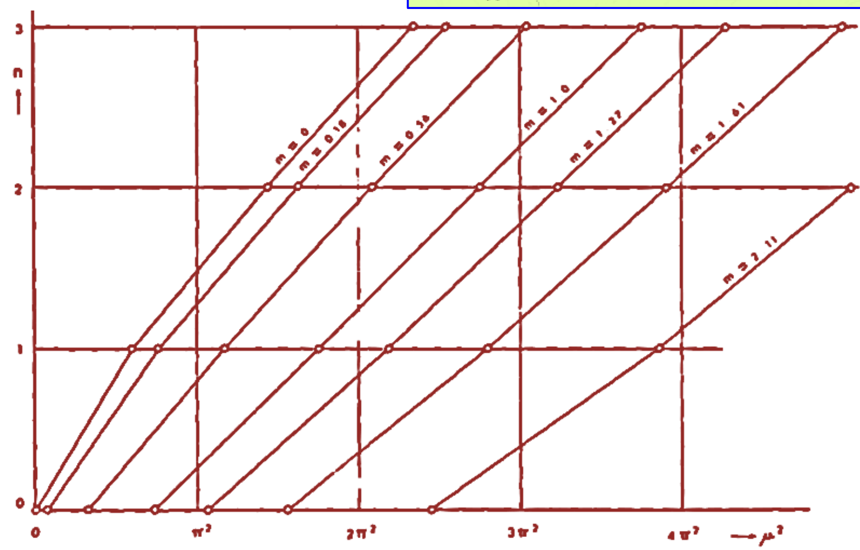


Fig. 5. "Regge trajectories" for mesons built from a quark-antiquark pair with equal mass, m, varying from 0 to 2.11 in units of $g/\sqrt{\pi}$. The squared mass of the bound states is in units g^2/π.

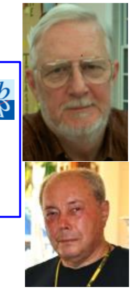
- In general, it is possible to **find/build** *potential* which will provide Ln(n) behavior or *radial excitation*.

- Problem is that the same *potential* should describe mass-spin dependence (*Regge trajectories*). For *Regge trajectory*, we need linear increasing V(r) = C*r but this will not give *Logarithm*.

Roper Formula

Two-Parameter Logarithm Function

L. David Roper & IIS, arXiv: 2410.11196 [hep-ph]

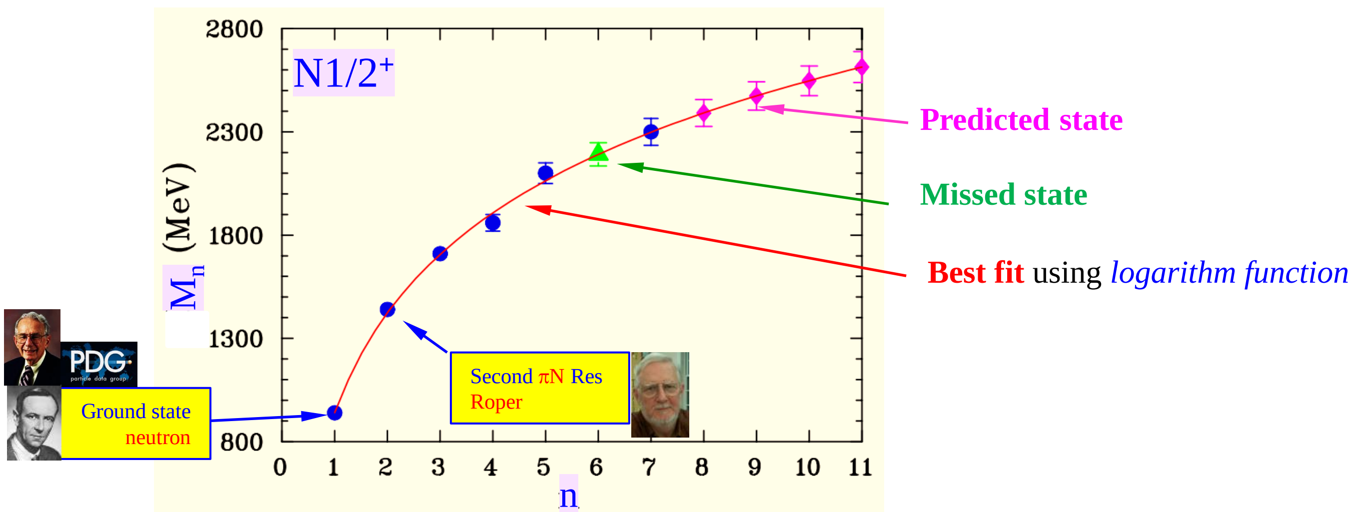


- Conjecture is made that accurately measured masses of all equal-quantum *baryon* (including exotic $P^{+}_{c\bar{c}}$s) & *meson* (including $s\bar{s}$, $c\bar{c}$, & $b\bar{b}$) excited states are related by *logarithm function* used here; at least for mass range of currently known excited states.

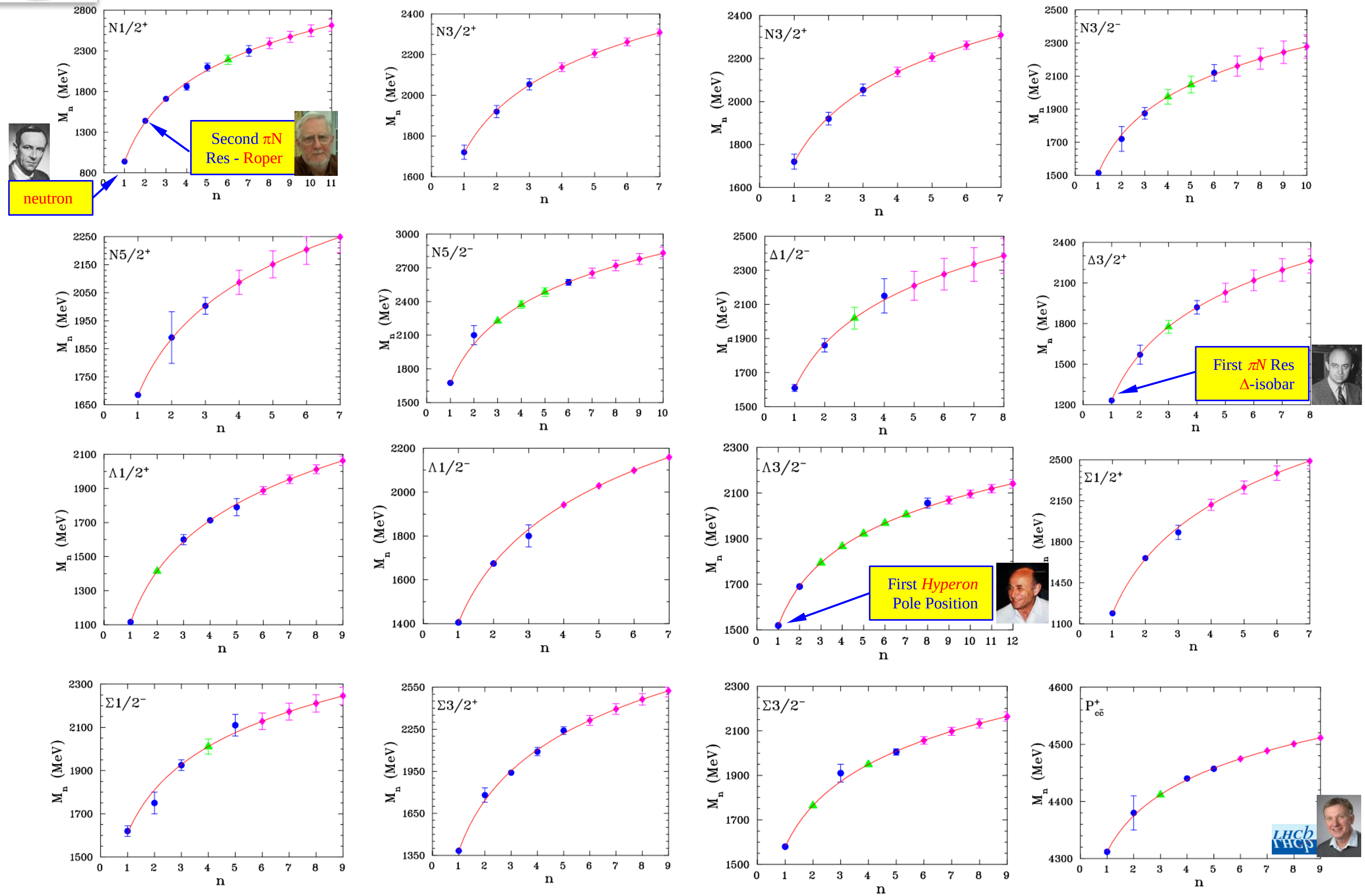
- *Logarithmic* fit to PDG BW masses of 3+ known excited states.

$$M_n = \alpha\ \mathbf{Ln(n)} + \beta$$

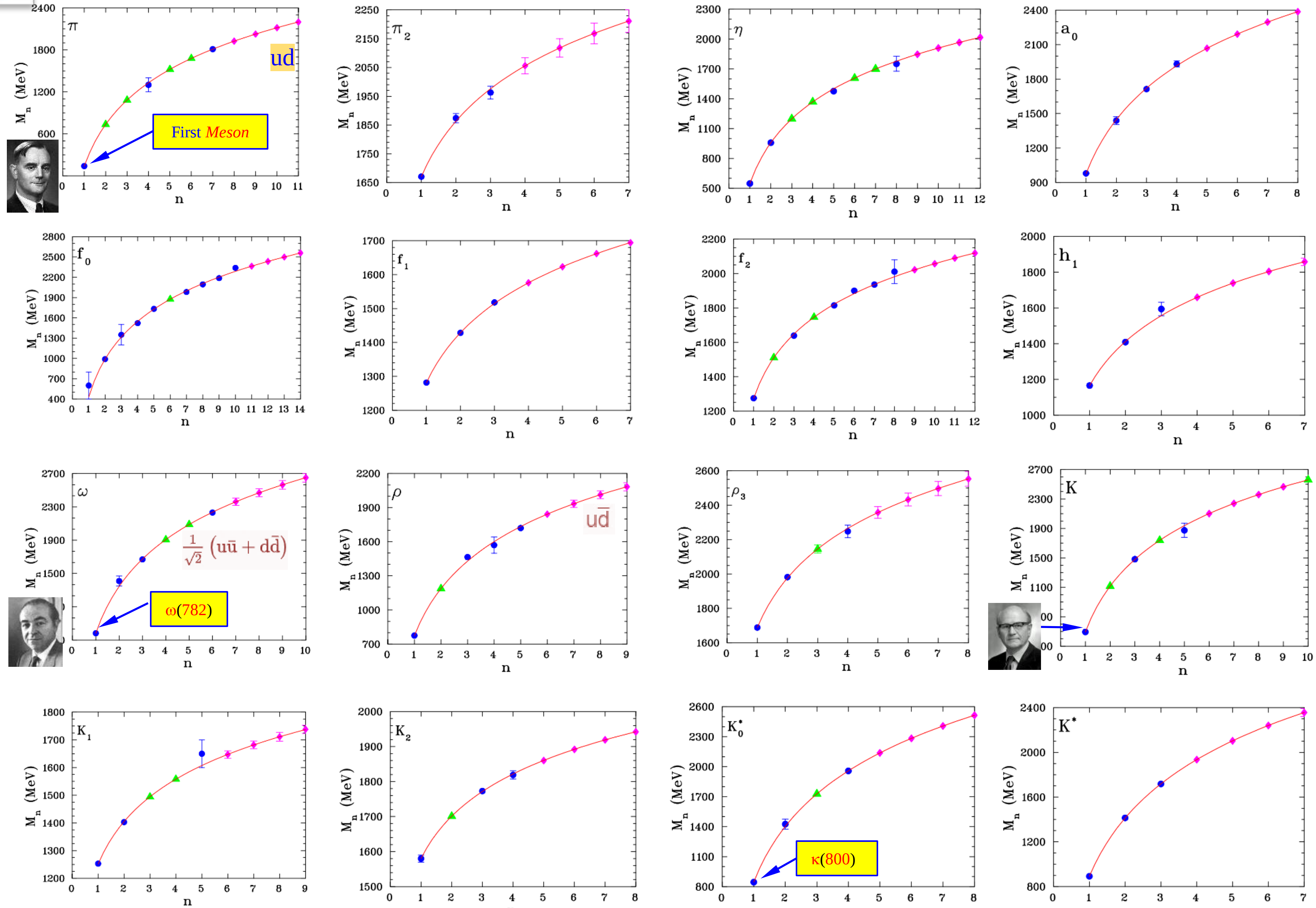
n is ***radial excitation level*** & **α** with **β** are *free parameters*.
Parameter **α** is *logarithmic slope.*
Parameter **β** is essentially *ground mass* in data set
[**β** = **M_1**, since Ln(**1**) = **0**].



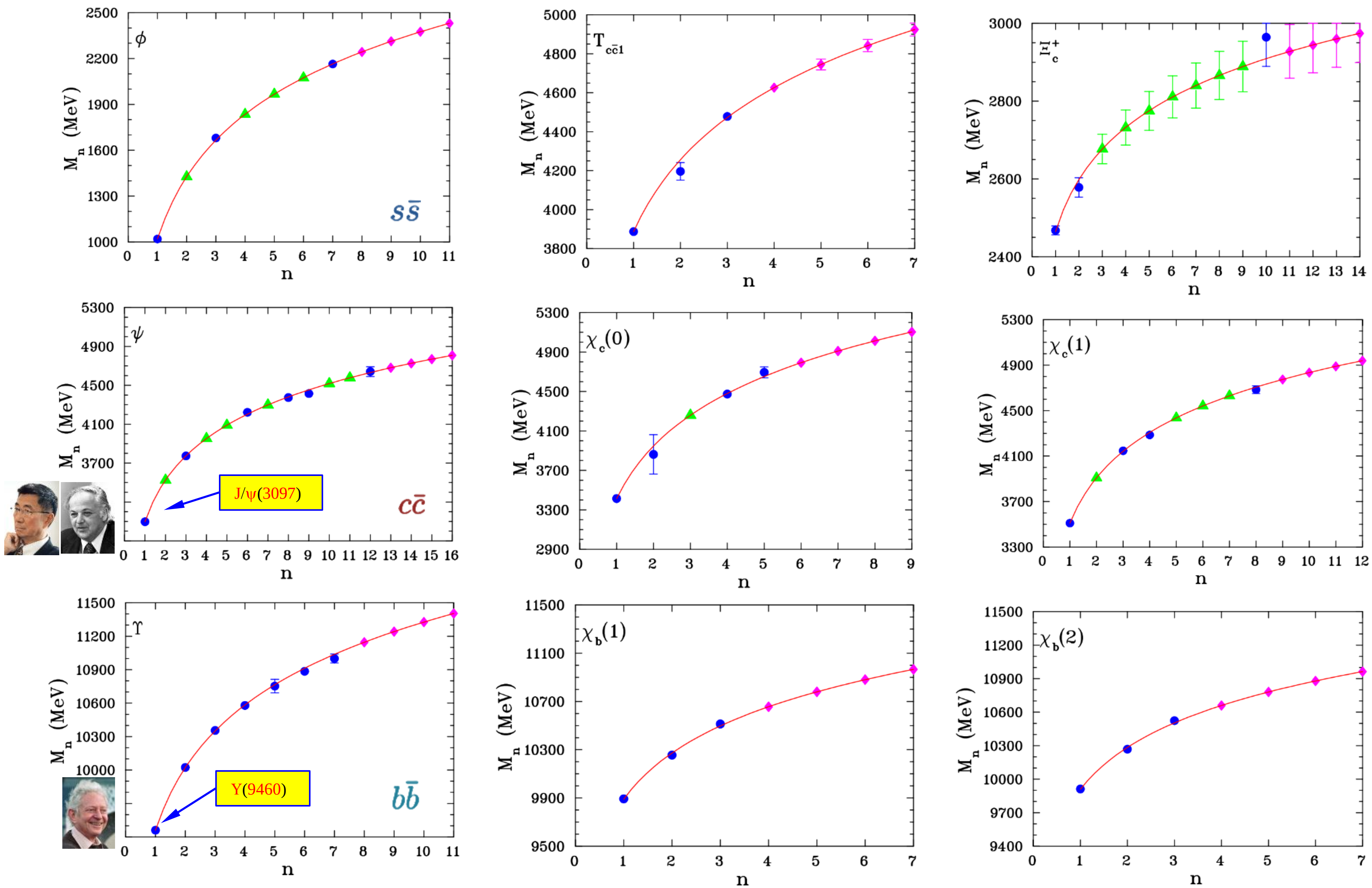
Logarithm Function for Baryons



Logarithm Function for Mesons - I



Logarithm Function for Mesons - II



Why Logarithm Function ?

$$M_n = \alpha \, \mathbf{Ln(n)} + \beta$$

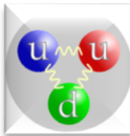
$$\frac{dM_n}{dn} = \frac{\alpha}{n}$$

$$M_n = \alpha \int \frac{1}{n} dn + \beta \cong \alpha \sum_{n=1}^{N} \frac{\delta n}{n} + \beta$$ for large **N**

- So, **M_n** can be approximated as *sum over large number* of **1/n** terms, such as in *energy levels*.
- Perhaps this is reason why *logarithm* function works so well in these mass fits.

Logarithm Function

- *Logarithm function* works very well for fitting masses for excited states of many other equal-quantum *baryon* & *meson* data sets with input n = 3+.

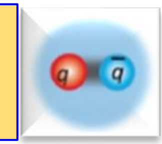


17 Data sets (60 states)
28 Missed states
68 Predicted states

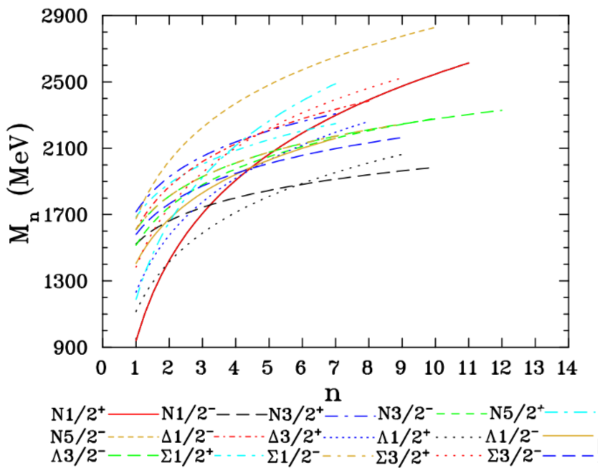
- Input PDG

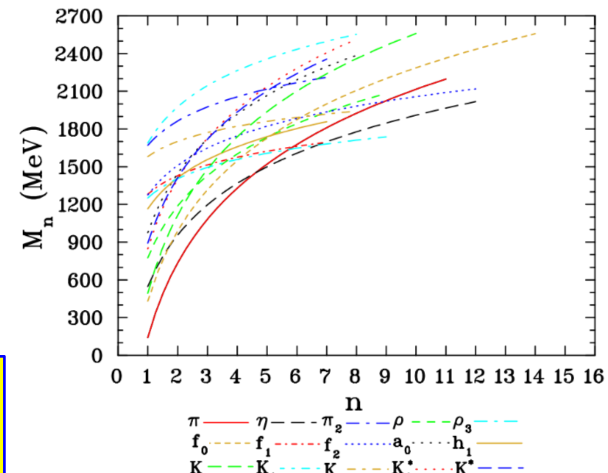
PDG Input -

24 Data sets (94 states)
36 Missed states
96 Predicted states

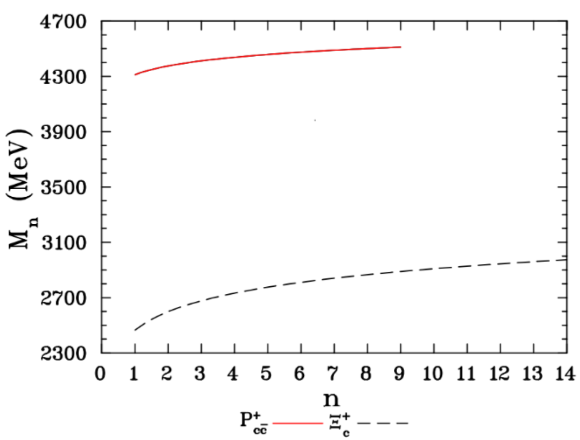


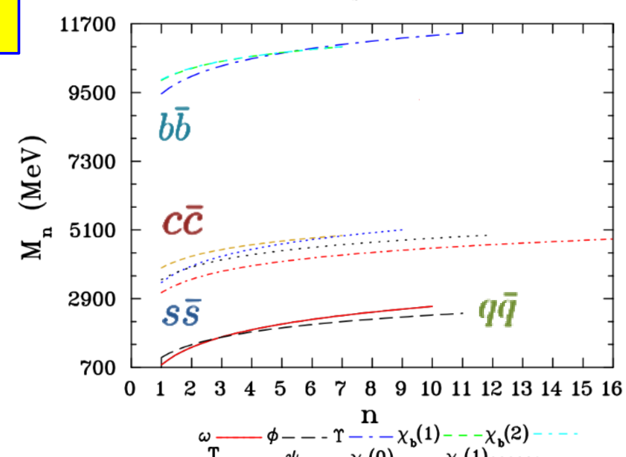
$$M_n = \alpha\, \mathbf{Ln(n)} + \beta$$



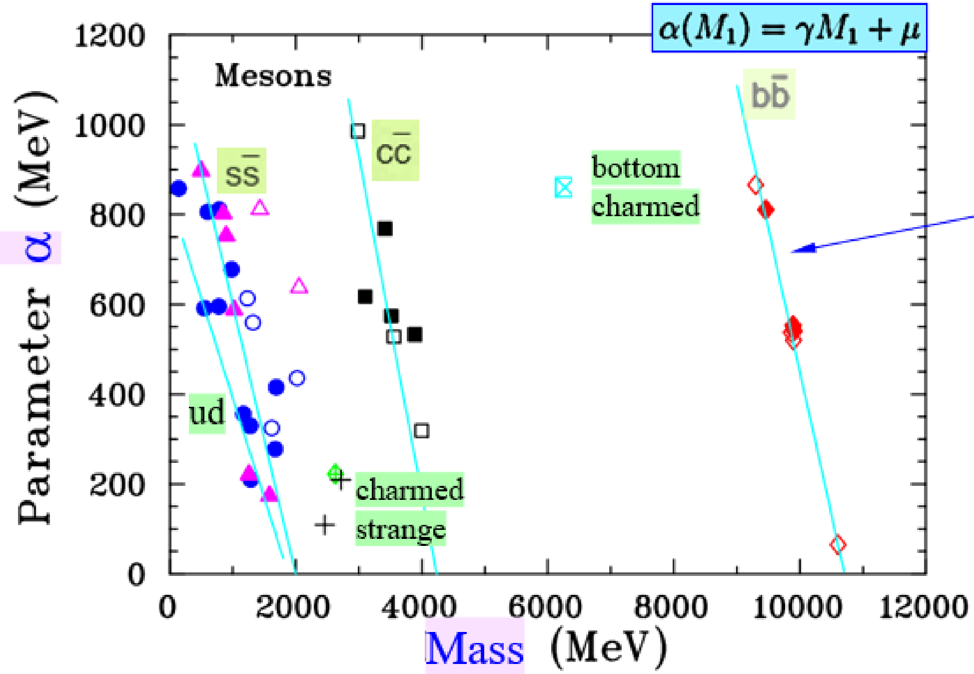


- *logarithmic slope*, α, usually, decreases as *ground-state* mass increases.



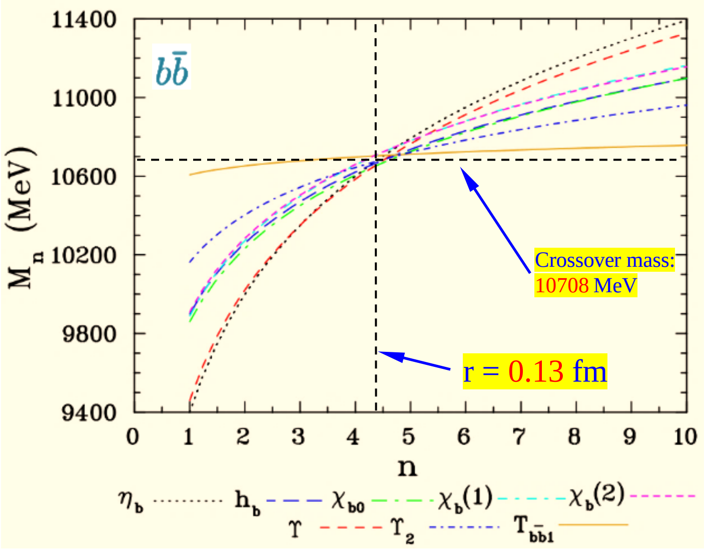


Logarithm Function for $b\bar{b}$



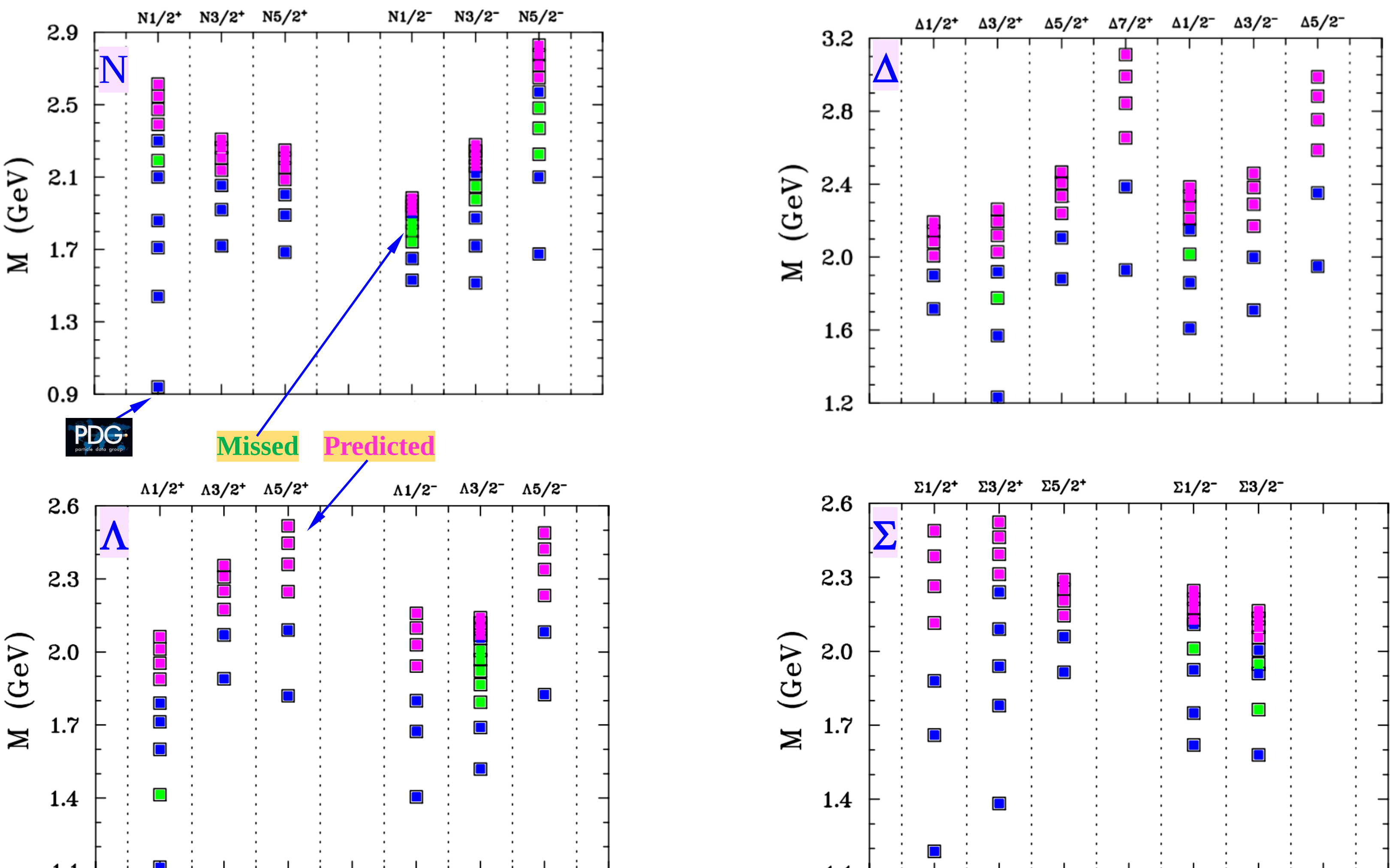
- State masses calculated for 7 $b\bar{b}$ sets differ @ most by 0.8%

Corresponds to mass of ground state M_1 or β



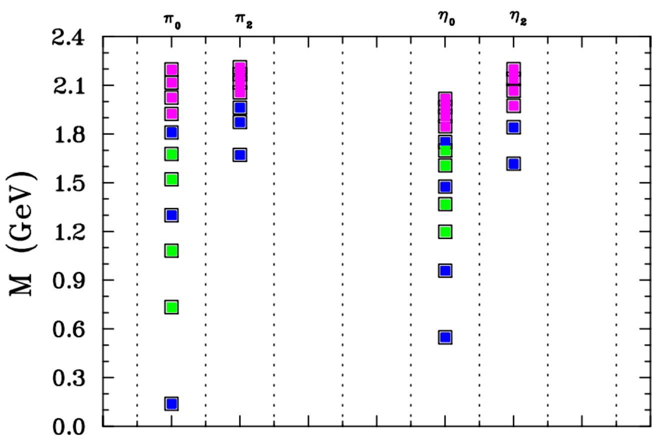
Baryon Spectra

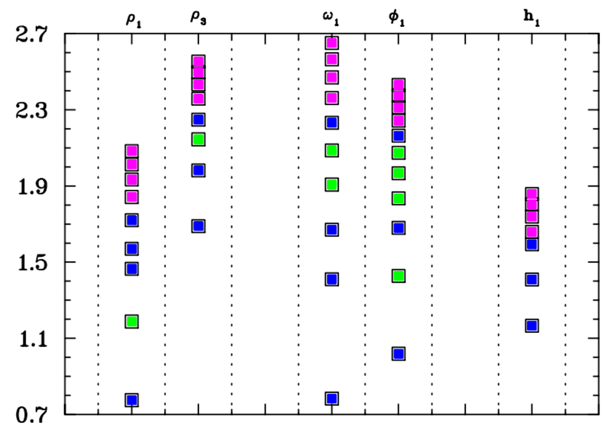
- Spectra of **N**, **Δ**, **Λ**, & **Σ** families of baryons for *spins* up to **5**/**2** & both *parities*.

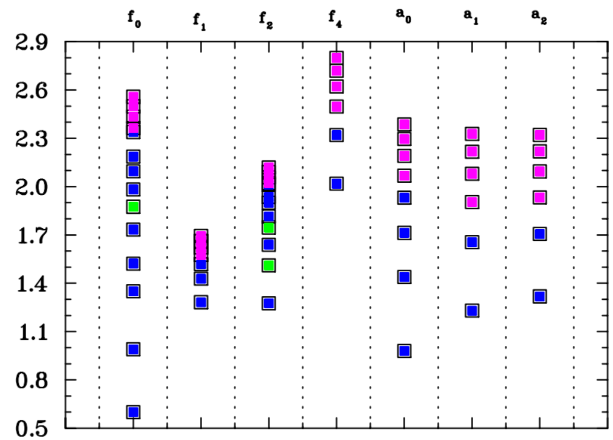


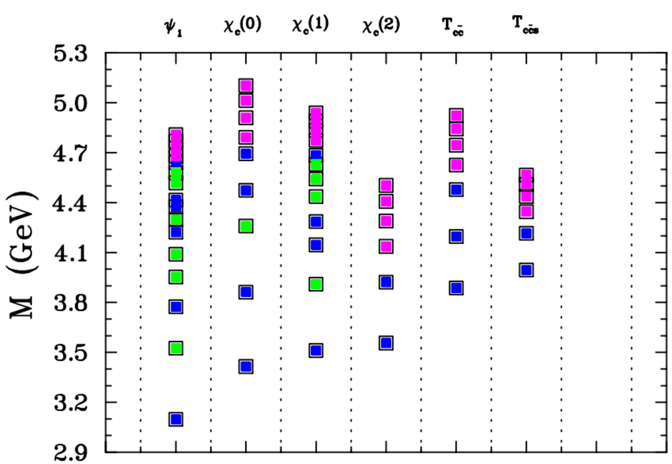
Meson Spectra

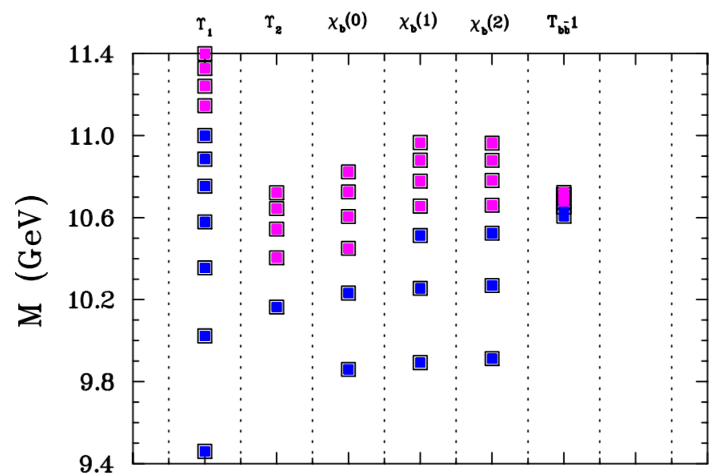
- Spectra of *π*, *η*, *ρ*, ω, *ϕ*, *f*, *a*, *etc.* families of mesons.



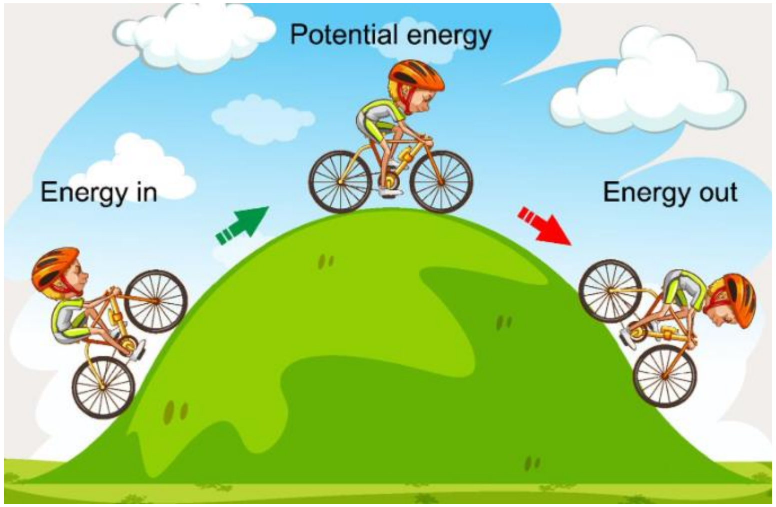








Potential Energy Approximation



Potential Energy Approximation

de Broglie wavelength $\lambda_n = \frac{hc}{M_n} = 2\pi\frac{\hbar c}{M_n}$

Radius of excited state $r_n = n\frac{\lambda_n}{4} = n\frac{\pi}{2}\frac{\hbar c}{M_n}$

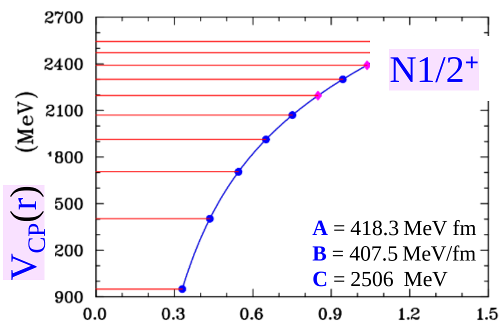
Cornell Potential $V_{CP} = -\frac{4}{3}A/r + Br + c$

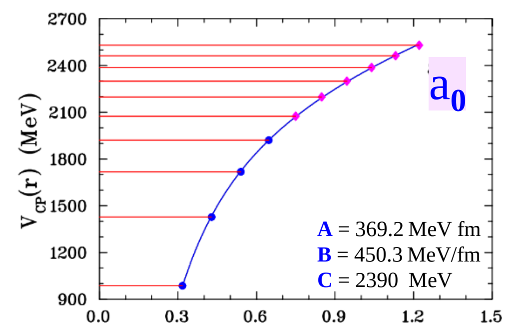
$V_{GCP} = -\frac{4}{3}A/r + Br^{D} + c$

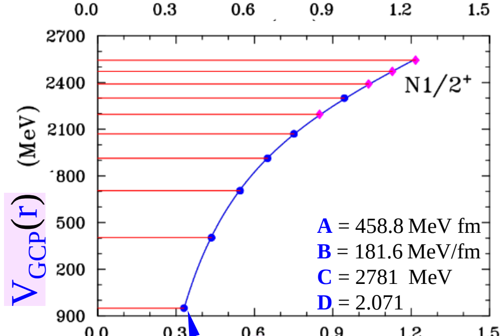
E. Eichten *et al* , Phys Rev Lett **34**, 369 (**1975**)
Phys Rev D **17**, 3090 (**1978**)
N. Brambilla & A. Vairo, [arXiv:hep-ph/9904330 [hep-ph]].

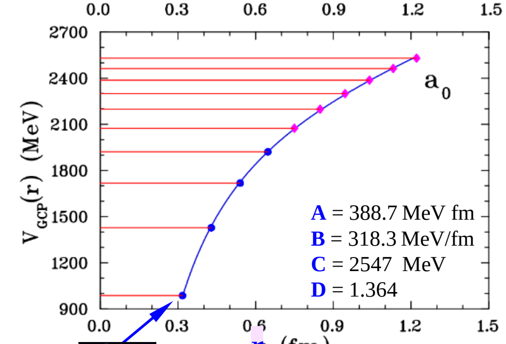
r is effective radius of resonance state,
A is QCD running coupling,
B is QCD string tension,
C & **D** are constant.

- Approximate *Potential* shape our data show & indicates that somehow that potential yields log behavior for *baryons* & *mesons* & for *light* & *heavy* quarks..
- $\mathbf{V_{GCP}}$ is always better fit than $\mathbf{V_{CP}}$.
- While $\mathbf{V_{CP}}$ gives *strict linear radial behavior* of is necessary to obtain *Regge* trajectories.

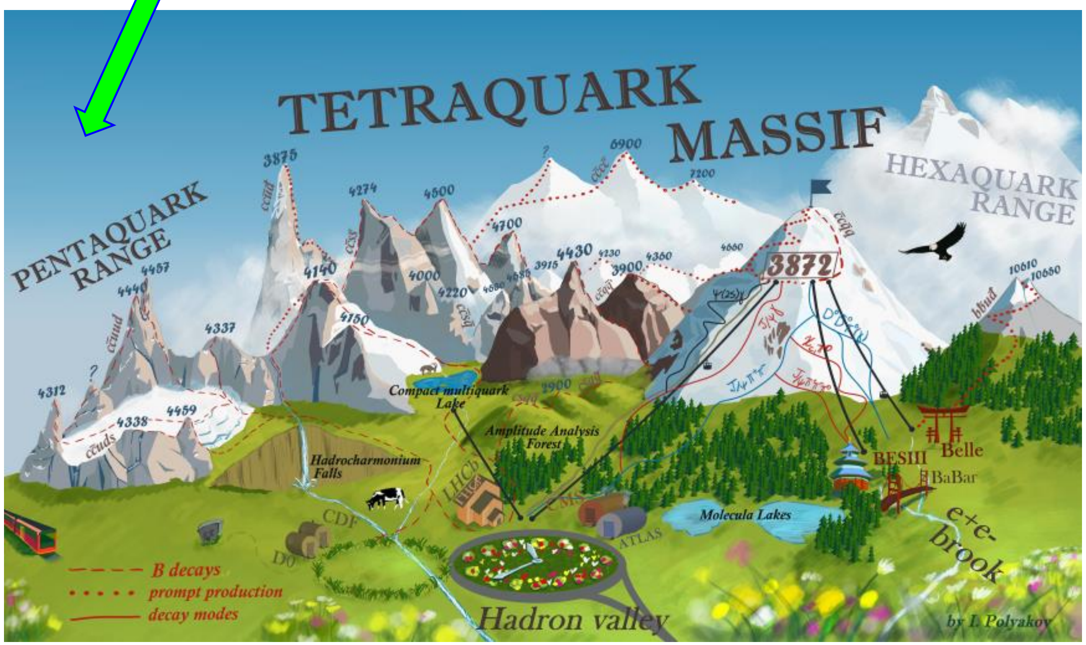








LHCb Pentaquarks



Nils Huesken *et al* arXiv:2410.06923 [hep-ph]

QCD & Hadron Spectrum

- QCD gives rise to *Hadron Spectrum*.

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

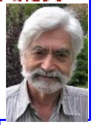
A SCHEMATIC MODEL OF BARYONS AND MESONS

M. GELL-MANN

If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" ... Baryons can now be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc.

CERN-TH-412 (Feb 21, 1964)

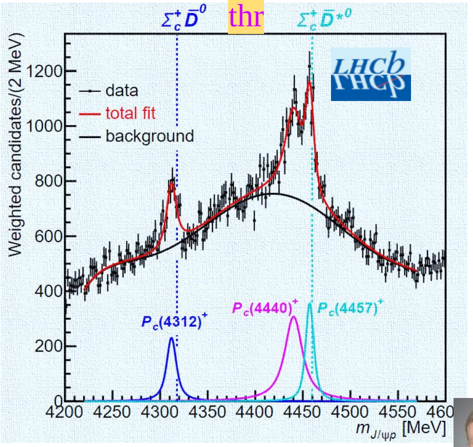
AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING



G. Zweig *)

CERN – Geneva

Both mesons and baryons are constructed from a set of three fundamental particles called aces. The aces break up into an isospin doublet and singlet. Each ace carries baryon number $\frac{1}{3}$ and is consequently fractionally charged. SU_3 (but not the Eightfold Way) is adopted as a higher symmetry for the strong interactions. The breaking of this symmetry is assumed to be universal, being due to mass differences among the aces. Extensive space-time and group theoretic structure is then predicted for both mesons and baryons, in agreement with existing experimental information. An experimental search for the aces is suggested.



R. Aaij *et al*, Phys Rev Lett **122**, 222001 (**2019**)
R. Aaij *et al*, Phys Rev Lett **115**, 072001 (**2015**)

- Many $\bar{q}q$ & qqq states have been *observed*.

PDG 220 & 100.

- $\bar{q}q\bar{q}q$, $qqq\bar{q}q$, … are *not forbidden* or *we do not know it yet*.

Narrow Pentaquarks from $\Lambda_b^0 \rightarrow P_{c\bar{c}}^+ K^- \rightarrow (J/\psi p) K^-$

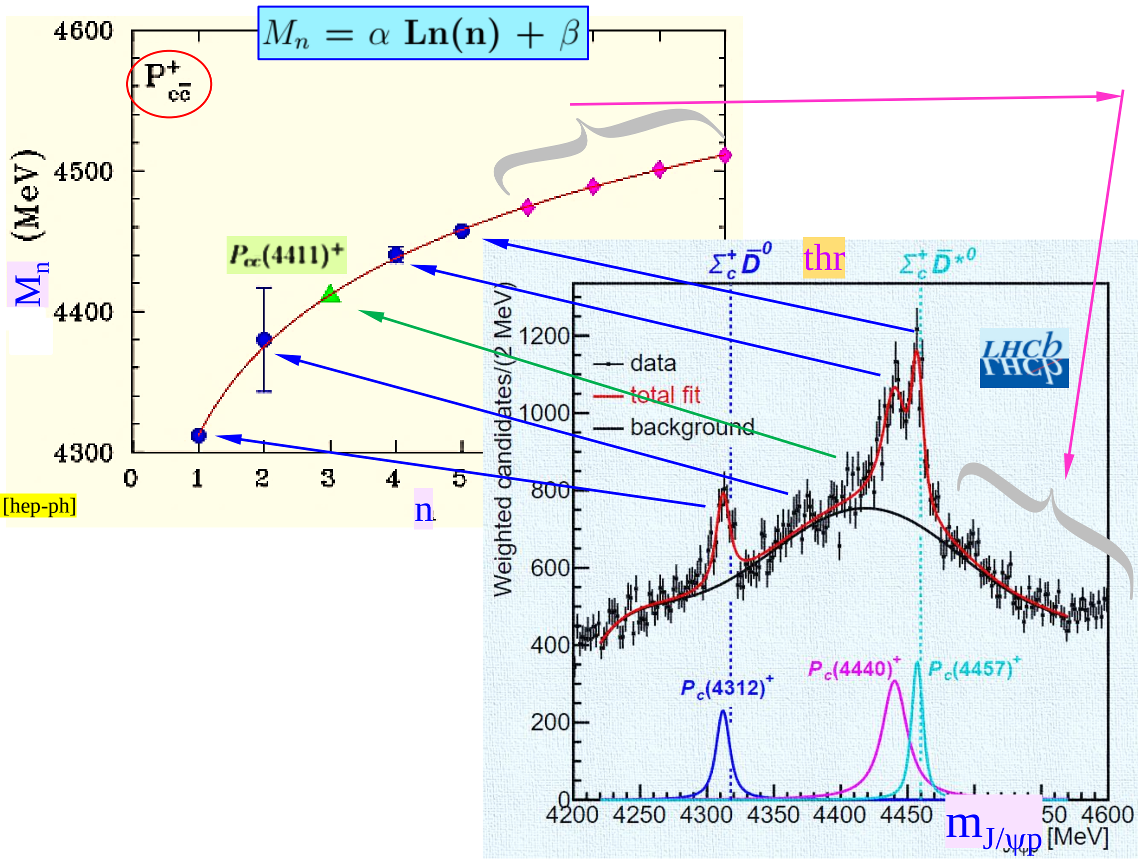
Bump Hunting

PDG $I(J^P) = ½(?^?)$

State	Mass (MeV)	$\Gamma[P_{c\bar{c}} \to J/\psi p]$ (MeV)	Significance (σ)
$P_{c\bar{c}}(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	7.3
$P_{c\bar{c}}(4380)^+$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$	...
$P_{c\bar{c}}(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	5.4
$P_{c\bar{c}}(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	5.4

"Not every bump is a resonance, not every resonance is a bump"
R. Gordon Moorhouse (**1960s**)

R. Aaij *et al*, Phys Rev Lett **122**, 222001 (**2019**)
R. Aaij *et al*, Phys Rev Lett **115**, 072001 (**2015**)



L. David Roper & IIS, arXiv: **2410**.11196 [hep-ph]

Narrow Pentaquarks from $\Xi_b^- \rightarrow P^0_{ccs} K^- \rightarrow (J/\psi\Lambda)K^-$ & $B^- \rightarrow P^0_{ccs}\bar{p} \rightarrow (J/\psi\Lambda)\bar{p}$

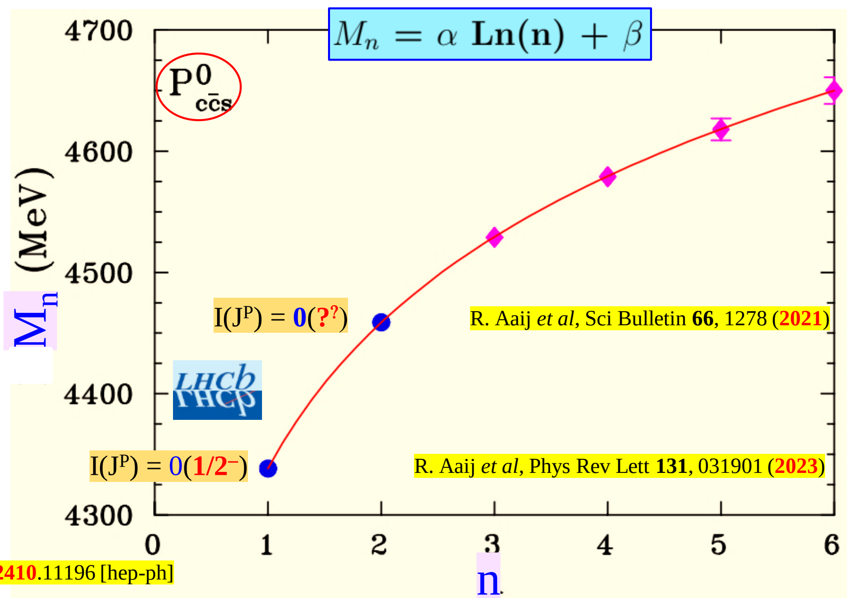
Bump Hunting

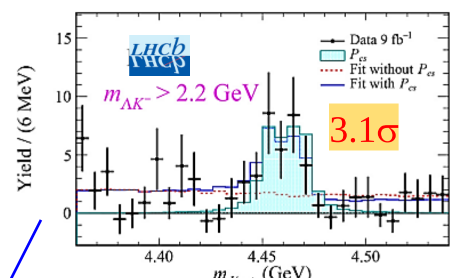
- First evidence of a structure in the $J/\psi\Lambda$ invariant mass distribution is obtained from an amplitude analysis of $\Xi_b^- \rightarrow J/\psi\Lambda K^-$ decays. The observed structure is consistent with being due to a charmonium pentaquark with strangeness with a significance of 3.1σ including systematic uncertainties and look-elsewhere effect. Its mass and width are determined to be $4458.8 \pm 2.9^{+4.7}_{-1.1}$ MeV and $17.3 \pm 6.5^{+8.0}_{-5.7}$ MeV, respectively, where the quoted uncertainties are statistical and systematic. The structure is also consistent with being due to two resonances. In addition, the narrow excited Ξ^- states, $\Xi(1690)^-$ and $\Xi(1820)^-$, are seen for the first time in a Ξ_b^- decay, and their masses and widths are measured with improved precision. The analysis is performed using pp collision data corresponding to a total integrated luminosity of 9 fb^{-1}, collected with the LHCb experiment at centre-of-mass energies of 7, 8 and 13 TeV.

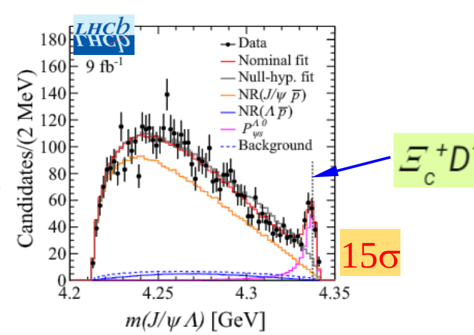
R. Aaij *et al*, Sci Bulletin **66**, 1278 (2021)

- An amplitude analysis of $B^- \rightarrow J/\psi\Lambda\bar{p}$ decays is performed using 4400 signal candidates selected on a data sample of pp collisions recorded at center-of-mass energies of 7, 8, and 13 TeV with the LHCb detector, corresponding to an integrated luminosity of 9 fb^{-1}. A narrow resonance in the $J/\psi\Lambda$ system, consistent with a pentaquark candidate with strangeness, is observed with high significance. The mass and the width of this new state are measured to be $4338.2 \pm 0.7 \pm 0.4$ MeV and $7.0 \pm 1.2 \pm 1.3$ MeV, where the first uncertainty is statistical and the second systematic. The spin is determined to be 1/2 and negative parity is preferred. Because of the small Q-value of the reaction, the most precise single measurement of the B^- mass to date, $5279.44 \pm 0.05 \pm 0.07$ MeV, is obtained.

R. Aaij *et al*, Phys Rev Lett **131**, 031901 (2023)

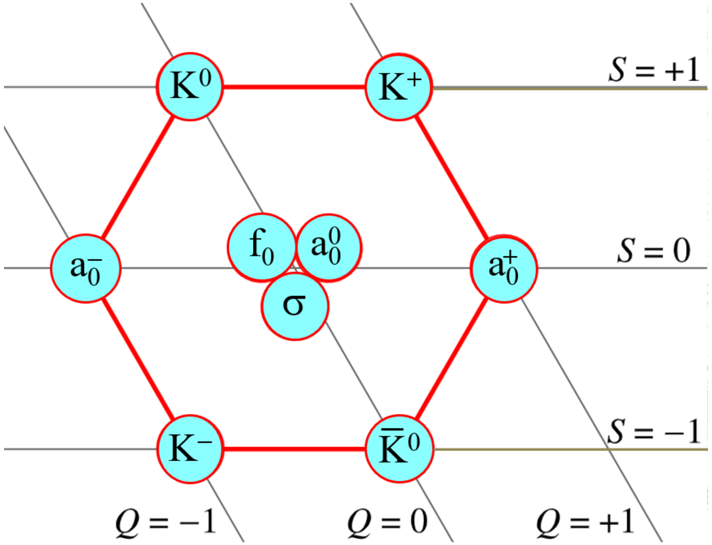




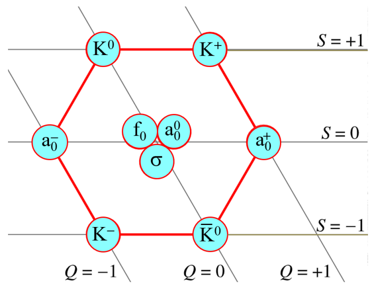


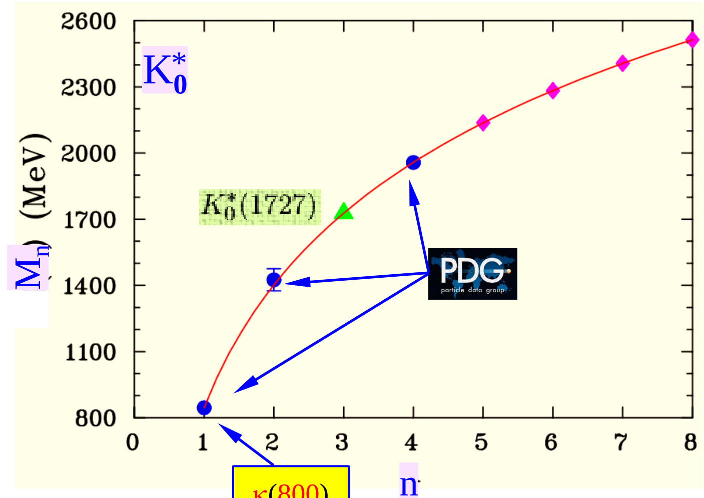
L. David Roper & IIS, arXiv: 2410.11196 [hep-ph]

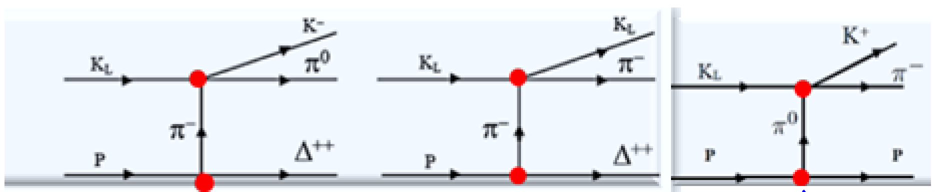
Kaon-Pion Spectroscopy

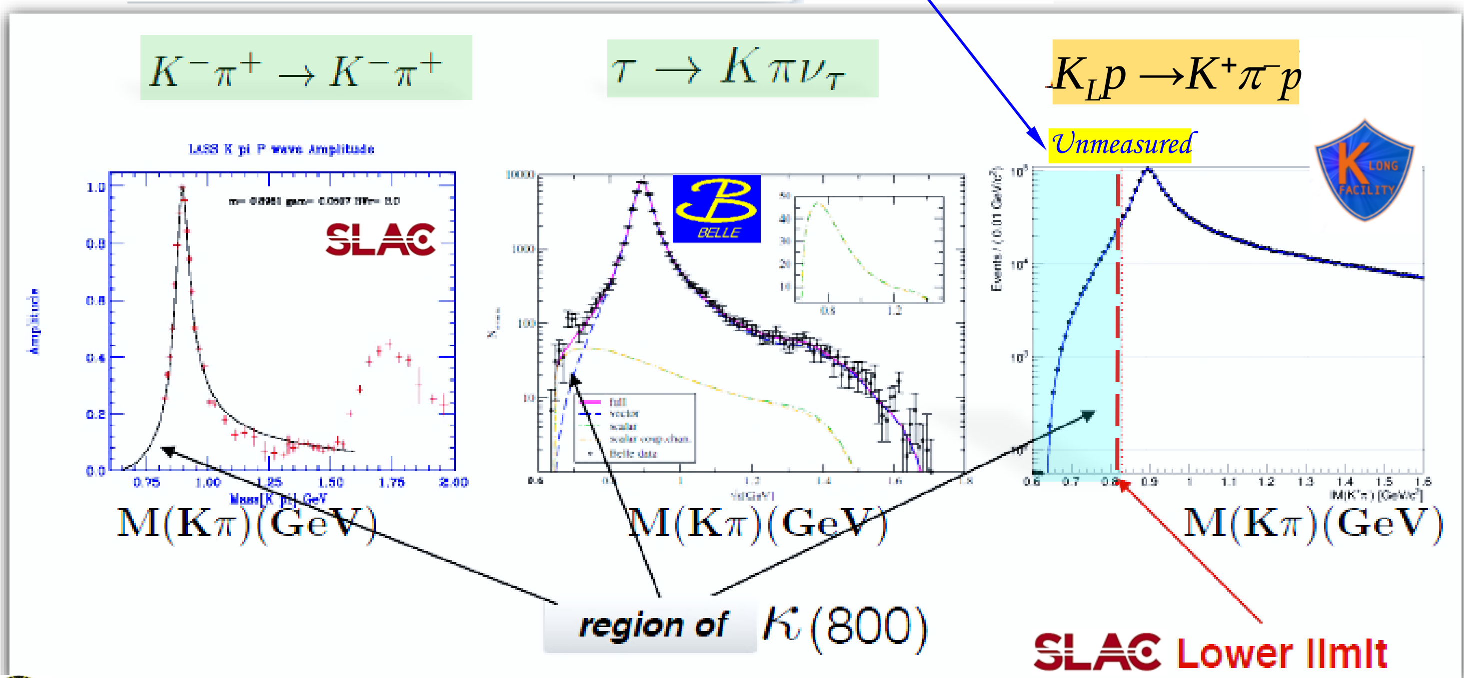


Proposed Measurements for $K\pi$ Scattering

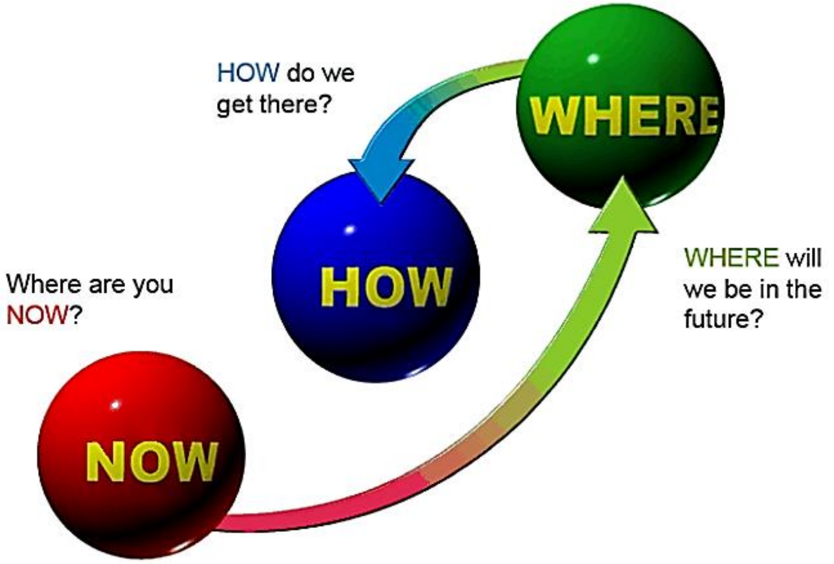


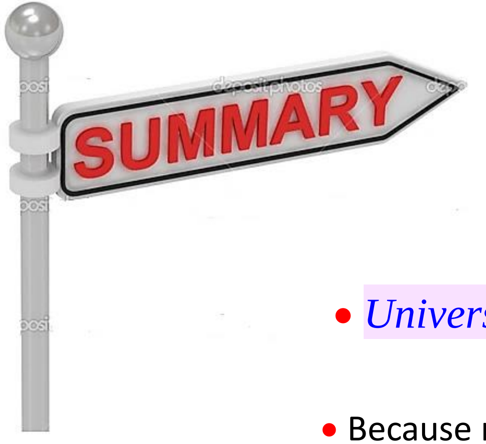






Where are We Now & . . .





- *Universal Mass Equation* (*UME*) for *equal–quantum excited states* is presented.

- Because many states with copious data points are so well fitted with our **UME**, we are confident that **UME** calculations of missing states & predicted states are reasonably accurate.
  It is not surprising that *baryons* & *mesons* look similar because *baryon* can be considered as *meson* (**qq**) plus one more **q**.

- Some interesting results of this study are:

1. *Logarithmic behavior* of masses of resonances with same quantum numbers,
2. Prediction of *four* higher-mass excited states for each of the **41** data sets; *ie*,
   41 x **4** = **164** higher-mass excited-states are predicted.
3. In addition, our fits allows us to determine *lesser* masses of **64** states missing in PDG.
4. For *light* quarks @ large **n** from *quasi-classics*, we expect $\Delta M_n \sim 1/n$.
   We can see that *logarithm* behavior gives stronger behavior than *quasi-classical* case & it works for *light* & *heavy* quarks.
5. *Cornell potential* is example of how such *logarithm* behavior can be explained by appropriate potential.

- Roper formula is opportunity to look for missed *Baryon* & *Meson* resonances predicted by QCD models & LQCD calculations.
  That is one of goals of experiment @ Jefferson Lab.

- *Logarithmic* fit to BW masses of *two known excited states* – next step.

Two- & One-Parameter Logarithm Function

L. David Roper & IIS, arXiv: 2410.11196 [hep-ph]

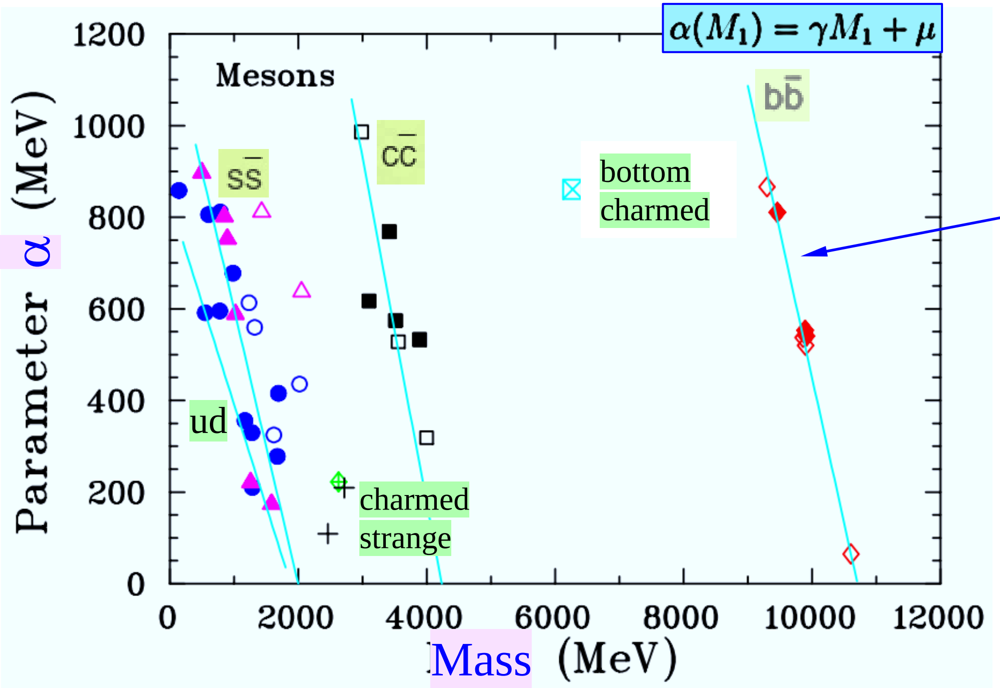
In progress

	Baryons	Mesons	Total
3+-states	17;	24;	41
2-states	12;	17;	29
Total:	29;	41;	70
Predicted missing states:	29;	35;	64
Predicted excited states:	280		350

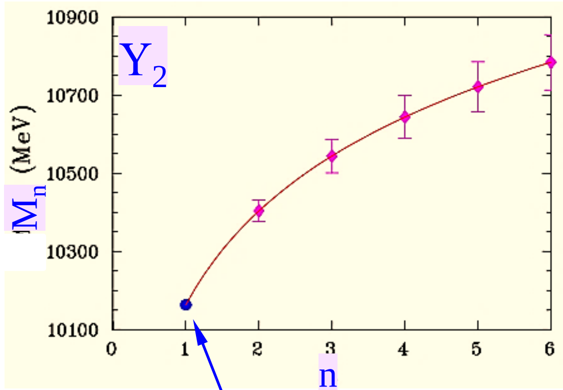
Filled symbols $\beta = M_1$

Open symbols

Corresponds to QCD predictions & LQCD calculations

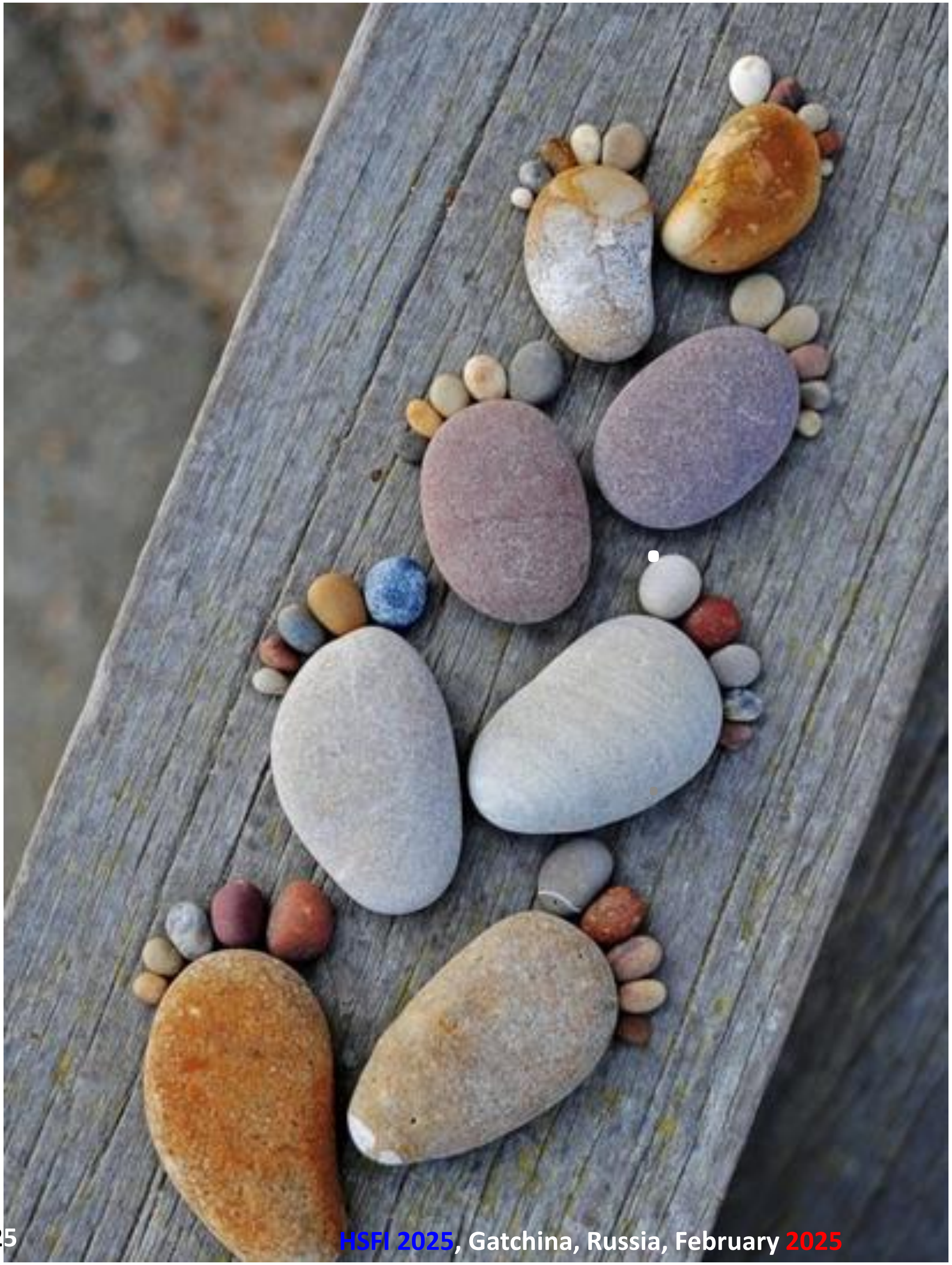


- State masses calculated for 7 $b\bar{b}$ sets differ @ most by 0.8%



PDG $\Upsilon_2(1D)$ $I^G(J^{PC}) = 0^-(2^{--})$

Corresponds to mass of ground state M_1 or β



Tribute to Dick Arndt

As *Dick* said in his autobiography, he had a “gift”.

Dave had the good fortune to closely observe him exercise his “gift” in many ways and to greatly benefit from his “gift”.

igor@gwu.edu
Igor Strakovsky

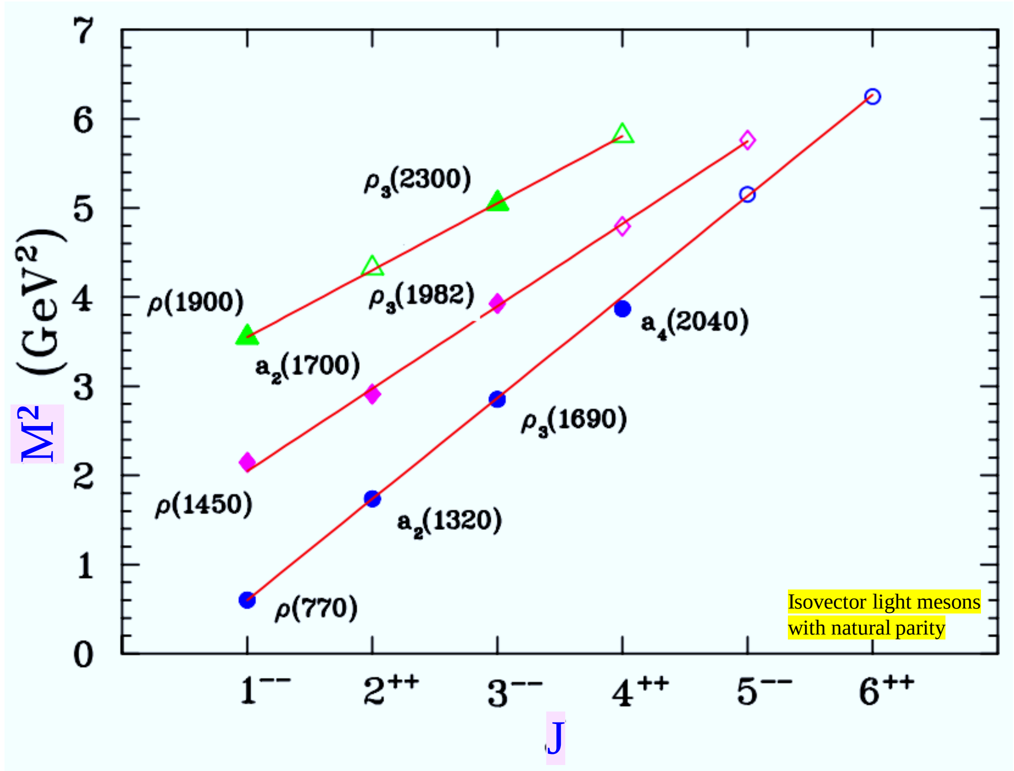
Thank You

We appreciate conversations with

Claude Amsler
Vanya Belyaev
Yury Dokshitzer
Nils Huesken
Kevin Pitts
Pavel Pobylitsa
Ivan Polyakov
Arkaitz Rodas
Misha Ryskin
Tomasz Skwarnicki
Lothar Tiator
Arkady Vainshtein
Liming Zhang


BACKUP

Cornell Potential & Chew-Frautschi Plot



- First, we have interesting observation about Ln[M(n)] dependence.
- Next, we demonstrate that this dependence may be explained if **qq** potential has *Cornell* form.
- That is our Ln[M(n)] dependence is strong argument in favor of *Cornell* potential.
- Finally, we must check that the SAME potential (with {more or less} same parameters) explains spin/(orbital moment) mass dependence (*i.e.*, results in experimentally observed *Regge* trajectories).
- In case of correct *Regge* trajectories, then we may write that *hadron spectroscopy* allows us to "measure"/extract **qq** potential **!!**