

Недиагональные жесткие эксклюзивные реакции и структура адронов

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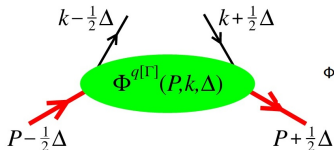


План на сегодня

- 1 Introduction and (some) general motivation
- 2 Kinematics of non-diagonal DVCS
- 3 $N \rightarrow \Delta$ and $N \rightarrow N^*$ non-diagonal DVCS
- 4 $N \rightarrow \pi N$ transition GPDs
- 5 $\pi \rightarrow \pi\pi$ transition GPDs and the Omnes solution for dispersion relation;
- 6 Use of the Gribov-Froissart projection;
- 7 Conclusions and Outlook.

In collaboration with J.-Y. Kim, S. Son, P. Sznajder, M. Vanderhaeghen, C. Weiss, H.-Y. Won

Структура адронов в QCD I



$$\Phi^{q[\Gamma]}(P, k, \Delta) = \frac{1}{2} \int \frac{d^4 z}{(2\pi)^4} e^{ik \cdot z} \langle p', S' | \bar{\psi}_q \left(-\frac{z}{2} \right) \Gamma \mathcal{W} \left[-\frac{z}{2}, \frac{z}{2} \right] \psi_q \left(\frac{z}{2} \right) | p, S \rangle.$$

$$x = \frac{k^+}{P^+}, \quad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$$

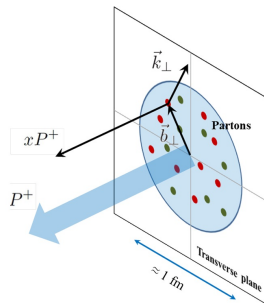
$$\Phi_{\text{FF}}^{q[\Gamma]}(P, \Delta) = \int d^4 k \Phi^{q[\Gamma]}(P, k, \Delta),$$

$$\Phi_{\text{PDF}}^{q[\Gamma]}(P, x) = \int d^4 k \Phi^{q[\Gamma]}(P, k, 0) \delta(k^+ - xP^+),$$

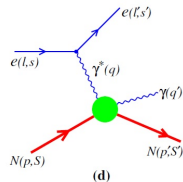
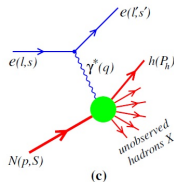
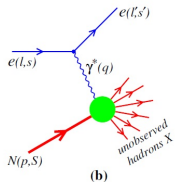
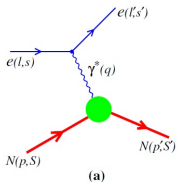
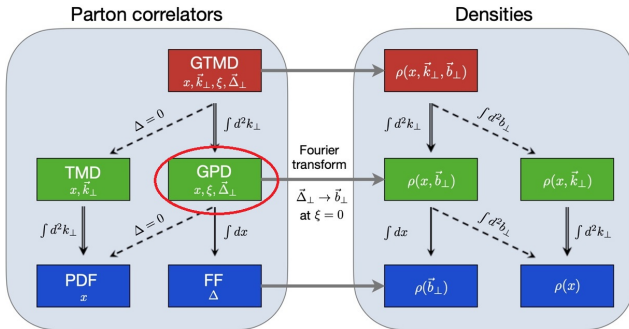
$$\Phi_{\text{TMD}}^{q[\Gamma]}(P, x, \vec{k}_\perp) = \int dk^+ dk^- \Phi^{q[\Gamma]}(P, k, 0) \delta(k^+ - xP^+),$$

$$\Phi_{\text{GPD}}^{q[\Gamma]}(P, x, \xi, \vec{\Delta}_\perp) = \int d^4 k \Phi^{q[\Gamma]}(P, k, \Delta) \delta(k^+ - xP^+),$$

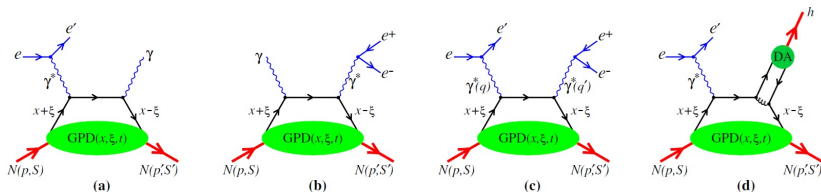
$$\Phi_{\text{GTMD}}^{q[\Gamma]}(P, x, \vec{k}_\perp, \xi, \vec{\Delta}_\perp) = \int dk^+ dk^- \Phi^{q[\Gamma]}(P, k, \Delta) \delta(k^+ - xP^+).$$



Структура адронов в QCD II



GPD: определение и свойства



$$\Phi_{\text{GPD}}^{q[\gamma^+]}(P, x, \xi, \vec{\Delta}_\perp) = \frac{1}{2P^+} \bar{u}(p', S') \left[\gamma^+ H^q(x, \xi, t) + \frac{i\sigma^{+\mu} \Delta_\mu}{2M} E^q(x, \xi, t) \right] u(p, S)$$

$$H^q(x, 0, 0) = f_1^q(x), \quad F_1^q(t) = \int_{-1}^1 dx H^q(x, \xi, t), \quad F_2^q(t) = \int_{-1}^1 dx E^q(x, \xi, t).$$

- Правило сумм Ji'97:

$$J^q = \frac{1}{2} \int_{-1}^1 dx x [H^q(x, 0, 0) + E^q(x, 0, 0)]; \quad J^q + J^g = \frac{1}{2}.$$

GPD: связь с тензором ЭИ

Kobzarev, Okun'62; Leader, Lorce'14

$$\langle p', S' | T_a^{\mu\nu}(0) | p, S \rangle = \bar{u}(p', S') \left[P^{[\mu} \gamma^{\nu]} A_a(t) + \frac{P^{[\mu} i \sigma^{\nu]\lambda} \Delta_\lambda}{2M} B_a(t) + M g^{\mu\nu} \bar{C}_a(t) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} D_a(t) - \frac{P^{[\mu} i \sigma^{\nu]\lambda} \Delta_\lambda}{M} S_a(t) \right] u(p, S)$$

$$\int dx x H^q(x, \xi, t) = A_q(t) + \xi^2 D_q(t), \quad \int dx x E^q(x, \xi, t) = B_q(t) - \xi^2 D_q(t).$$

Breit frame $\vec{p}' = -\vec{p} = \vec{\Delta}/2$

M. Polyakov'03:

Тензор напряжений для частицы со спином-1/2:

$$\mathcal{T}_a^{\mu\nu}(\vec{r}, \vec{s}) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r}} \frac{\langle p', \vec{s} | T_a^{\mu\nu}(0) | p, \vec{s} \rangle}{2E}.$$

$$\mathcal{T}^{ij}(\vec{r}) = \delta^{ij} p(r) + \left(\frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij} \right) s(r).$$



"давление"



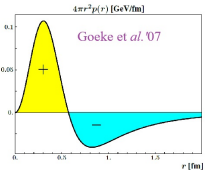
"анизотропия давления"

$$D = M \int d^3r r^2 p(r) = -\frac{4}{15} M \int d^3r r^2 s(r)$$

Условие стабильности фон Лауэ:

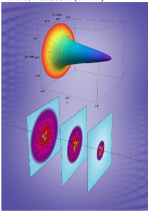
$$\int d^3r p(r) = 0$$

См. C. Lorce and P. Schweitzer, Acta Phys.Polon.B 56 (2025)

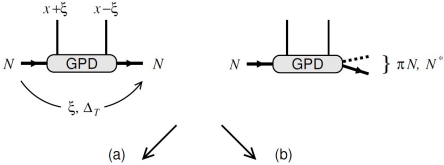


GPDs and Transition GPDs

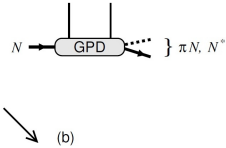
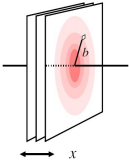
Burkardt (2000, 2003)
Belitsky, Ji, Yuan (2004)
Pire, Ralston (2001)



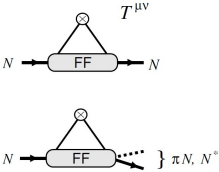
tomographic imaging
of nucleon/resonance



(a)

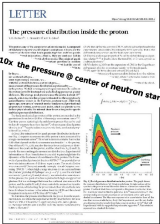


(b)



QCD energy-momentum tensor
angular momentum, mass

Burkert, Elouadrhiri, Giron,
Nature 557(2018)



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<https://doi.org/10.1140/epja/10050-025-01552-2>

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Regular Article - Experimental Physics

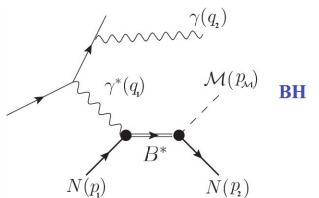
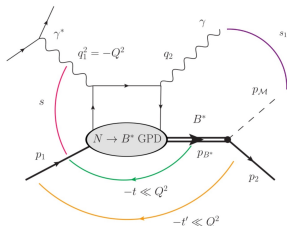
Exploring baryon resonances with transition generalized parton distributions: status and perspectives

S. Diehl^{1,2,3}, K. Joo⁴, K. Semenov-Tian-Shansky^{1,4}, C. Weiss⁵, V. Braun⁶, W. C. Chang⁷, P. Chatagnon⁸, M. Constantinou⁹, Y. Guo⁹, P. T. P. Hutaurok¹⁰, H.S. Jo¹, A. Kim³, J.-Y. Kim³, P. Kroll¹¹, S. Kumano^{12,13}, C.-H. Lee^{14,15}, S. Liuti¹⁶, R. McNulty¹⁷, H.-D. Son¹⁸, P. Sznajder¹⁹, A. Usman²⁰, C. Van Hulse²¹, M. Vanderhaeghen²², M. Winn²³

Eur.Phys.J.A 61 (2025) 6, 131

What is non-diagonal DVCS?

$$\gamma^*(q_1) + N(p_1) \rightarrow \gamma(q_2) + [\mathcal{M}(p_{\mathcal{M}})N(p')]; \mathcal{M} = \pi, \eta, \rho, \omega \dots$$



- Factorized description in terms of $N \rightarrow B^*$ GPDs in the generalized Bjorken kinematics:

$$\begin{aligned} -q_1^2; s = (p_1 + q_1)^2; s_1 = (p_{\mathcal{M}} + q_2)^2 - \text{large}; \quad x_B = \frac{-q_1^2}{2p_1 \cdot q_1} - \text{fixed}; \\ -t = -(p_{B^*} - p_1)^2; \quad -t' = -(p_2 - p_1)^2; \quad W_{\mathcal{M}N}^2 = (p_1 + p_{\mathcal{M}})^2 \text{ of hadronic scale.} \end{aligned}$$

- Meson-nucleon system resonates at $W_{\mathcal{M}N}^2 = M_{B^*}^2$.
- Status of factorization: same as for the DVCS&DVMP: X. Ji et al.'98, J. Collins et al.'97,99.
- Rates are the same order as in usual DVCS.

Some motivation

E.m./weak probe :

QCD structure :

$$\begin{aligned}\gamma &\Leftrightarrow \langle B^* | \bar{q} \hat{Q}_{\text{e.m.}} \gamma_\mu q | N \rangle \\ W^\pm, Z^0 &\Leftrightarrow \langle B^* | \bar{q} \hat{Q}_w \gamma_\mu (1 - \gamma_5) q | N \rangle\end{aligned}$$

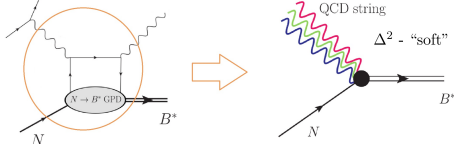
- Only $C = -1$ probe;
- Local in space-time;
- No direct access to gluons;

Hadronic probe :

QCD structure :

$$\pi, K \Leftrightarrow \langle B^* | ??? | N \rangle$$

- DVCS creates a low-energy QCD string = a tower of local probes of arbitrary spin- J .



$$\bar{\Psi}(n) P \exp \left(i \int_{-n}^n dz^\mu A_\mu(z) \right) \Psi(-n) = \text{diagram of a line between two dots} = \sum_{J=0}^{\infty} \left[\text{diagram of a line between two dots with a red segment} \right]_J Y_{JM}$$

E.g. “G probe” :

QCD structure :

$G \Leftrightarrow$

$$\langle B^* | \underbrace{\bar{\Psi} \gamma_\mu (\partial_\nu - A_\nu) \Psi + \frac{1}{4} F_{\mu\alpha}^a F_{\nu\alpha}^a}_{\text{QCD EMT}} | N \rangle$$

Physical contents I: a unique option for baryon spectroscopy

2008 White paper

Baryon spectroscopy in non-diagonal DVCS

M. Amarian¹, M.V. Polyakov², K.M. Semenov-Tian-Shansky², I. Strakovsky³

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and

³Center for Nuclear Studies, Department of Physics, The George Washington University,
Washington, DC, 20052, USA

- 1 Excitation of resonances by non-local QCD quark light-cone operators:

$$\left\langle N^* \left| \bar{\Psi}_\alpha(0) P e^{ig \int_0^z dx_\mu \lambda^c A_c^\mu} \Psi_\beta(z) \right| N \right\rangle$$

★ excitation by probes of arbitrary spin (not just $J = 1$);

- 2 Possible generalization to the gluon light-cone operators. ★ explicit access to the gluonic DOFs:

$$\langle N^* | G_{\alpha\beta}^a(0) [0, z]^{ab} G_{\mu\nu}^b(z) | N \rangle$$

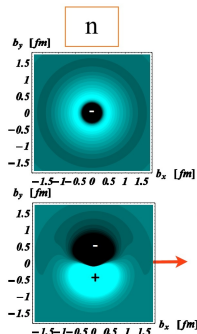
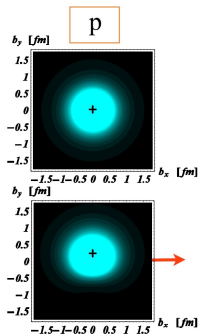
- 3 Direct access to **Im** (beam spin asymmetry) and **Re** (charge asymmetry) of the amplitude $A_{N \rightarrow B^*}^{\text{DVCS}}$. **May be simpler than usual PWA.**

- Hunt for exotics: possible access to non-usual spin-flavor configurations: e.g. $SU(6)$ $[20, 1^+]$: $N = 2$ orbital excitation of the $SU(6)$ 20-plet.
- Symmetry argument by **R. Feynman'1972**: *"Two quark at least must have their motion changed to get to the $[20, 1^+]$ from the fundamental $[56, 0^+]$."*

Hadronic imaging: nucleon FF case

- FFs as quark transverse charge densities in the IMF, J. Miller'07, C. Carlson, M. Vanderhaeghen'07;

$$\begin{aligned}\rho_T^N(\vec{b}) &\equiv \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \left\langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} \left| J^+(0) \right| P^+, -\frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} \right\rangle \\ &= \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F_1(Q^2) + \sin(\phi_b - \phi_S) \int_0^\infty \frac{dQ}{2\pi} \frac{Q^2}{2M} J_1(bQ) F_2(Q^2).\end{aligned}$$



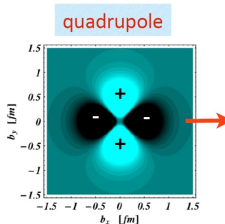
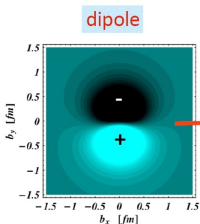
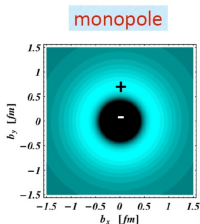
A. Einstein and J. Laub, 1908

$$d_y = \kappa \frac{e}{2m_N}$$

Hadronic imaging: FF case, $N \rightarrow \Delta(1232)$ transition charge densities

- $N \rightarrow \Delta$ transition: 3 FFs, H. Jones, M. Scadron '73:
 G_M -Magnetic, G_E -Electric, G_C -Coulomb quadrupole;

$$\begin{aligned} \rho_T^{N\Delta}(\vec{b}) &\equiv \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \left\langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp^\Delta = +\frac{1}{2} \left| J^+(0) \right| P^+, -\frac{\vec{q}_\perp}{2}, s_\perp^N = +\frac{1}{2} \right\rangle \\ &= \int_0^\infty \frac{dQ}{2\pi} \frac{Q}{2} \left\{ J_0(bQ) G_{+\frac{1}{2}+\frac{1}{2}}^+ \rightarrow \text{monopole} \right. \\ &\quad - \sin(\phi_b - \phi_S) J_1(bQ) \left[\sqrt{3} G_{+\frac{3}{2}+\frac{1}{2}}^+ + G_{+\frac{1}{2}-\frac{1}{2}}^+ \right] \rightarrow \text{dipole} \\ &\quad \left. - \cos 2(\phi_b - \phi_S) J_2(bQ) \sqrt{3} G_{+\frac{3}{2}-\frac{1}{2}}^+ \right\} \rightarrow \text{quadrupole} \end{aligned}$$

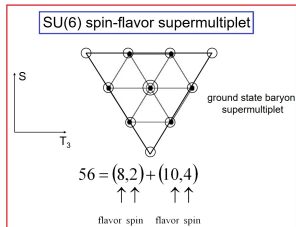


C. Carlson, M.Vanderhaeghen'2007



How to interpret hadron imaging with transition FFs (and tGPDs)

- A correspondence between $N \rightarrow \Delta$ transitional FFs and quantities, admitting a probabilistic interpretation, is based on the $SU(2N_f)$ spin-flavor symmetry (symmetry of QCD at large N_c).



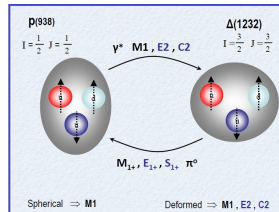
- $SU(6)$ symmetry predictions for $G_{M,E,C}(t)$:

$$G_M(0) = \frac{2\sqrt{2}}{3} \mu_p = 2.63 \mu_N \quad \text{Exp: } (3.46 \pm 0.03) \mu_N$$

Clebsch-Gordan
coefficient

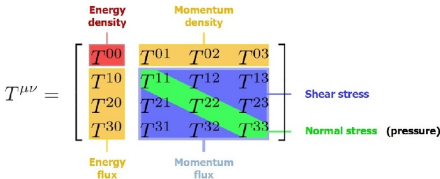
$G_{E,C} \sim SU(6)$ breaking effects;

- Non-zero $G_{E,C}$ signal deviation from $SU(6)$ caused by
 - quark mass differences;
 - relativistic effects;
 - long-range QCD effects (pion cloud);
- One may compute systematically the $1/N_c$ corrections;
- Try lattice QCD and/or functional approach;



QCD EMT $N \rightarrow B^*$ transition FFs

Gravitational FFs of the proton, see e.g. [V.D. Burkert et al. 2303.08347](#) [Burkert, Elouadrhiri, Girod, Nature 557\(2018\)](#)



M. Polyakov' 03:

$$\mathcal{T}^{ij}(\vec{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij} \right) s(r) + \delta^{ij} p(r)$$

$$s(r) = -\frac{1}{4M_N} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \tilde{D}(r)$$

$$p(r) = \frac{1}{6M_N} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \tilde{D}(r)$$

$$\tilde{D}(r) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r}} D(-\vec{\Delta}^2).$$

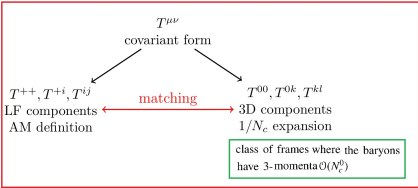
- Study of QCD EMT $N \rightarrow B^*$ matrix elements complements studies of e.m. transition FFs;
- Possible access to transition spin contents (for $N \rightarrow N^*$, Δ);
- AM as a transverse density at fixed light-front time [C. Lorce, L. Mantovani, B. Pasquini'18](#);
- Transition angular momentum $N \rightarrow \Delta$: [J.-Y. Kim et al.'23](#);

For large- N_c :

$$J_{p \rightarrow p}^V = \frac{1}{\sqrt{2}} J_{p \rightarrow \Delta^+}^V = 5J_{\Delta^+ \rightarrow \Delta^+}^V = O(N_c);$$

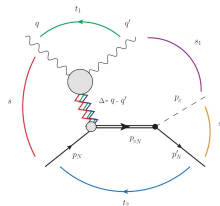
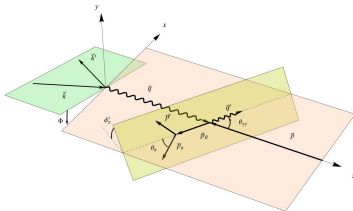
Large J^{u-d} flavor asymmetry!

$$\int_{-1}^1 dx \, x H_M(x, \xi, 0) = 2J_{p \rightarrow \Delta^+}^V;$$



Kinematics and decay angular distribution

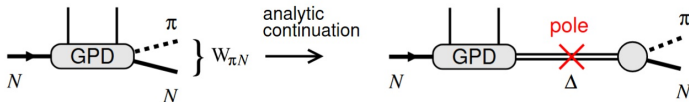
$$e(k) + N(p_N) \rightarrow e'(k') + \gamma^*(q) + N(p_N) \rightarrow e'(k') + \gamma(q') + \pi(p_\pi) + N'(p'_N)$$



- $\gamma^* N \rightarrow B^* \gamma$: $\gamma^* N$ CMS;
- $B^* \rightarrow \pi N'$: $\pi N'$ CMS $\equiv (\pi N')$ at rest;
 - $W_{\pi N}^2$: πN invariant mass;
 - $\theta_\pi^*, \varphi_\pi^*$: decay angles in πN rest frame; $\Leftrightarrow s_1, t_2$

$$\frac{d^7 \sigma}{\underbrace{dQ^2 dx_B}_{\text{lepton side } \gamma^* N \rightarrow \gamma B^*} \underbrace{dt d\Phi}_{B^* \rightarrow \pi N} \underbrace{dW_{\pi N}^2 d\Omega_\pi^*}_{B^* \rightarrow \pi N}}$$

Non-diagonal reactions: integrated cross section



- Narrow resonance limit: $\Gamma_R \ll M_R$;
- The Breit-Wigner spectral function with the energy-dependent decay width;

$$\int dW_{\pi N}^2 \int d\Omega_{\pi}^* \frac{d^7\sigma}{dQ^2 dx_B dt d\Phi dW_{\pi N}^2 d\Omega_{\pi}^*}$$

$$\approx \frac{1}{(2\pi)^4} \frac{x_B y^2}{32 Q^4 \sqrt{1 + \frac{4M_{N \times B}^2}{Q^2}}} \sum_i \sum_f |\mathcal{M}(e^- N \rightarrow e^- \gamma R(M_R, \lambda_R))|^2.$$

- Fully consistent with the result for a stable particle.

Compton FFs for $N \rightarrow \pi N$ DVCS

- LO, leading twist-2: same hard part

$$C^{\pm}(x, \xi) = \frac{1}{x - \xi + i\varepsilon} \pm \frac{1}{x + \xi + i\varepsilon};$$

Diagonal DVCS

$$\{\mathcal{H}, \tilde{\mathcal{H}}\}(\xi, t);$$

Non-diagonal DVCS

$$\{\mathcal{H}, \tilde{\mathcal{H}}\}(\xi, t; W_{\pi N}^2, \theta_{\pi}^*, \varphi_{\pi}^*);$$

A dream of an ideal experiment:

- Good coverage in $\theta_{\pi}^*, \varphi_{\pi}^*$;
- Scan over $W_{\pi N}^2$.

A test ground: $N \rightarrow \Delta(1232)$ DVCS

$$\gamma^*(q) + N^P(p_N) \rightarrow \gamma(q') + \Delta^+(p_\Delta) \rightarrow \gamma(q') + \pi^0(p_\pi) + N^P(p'_N)$$

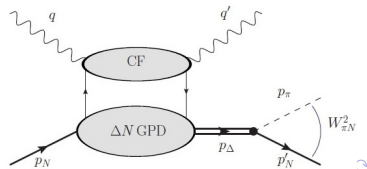
K. Goeke, M. Polyakov and M. Vanderhaeghen'01:

- 4 vector $N \rightarrow \Delta$ GPDs $H_{E,M,C,X}$ (and also 4 axial-vector $C_{1,2,3,4}$):

$$\begin{aligned} & \frac{1}{2\pi} \int dy^- e^{ixP^+y^-} \langle \Delta(p_\Delta) | \bar{\Psi}(-y/2) \gamma \cdot n \tau_3 \Psi(y/2) | N(p_N) \rangle \Big|_{y^+=\vec{y}_\perp=0} \\ &= \sqrt{\frac{2}{3}} \bar{U}^\beta(p_\Delta) \left\{ H_M(x, \xi, t) (-\mathcal{K}_{\beta\mu}^M) n^\mu + H_E(x, \xi, t) (-\mathcal{K}_{\beta\mu}^E) n^\mu \right. \\ & \quad \left. + H_C(x, \xi, t) (-\mathcal{K}_{\beta\mu}^C) n^\mu + \underbrace{H_X(x, \xi, t) (-\mathcal{K}_{\beta\mu}^X) n^\mu}_{\text{omitted structure, see J.-Y. Kim, H.-Y. Won et al.'25}} \right\} u(p_N), \end{aligned}$$

omitted structure, see J.-Y. Kim, H.-Y. Won et al.'25

- 1 + 2 relevant in the large N_c limit;
- Early analysis: P. Guichon, L. Mossé and M. Vanderhaeghen'03;
- K.S. and M. Vanderhaeghen, PRD 108 (2023): study for CLAS@12 conditions;



Sum rules and large- N_c relations

- Unpolarized GPDs are related to e.m. form factors [Jones and Scadron '73](#), see a review [G. Ramalho, M.T. Peña '24](#):

$$\int_{-1}^1 dx H_{M,E,C} = 2G_{M,E,C}(t); \quad \int_{-1}^1 dx H_X = 0;$$

- Polarized transition GPDs are related to axial form factors [Adler '75](#); (can be accessed in neutrino-production reactions):

$$\int_{-1}^1 dx C_{1,2,3} = 2C_{5,6,3}^A(t).$$

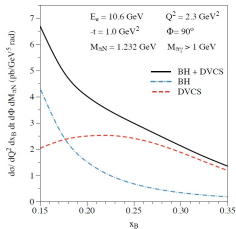
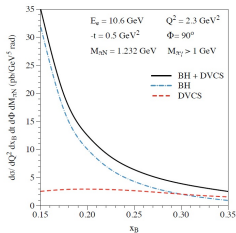
- Large- N_c relations for octet-to-decuplet transition GPDs, [Goeke et al. '01](#):

$$H_M = \frac{2}{\sqrt{3}} [E^u - E^d]; \quad C_1 = \sqrt{3} [\tilde{H}^u - \tilde{H}^d]; \quad C_2 = \frac{\sqrt{3}}{4} [\tilde{E}^u - \tilde{E}^d];$$

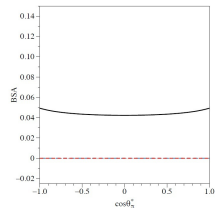
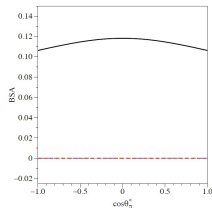
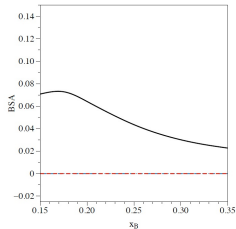
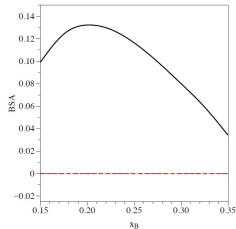
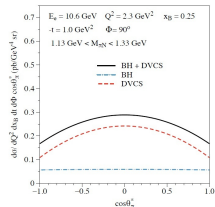
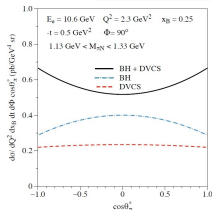
- Pion pole contribution into C_2 :

$$\lim_{t \rightarrow m_\pi^2} C_2(x, \xi, t) = \sqrt{3} \frac{g_A m_N^2}{m_\pi^2 - t} \theta[\xi - |x|] \frac{1}{\xi} \Phi_\pi \left(\frac{x}{\xi} \right);$$

Cross sections and BSA for JLab@12 GeV



- Δ in helicity $\pm 1/2$ state: $\frac{1}{4} (1 + 3 \cos^2 \theta_\pi^*)$
- Δ in helicity $\pm 3/2$ state: $\frac{3}{4} \sin^2 \theta_\pi^*$



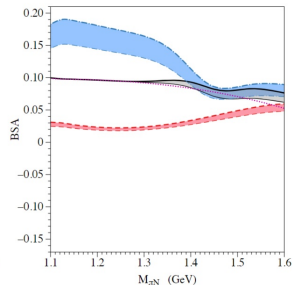
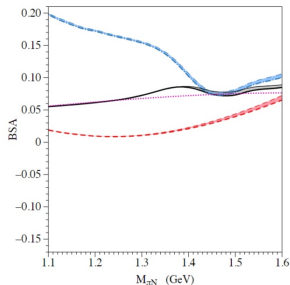
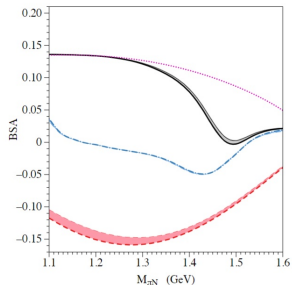
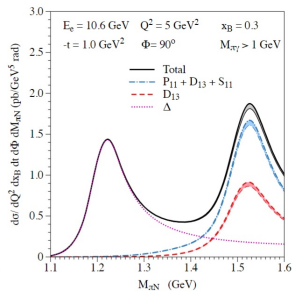
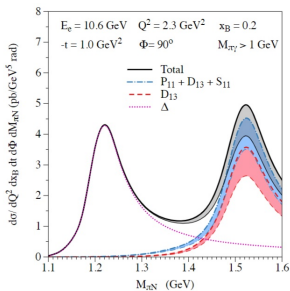
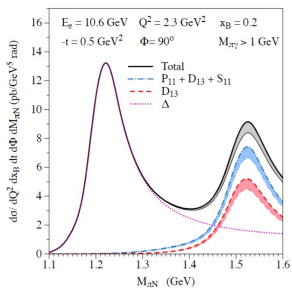
Going to the 2nd resonance region

- Formalism extended to $N \rightarrow N^*$ DVCS for $N^* = P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$:
 - for spin- $\frac{1}{2}$ resonances at twist-2: 2 unpolarized GPDs (vector operator), 2 polarized GPDs (axial-vector operator);
 - for spin- $\frac{3}{2}$ resonances at twist-2: 4 unpolarized GPDs (vector operator), 4 polarized GPDs (axial-vector operator);

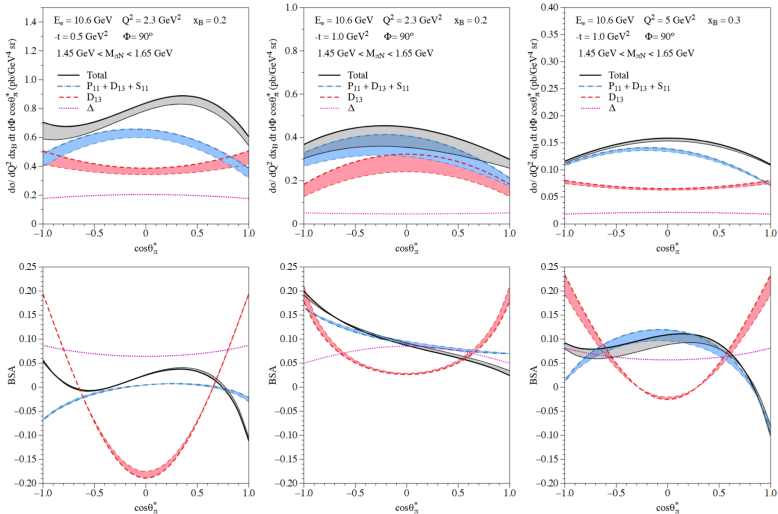
$$\begin{aligned} & \int \frac{d\lambda}{2\pi} e^{i\lambda x} \sum_q e_q^2 \langle R(p_R, s_R) | \bar{q} \left(-\frac{\lambda n}{2} \right) \gamma \cdot n \gamma_5 q \left(\frac{\lambda n}{2} \right) | N(p, s_N) \rangle \\ &= \bar{R}_\beta(p_R, s_R) \left\{ \tilde{H}_1^{pD_{13}}(x, \xi, \Delta^2) n^\beta + \tilde{H}_2^{pD_{13}}(x, \xi, \Delta^2) \left(\frac{\Delta \cdot n}{M_N^2} \right) \Delta^\beta \right. \\ & \quad \left. + \tilde{H}_3^{pD_{13}}(x, \xi, \Delta^2) \frac{1}{M_N} (n^\beta \gamma \cdot \Delta - \Delta^\beta \gamma \cdot n) + \tilde{H}_4^{pD_{13}}(x, \xi, \Delta^2) \frac{2}{M_N^2} (n^\beta \bar{P} \cdot \Delta - \Delta^\beta) \right\} \gamma_5 N(p, s_N) \end{aligned}$$

- t -dependence of GPDs (first moments):
 - unpolarized GPDs: first moments constrained by data on e.m. transition FFs (CLAS@6 GeV)
 - polarized GPDs: 2 dominant axial FFs constrained using PCAC + pion pole dominance:
 - normalization at $t = 0$ given by $(f_{\pi NN^*}/m_\pi) 2f_\pi$ (Goldberger-Treiman relation);
 - t -dependence: dipole ($M_A = 1$ GeV) and pion-pole $\sim 1/(t - m_\pi^2)$;
 - isoscalar axial FF neglected;
- x & ξ dependence of GPDs: RDDA $b = 1$ and $b = \infty$ with $q(x) \sim x^{-0.5}(1-x)^3$

Cross section and BSA



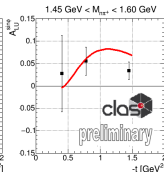
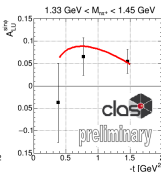
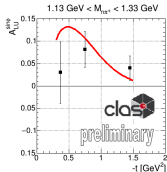
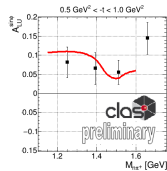
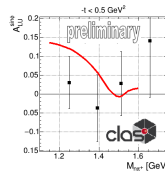
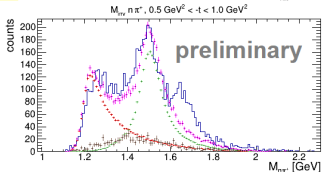
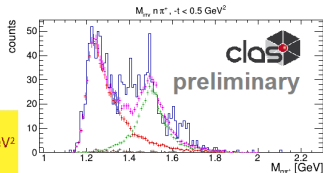
Pion angular distribution



- Forward-backward asymmetry: due to interference between even/odd partial waves.

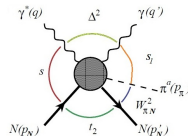
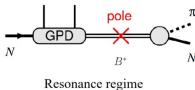
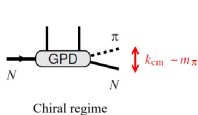
$e^-p \rightarrow e^- \gamma \pi^+ n$ with CLAS@12 GeV

$W > 2 \text{ GeV}$
 $Q^2 \approx 2.3 \text{ GeV}^2$
 $x_B = 0.25$



- Preliminary data confirm that with increasing of $-t$ the 2nd resonance region becomes more pronounced. S. Diehl et al., Eur.Phys.J.A 61 (2025) 6, 131;
- BSA compared to predictions of K.S. and M.Vanderhaeghen, Phys.Rev.D 108(2023);
- Possible JLab@20 upgrade: statistics may increase by a factor 100 - 1000;
- Access to the complete angular distribution of the cross section $\Rightarrow$ PW analysis?

A unified description: $N \rightarrow \pi N$ transition GPDs



- Unpolarized $N \rightarrow \pi N$ GPDs [M. Polyakov and S. Stratmann, arXiv:hep-ph/0609045](#) :

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x \bar{P} \cdot n} \left\langle N(p'_N) \pi^a(k_\pi) | \bar{\Psi}(-\frac{\lambda n}{2}) \not{n} \Psi(\frac{\lambda n}{2}) | N(p_N) \right\rangle = \frac{ig_A}{m_N f_\pi} \sum_{i=1}^4 \bar{u}(p'_N) \Gamma_i \tau^a H_i^{(0)}(x, \xi, t_1, \alpha, W_{\pi N}^2, t_2) u(p_N)$$

$$\Gamma_1 = \gamma_5; \quad \Gamma_2 = \frac{m_N \not{k}_\pi}{n \cdot \bar{P}} \gamma_5; \quad \Gamma_3 = \frac{\not{k}_\pi}{m_N} \gamma_5; \quad \Gamma_4 = \frac{\not{k}_\pi \not{n}}{m_N} \gamma_5; \quad \alpha = \frac{n \cdot k_\pi}{n \cdot (p'_N + k_\pi)}$$

- Polynomiality of the Mellin moments; access to the EMT FFs;
- In chiral regime $N \rightarrow \pi N$ GPDs from χ PT: soft pion theorems; **First principle calculations!**
- $\Delta(1232)$ can be included in χ PT;

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Chiral theory of nucleons and pions in the presence of an external gravitational field

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Gravitational $p \rightarrow \Delta^+$ transition form factors in chiral perturbation theory

H. Alharazin,^a B.-D. Sun,^a E. Epelbaum,^a J. Gegelia^{a,b} and U.-G. Meißner^{a,b}

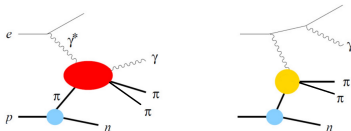
- In resonance regime: constraints from unitarity ([Watson '54](#) final state interaction theorem);

A test ground: $\pi \rightarrow \pi\pi$ ND DVCS

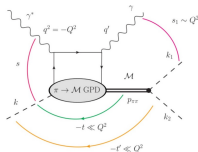
S. Son and K.S., JHEP 01 (2025);

- The Sullivan-type process:

$$e(l) + p(p) \rightarrow e(l') + \gamma(q') + \pi^+(k) + n(p') \rightarrow e(l') + \gamma(q') + \pi^+(k_1) + \pi^0(k_2) + n(p')$$



- No complications due to spin- $\frac{1}{2}$;
- Access to the meson spectrum: $\rho(770)$, $f_2(1270)$ etc;
- An option for the EIC?



$$Y_\ell^m(\theta_\pi^*, \varphi_\pi^*) = (-1)^m \sqrt{\frac{(2\ell+1)(\ell-m)!}{4\pi(\ell+m)!}} P_\ell^m(\cos\theta - \pi^*) e^{im\varphi_\pi^*};$$

the real-valued spherical harmonics read :

$$Y_\ell^m = \begin{cases} \frac{1}{\sqrt{2}} (Y_{\ell,|m|} - i Y_{\ell,-|m|}) & \text{if } m < 0; \\ Y_{\ell,0} & \text{if } m = 0; \\ \frac{(-1)^m}{\sqrt{2}} (Y_{\ell,|m|} + i Y_{\ell,-|m|}) & \text{if } m > 0; \end{cases}$$

- N.B.** $\gamma^* N \rightarrow \rho N' \rightarrow \pi\gamma N'$ a background for $N \rightarrow \Delta$ DVCS.

A test ground: $\pi \rightarrow \pi\pi$ ND DVCS

$$\frac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x \bar{P} \cdot n} \langle \pi(k_1) \pi(k_2) | \bar{\psi} \left(-\frac{\lambda n}{2} \right) \left[\not{n} \gamma_5 \right] \psi \left(\frac{\lambda n}{2} \right) | \pi(p_\pi) \rangle = \left[\begin{aligned} & \frac{1}{2\bar{P} \cdot n} i\varepsilon(n, \bar{P}, \Delta, k_1) \frac{1}{f_\pi^3} H_{\pi \rightarrow \pi\pi}(x, \xi, \Delta^2, W_{\pi\pi}^2, \theta_\pi^*, \varphi_\pi^*); \\ & \frac{1}{2\bar{P} \cdot n} (\bar{P} \cdot n) \frac{1}{f_\pi} \tilde{H}_{\pi \rightarrow \pi\pi}(x, \xi, \Delta^2, W_{\pi\pi}^2, \theta_\pi^*, \varphi_\pi^*); \end{aligned} \right]$$

- PW expansion in angles θ_π^* and φ_π^* for vector GPD:

$$H_{\pi \rightarrow \pi\pi}(x, \xi, t, W_{\pi\pi}^2, \theta_\pi^*, \varphi_\pi^*) = \frac{1}{\sqrt{1 - \cos^2 \theta_\pi^* \sin \varphi_\pi^*}} \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{-1} H_{\pi \rightarrow \pi\pi}^{\ell m}(x, \xi, t, W_{\pi\pi}^2) Y_{\ell m}(\theta_\pi^*, \varphi_\pi^*);$$

N.B. Spherical harmonics in are odd under $\varphi_\pi^* \rightarrow -\varphi_\pi^*$.

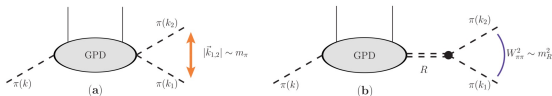
- PW expansion in angles θ_π^* and φ_π^* for axial-vector GPD:

$$\tilde{H}_{\pi \rightarrow \pi\pi}(x, \xi, t, W_{\pi\pi}^2, \theta_\pi^*, \varphi_\pi^*) = \sum_{\ell=0}^{\infty} \sum_{m=0}^{\ell} \tilde{H}_{\pi \rightarrow \pi\pi}^{\ell m}(x, \xi, t, W_{\pi\pi}^2) Y_{\ell m}(\theta_\pi^*, \varphi_\pi^*);$$

N.B. Spherical harmonics in are even under $\varphi_\pi^* \rightarrow -\varphi_\pi^*$.

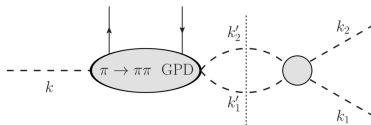
How to go beyond the threshold?

- Two regimes



(a) $\pi \rightarrow \pi\pi$ transition GPD in the chiral regime; (b) $\pi \rightarrow \pi\pi$ transition GPD in the resonance region;

- The **Watson'54** final state interaction theorem for $\pi \rightarrow \pi\pi$ transition GPD:



- The Omnes solution (for $N = 0$) $\delta_\ell^I(W_{\pi\pi}^2)$ are the $\pi\pi$ scattering phases:

$$\tilde{H}_{\ell,m}^I(x, \xi, W_{\pi\pi}^2) = \tilde{H}_{\ell,m}^I(x, \xi, W_{\pi\pi}^2 = 0) \exp \left[\frac{1}{\pi} \int_{4m_\pi^2}^{\infty} d\omega \frac{\delta_\ell^I(\omega)}{\omega - m_{\pi\pi}^2 - i\epsilon} \right]$$

- $\pi \rightarrow \pi\pi$ (and $N \rightarrow \pi N$) transition GPDs are complex functions above threshold!

What we want?

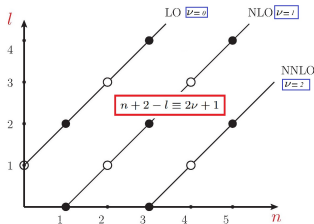
$$\int_{-1}^1 dx \, x H(x, \xi, t) = M_2(t) + \xi^2 \frac{4}{5} d_1(t);$$

What we get from experiment:

$$D(t) = \sum_{i \text{ odd}} d_i(t);$$

- Mellin moments expanded in PWs of the t -channel reaction $\gamma^* \gamma \rightarrow h \bar{h}$: M. Polyakov, A. Shuvaev'02, K. Kumericki, D. Müller, and K. Passek-Kumericki'08, D. Müller, M. Polyakov and K.S.'15;
Conformal PW expansion:

$$H_+(x, \xi, t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}(t) \theta \left(1 - \frac{x^2}{\xi^2} \right) \left(1 - \frac{x^2}{\xi^2} \right) C_n^{\frac{3}{2}} \left(\frac{x}{\xi} \right) P_l \left(\frac{1}{\xi} \right)$$

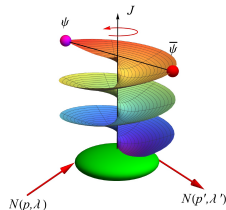


- Polynomiality implemented via the Wigner-Eckart theorem ($l \leq n+1$).
- Discrete symmetries (C, T) through the selection rules for I^{PC} (X. Ji, R. Lebed'01).
- Generalized FFs $B_{nl}(t)$ are renormalized multiplicatively.

Can we handle with the QCD string?

- How to decompose QCD string into probes of different spin- J ?

$$\bar{\Psi}(z)\gamma_{\mu}[z; 0]\Psi(0)|_{z \rightarrow 0} = z^{\nu} \underbrace{\bar{\Psi}\gamma_{\mu}\nabla_{\nu}\Psi}_{\text{Spin}-2} + z^{\nu}z^{\rho} \underbrace{\bar{\Psi}\gamma_{\mu}\nabla_{\nu}\nabla_{\rho}\Psi}_{\text{Spin}-3} + \dots$$



- Expansion of the Compton FFs in the $SO(3)$ PWs of the cross channel ($\gamma^*\gamma \rightarrow h\bar{h}$):

$$F_J(t) \equiv \frac{2J+1}{2} \int_{-1}^1 d(\cos \theta_t) P_J(\cos \theta_t) \mathcal{H}^{(+)}(\cos \theta_t, t); \quad \cos \theta_t = -\frac{1}{\xi\beta} + \mathcal{O}(1/Q^2).$$

- Froissart-Gribov projection (Gribov'61, Froissart'61) of the Compton FFs allows the calculation of $F_J(t)$ from the absorptive part of the amplitude (GPDs on the $x = \xi$ line);

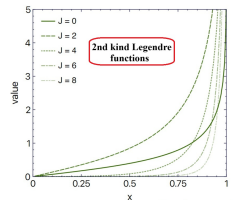
$$\text{Re } \mathcal{H}_+(\xi, t) \stackrel{\text{LO}}{=} \mathcal{P} \int_0^1 dx \frac{2x H_+(x, x, t)}{\xi^2 - x^2} + 4D(t).$$

- Froissart-Gribov projection in the context of DVCS: K. Kumericki, D. Müller, and K.

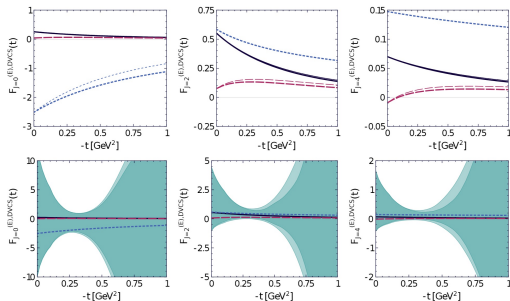
Passek-Kumericki, Eur. Phys. J. C 58, (2008); M. Polyakov, Phys. Lett. B 659, (2008);

$$F_J(t) = 2(2J+1) \int_0^1 dx \left[\frac{Q_J(1/x)}{x^2} - \frac{1}{x} \delta_{J,0} \right] H_+(x, x, t) + 4D(t) \delta_{J,0}.$$

- Possible benefits of $F_J(t)$ as observables?
- Shortcomings:
 - Analytic continuation in t ;
 - Target mass corrections ($\beta \neq 1$) mix J s;



First weight functions $2(2J+1) \left(\frac{Q_J(1/x)}{x^2} - \delta_{J,0} \frac{1}{x} \right)$



- GK (solid black), MMS (dashed red) and KM (dotted blue) GPD models;
- V.s. the global analysis of DVCS data, H. Moutarde, P. Sznajder, and J. Wagner,

◀ ◻ ▶ Eur. Phys. J. C 79, (2019);



Sum rules for the Mellin moments of GPDs

- Polynomiality property of GPDs ($h_{N,k}(t)$ are in one-to-one correspondence with $B_{n,l}(t)$)

$$\int_0^1 dx x^N H_+(x, \xi, t) = \sum_{\substack{k=0 \\ \text{even}}}^{N+1} h_{N,k}(t) \xi^k, \quad \text{for odd } N;$$

- The sum rule for $J = 0, 2$ FG projection truncated at $\nu = 1$ (“next-to-minimalist” contribution):

$$F_{J=0}(t) = 4(B_{1,0}(t) + \dots) = \frac{5}{3}h_{1,0}(t) + 5h_{1,2}(t) + \left\{ \begin{array}{l} \text{contribution of conformal PWs} \\ \text{with } \nu \geq 2 \end{array} \right\};$$

$$\begin{aligned} F_{J=2}(t) &= 4(B_{1,2}(t) + B_{3,2}(t) + \dots) \\ &= -\frac{7}{6}h_{1,0}(t) + 9h_{3,0}(t) + \frac{21}{2}h_{3,2}(t) + \left\{ \begin{array}{l} \text{contribution of conformal PWs} \\ \text{with } \nu \geq 2 \end{array} \right\}, \end{aligned}$$

- Coefficients $h_{N,k}$ can be studied with methods of lattice QCD. [A new way to connect lattice results to the data!](#)

On mixing due to target mass $\neq 0$ I

- The modification of the double PW expansion (dual parametrization) explicit inclusion of threshold $\beta \equiv \sqrt{1 - \frac{4m^2}{t}}$ corrections in the summation of t -channel spin- l exchanges:

$$H_+(x, \xi, t) = 2 \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} \beta^l \bar{B}_{n,l}(t) \theta \left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right) C_n^{3/2} \left(\frac{x}{\xi}\right) P_l \left(\frac{1}{\xi\beta}\right).$$

- The appropriate system of orthogonal polynomials for the CFF is still $P_l(\beta \cos \theta_t)$, (see the integration limits in the dispersive integral for the LO CFF: $[0; 1]$ and not $[0; 1/\beta]$).
- Reexpansion of $P_l(\cos \theta_t)$ back to $P_l(\beta \cos \theta_t) \equiv P_l\left(\frac{1}{\xi}\right)$.
- Some guiding principles:
 - Two expansions give the same coefficients $h_{N,k}(t)$ at power ξ^k of N -th Mellin moments of a GPD.
 - The coefficients $h_{N,k}(t)$ must be regular in the $t \rightarrow 0$ limit $\Rightarrow$ assume specific singularities for the generalized FFs $\bar{B}_{n,l}(t)$ at $t = 0$.

On mixing due to target mass $\neq 0$ II

- Consider $h_{N,k=0}(t)$: no modification for $\nu = 0$ contribution ($Q_0(y, t)$):

$$B_{n,n+1}(t) = \bar{B}_{n,n+1}(t);$$

- Consider $h_{N,k=2}(t)$: for $\nu = 1$ ($Q_2(y, t)$): admixture of higher $J = l + 2$ spin:

$$\underbrace{B_{n,n-1}(t)}_{\text{Occurs in } J = n - 1\text{-th FG projection}} = \underbrace{\bar{B}_{n,n-1}(t)}_{\text{spin } n-1} - (1 - \beta^2) \left(\frac{1}{2} - n \right) \underbrace{\bar{B}_{n,n+1}(t)}_{\text{spin } n+1}.$$

- In general, the mixing for $B_{l+2\nu-1,l}(t)$ involves higher spin contributions up to $l + 2\nu$.
- FG projections $F_J(t)$ get admixture from higher spins:

$$F_J(t) = 4 \int_0^1 dy x^{J-1} N(y, t) = 4 \left[\underbrace{B_{J-1,J}(t)}_{\text{pure spin-}J} + \underbrace{B_{J+1,J}(t)}_{\text{spin } J, J+2} + \underbrace{B_{J+3,J}(t)}_{\text{spin } J, J+2, J+4} + \dots \right]$$

- Mixing can be tamed once we may truncate summation in ν at some $\nu_{\max}$!
- This assumption can be tested through saturation of sum rules.

Can we treat the analytic properties in t better?



Dispersive evaluation of the D-term form factor in deeply virtual Compton scattering

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- Unsubtracted DR in t for the D -term FF at $\frac{s-u}{4m_N} = 0$:

$$4D^q(t) = \frac{1}{\pi} \int_{4m_\pi^2}^{+\infty} dt' \frac{\text{Im}_t F^q(0, t')}{t' - t} + \frac{1}{\pi} \int_{-\infty}^{-a} dt' \frac{\text{Im}_t F^q(0, t')}{t' - t};$$

- Phenomenological input for the absorptive part: $\pi\pi$ intermediate state

$$2 \text{Im } T^{\gamma^* \gamma \rightarrow N\bar{N}} = \frac{1}{(4\pi)^2} \frac{p_\pi}{\sqrt{t}} \int d\Omega_\pi \left[T^{\gamma^* \gamma \rightarrow \pi\pi} \right] \cdot \left[T^{\pi\pi \rightarrow N\bar{N}} \right]^*;$$

- Can one extend this kind of technique for the FG projection FFs?

Summary and Outlook

- 1 New tool for baryon spectroscopy: arbitrary spin- J probe and PW analysis of excited states;
- 2 Access to $N \rightarrow N^*$ EMT matrix elements: mechanical properties of resonances;
- 3 $N \rightarrow \pi N$, $N \rightarrow \eta N$ GPDs provide a lab for χ PT on the light cone: soft pion theorems and chiral expansion. Can we build a new bridge between the chiral regime and resonance physics?
- 4 GPD formalism worked out for $N \rightarrow \Delta(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$. Can be studied at JLab@12 GeV and an option for JLab@22 GeV;
- 5 Need to study for the kinematical conditions of EIC and EicC;
- 6 Froissart-Gribov projection as a means to focus on specific J in the cross channel.

Спасибо за внимание!