Связанно-связанные мюонные переходы в мюонных атомах с рождением электрон-позитронных пар

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Muon (free)

Muon is an elementary particle. It was descovered in 1936 by the American physicists Carl D. Anderson and Seth Neddermeyer.

$$\begin{aligned} \text{mass:} \quad & m_\mu = 207\, m_e = 106\,\mathrm{MeV}/c^2\,, \\ \text{spin:} \quad & s = \frac{1}{2}\,, \\ \text{lifetime:} \quad & \tau_\mu = 2 \times 10^{-6}\,\mathrm{s}\,. \end{aligned}$$

There is a negatively charged muon (μ^-) and its positevily charged antiparticle (μ^+). The charge of a muon is equal to the charge of an electron.

The muons are unstable and decay as

$$\mu^- \;\to\; e^- + \bar{\nu}_e + \nu_\mu\,, \qquad \mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu\,.$$

Muon (negatively charged) can be considered as a heavy electron.

Muon beams

- Muon station for sciEnce, technoLOgy and inDustrY (MELODY) at the China Spallation Neutron Source (CSNS)
  "Progress report on Muon Source Project at CSNS". Journal of Physics: Conference Series **2462**, 012034 (2023).
- The MICE Collaboration
  "Transverse emittance reduction in muon beams by ionization cooling". Nature Physics **20**, 1558 (2024).
- Petersburg Nuclear Physics Institute, Gatchina Synchrocyclotron SC 1000

Decay channels of bound muon ($2s$, $2p$ states)

- **Radiative transitions**

$$\mu_i \quad \rightarrow \quad \mu_f + \gamma \quad (+\gamma')$$

- **Transitions with production of an electron-positron pair (EPP)**

$$\mu_i \quad \rightarrow \quad \mu_f + e^- + e^+$$

- Auger transitions

$$\mu_i + e^-(\text{bound}) \quad \rightarrow \quad \mu_f + e^-(\text{continuum})$$

- Muon-induced electromagnetic excitation of an atomic nucleus
- Muon capture by an atomic nucleus

$$\mu_i + p^+ \quad \rightarrow \quad \nu_\mu + n \quad (+\gamma)\,.$$

Radiative bound-bound transitions in electron or muon ions

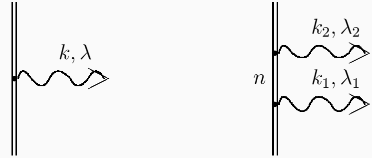
One- and two-photon transitions

Electron

$$e_i \rightarrow e_f + \gamma(\boldsymbol{k}, \epsilon)\,,$$
$$e_i \rightarrow e_f + \gamma(\boldsymbol{k}, \epsilon) + \gamma(\boldsymbol{k}', \epsilon')\,,$$

Muon

$$\mu_i \rightarrow \mu_f + \gamma(\boldsymbol{k}, \epsilon)\,,$$
$$\mu_i \rightarrow \mu_f + \gamma(\boldsymbol{k}, \epsilon) + \gamma(\boldsymbol{k}', \epsilon')\,,$$



(a) (b)

One-photon transitions

The photon wave function

$$A_0^{(k,\lambda)}(\boldsymbol{r}) = 0\,, \qquad \boldsymbol{A}^{(k,\lambda)}(\boldsymbol{r}) = \sqrt{\frac{2\pi}{\omega}}\boldsymbol{\epsilon}^{(k,\lambda)}e^{i\boldsymbol{kr}}\,.$$

The amplitude of one-photon emission

$$A_{fi}^{*(k,\lambda)} = -e\int d\boldsymbol{r}\,\psi_f^+(\boldsymbol{r})\boldsymbol{\alpha}\boldsymbol{A}^{*(k,\lambda)}(\boldsymbol{r})\psi_i(\boldsymbol{r})\,,$$

where $k = (\omega, \boldsymbol{k})$ – 4-vector of the photon momentum,
λ – 4-vector of polarization.

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix},$$

where $\boldsymbol{\sigma}$ are the Pauli matrices.

$$dW_{i\to f}^{(\lambda)} = 2\pi\left|A_{fi}^{*(k,\lambda)}\right|^2\delta(\varepsilon_i - \omega_1 - \varepsilon_f)\frac{d^3\boldsymbol{k}}{(2\pi)^3}\,,$$

$$dW_{i\to f}^{(\lambda)} = 2\pi\left|A_{fi}^{*(k,\lambda)}\right|^2\omega^2\frac{d^2\Omega_k}{(2\pi)^3}\,.$$

Two-photon transitions

The amplitude of two-photon emission

$$U_{i\to f}^{(k_1,\lambda_1;k_2,\lambda_2)} = \sum_n \frac{A_{fn}^{*(k_2,\lambda_2)} A_{ni}^{*(k_1,\lambda_1)}}{\varepsilon_i - \omega_1 - \varepsilon_n} .$$

The two-photon differential transition probability reads as

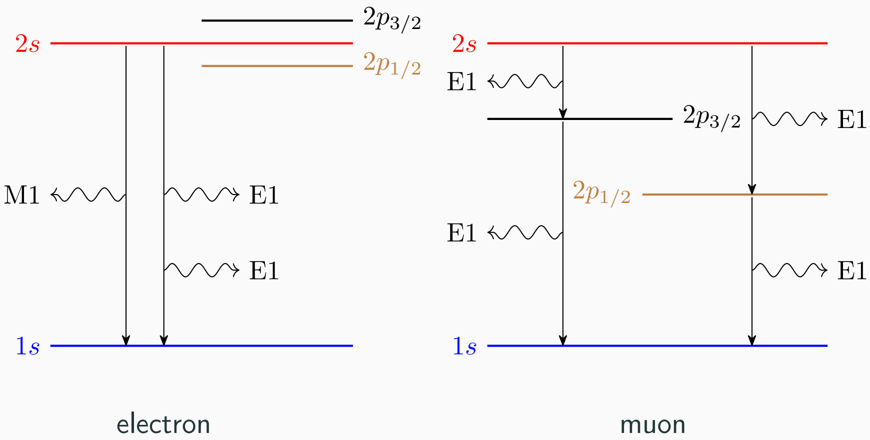
$$dW_{i\to f}^{(\lambda_1,\lambda_2)} = 2\pi \left| U_{i\to f}^{(k_1,\lambda_1;k_2,\lambda_2)} + U_{i\to f}^{(k_2,\lambda_2;k_1,\lambda_1)} \right|^2 \delta(\varepsilon_i - \omega_1 - \omega_2 - \varepsilon_f) \times \frac{d^3\boldsymbol{k}_1}{(2\pi)^3} \frac{d^3\boldsymbol{k}_2}{(2\pi)^3} .$$

After integration over one of the photon energies we obtain the following differential transition probability

$$\frac{dW_{fi}^{(\lambda_1,\lambda_2)}}{d^2\Omega_1 d^2\Omega_2 d\omega_1} = \frac{\omega_1^2\omega_2^2}{(2\pi)^5} \left| U_{fi}^{(k_1,\lambda_1;k_2,\lambda_2)} + U_{fi}^{(k_2,\lambda_2;k_1,\lambda_1)} \right|^2 ,$$

where $\Omega_{1,2}$ is the solid angle of the corresponding photon momentum. The energy conservation law: $\omega_1 + \omega_2 = \varepsilon_i - \varepsilon_f$.

Energy levels of electron and muon H-like ions



The energies and the electron orbital radii

TABLE VI. Bound energies and root-mean-square radii for electron ions. In the first two columns, the nuclear charge Z and the nuclear root-mean-square charge radii R (in fm) are given. In the next columns, the bound energies $E^{(e)} = \varepsilon^{(e)} - m_e c^2$ (in keV) and the root-mean-square radii $\langle\psi|r^2|\psi\rangle^{1/2}$ (in fm) are presented for the corresponding electron states.

Nucleus		1s		2s		$2p_{1/2}$		$2p_{3/2}$	
Z	R	$E^{(e)}$	$r^{(e)}$	$E^{(e)}$	$r^{(e)}$	$E^{(e)}$	$r^{(e)}$	$E^{(e)}$	$r^{(e)}$
1	0.8791	−1.360587283[−2]	91654.8	−3.401479529[−3]	342939.9	−3.401479530[−3]	289836.2	−3.401434246[−3]	289840.9
10	3.0053	−1.36238	9151.37	−0.34071	34233.1	−0.34071	28923.1	−0.34026	28970.1
50	4.6266	−3.52266[1]	1759.47	−8.88410	6543.62	−8.88437	5483.07	−8.57551	5725.50
92	5.860	−1.32081[2]	846.916	−3.41777[1]	3101.99	−3.42111[1]	2525.50	−2.96498[1]	3016.38
120	6.330	−2.59627[2]	543.913	−6.97852[1]	1960.51	−7.06350[1]	1500.79	−5.15841[1]	2236.49

TABLE VII. Bound energies and root-mean-square radii for muon ions. In the first two columns, the nuclear charge Z and the nuclear root-mean-square charge radii R (in fm) are given. In the next columns, the bound energies $E^{(\mu)} = \varepsilon^{(\mu)} - m_\mu c^2$ (in keV) and the root-mean-square radii $\langle\psi|r^2|\psi\rangle^{1/2}$ (in fm) are presented for the corresponding muon states.

Nucleus		1s		2s		$2p_{1/2}$		$2p_{3/2}$	
Z	R	$E^{(\mu)}$	$r^{(\mu)}$	$E^{(\mu)}$	$r^{(\mu)}$	$E^{(\mu)}$	$r^{(\mu)}$	$E^{(\mu)}$	$r^{(\mu)}$
1	0.8791	−2.53057	492.842	−6.32394[−1]	1844.61	−6.32192[−1]	1559.44	−6.32184[−1]	1559.46
10	3.0053	−2.77410[2]	44.9618	−6.97169[1]	167.328	−7.01762[1]	140.450	−7.00826[1]	140.677
50	4.6266	−5.23928[3]	11.6014	−1.53726[3]	37.7515	−1.81440[3]	27.0449	−1.76858[3]	27.8725
92	5.860	−1.21496[4]	8.88120	−4.32520[3]	24.4212	−5.93616[3]	15.4070	−5.70775[3]	16.0846
120	6.330	−1.68862[4]	8.16188	−6.65385[3]	20.5221	−9.46687[3]	12.7713	−9.10565[3]	13.3167

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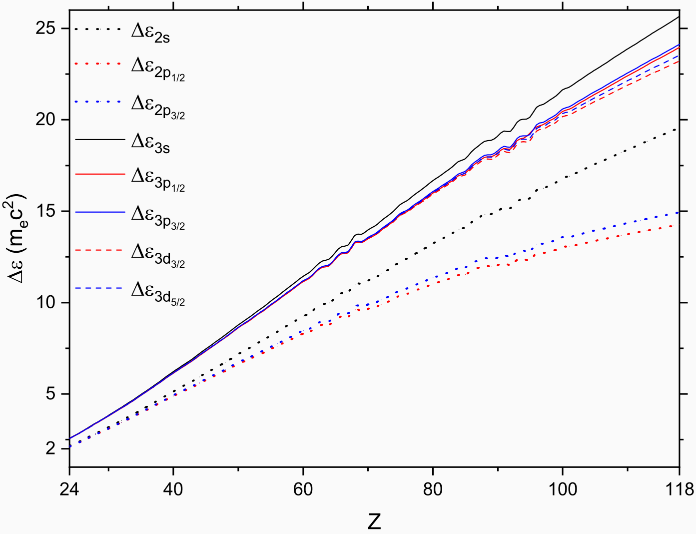
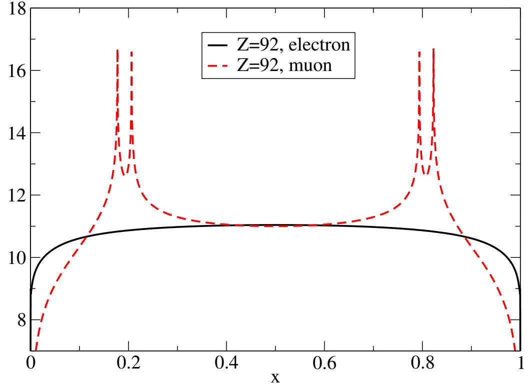


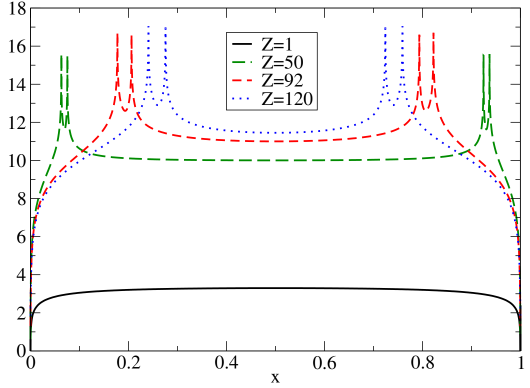
The energy released during the transitions to $1s$ in muon ions is represented (in units of $m_e c^2$) as a function of the atomic number Z. Bohr formula: $E_n = -m_e c^2 \frac{(\alpha Z)^2}{2n^2}$.

Two-photon transition probability



The differential transition probabilities (in $s^{-1}keV^{-1}$) for electron and muon ions for $Z = 92$ are presented as a function of the energy sharing parameter $x = \frac{\omega_1}{\varepsilon_{2s} - \varepsilon_{1s}}$. Differential transition probabilities are given on a logarithmic scale as $\log_{10}\left(\frac{dW^{(e,\mu)}}{d\omega_1}\right)$. $\omega_1 + \omega_2 = \varepsilon_{2s} - \varepsilon_{1s}$, $x_1 + x_2 = 1$,

Two-photon transition probability for muon



The differential transition probabilities for muon ions presented as a function of the energy sharing parameter $x = \frac{\omega_1}{\varepsilon_i - \varepsilon_f}$. The differential transition probabilities are given on a logarithmic scale as $\log_{10}\left(\frac{dW^{(\mu)}}{d\omega_1}\right)$.

Two-photon transition probabilities for electron

TABLE III. Transition probabilities $W^{(e)}$ (in s^{-1}) for one- and two-photon $2s \to 1s$ transitions in one-electron ions. The value of p_W shows the power dependence on Z of the corresponding transition probability ($W^{(e)} \sim Z^{p_W}$). In the first column the atomic number of the ion (Z) is indicated. In the next two columns the one-photon transition probabilities and their power dependence on Z are given. In the columns marked "$E1E1$: $2s \to 1s$" we give the two-photon transition probabilities with emission of $E1E1$ photons ($W^{(e)}$, in s^{-1}) and the corresponding results of Ref. [9] ($W^{(e)a}$) together with their power dependence on Z. The columns for "$2s \to 1s$, total 2γ" present the results of the exact calculation of transition probabilities: $W^{(e)}$, the total transition probability; non-spin-flip and spin-flip, transition probabilities in which the initial state does not change or changes the projection of the total angular momentum, respectively. We note that the spin flip for the one-photon $M1$ transition is $\frac{2}{3}$ of the total transition probability $W^{(e)}$, while the non-spin-flip is $\frac{1}{3}$ of $W^{(e)}$.

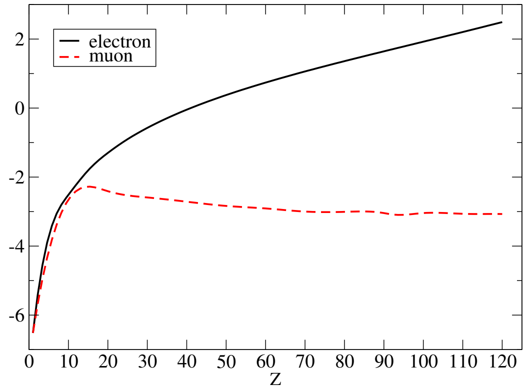
Nucleus	$M1$: $2s \to 1s$		$E1E1$: $2s \to 1s$			$2s \to 1s$, total 2γ					
Z	$W^{(e)}$	p_W	$W^{(e)}$	$W^{(e)a}$	p_W	$W^{(e)}$	p_W	Non-spin-flip	p_{nsf}	Spin flip	p_{sf}
1	2.49592[−6]	10.00	8.22906	8.22906[a]	6.00	8.22906	6.00	8.22906	6.00	3.88291[−9]	9.99
10	2.51003[4]	10.01	8.20064[6]	8.1923[6][a]	5.99	8.20065[6]	5.99	8.20061[6]	5.99	3.15349[1]	9.64
20	2.61488[7]	10.05	5.19513[8]	5.1901[8][a]	5.97	5.19515[8]	5.97	5.19492[8]	5.97	2.30865[4]	9.43
30	1.55241[9]	10.11	5.82109[9]	5.8151[9][a]	5.94	5.82125[9]	5.94	5.82019[9]	5.94	1.05200[6]	9.36
40	2.87414[10]	10.20	3.19862[10]	3.1954[10][a]	5.90	3.19889[10]	5.90	3.19735[10]	5.90	1.54080[7]	9.28
50	2.82905[11]	10.32	1.18662[11]	1.1854[11][a]	5.84	1.18686[11]	5.84	1.18565[11]	5.84	1.21404[8]	9.21
60	1.87950[12]	10.48	3.42645[11]	3.4229[11][a]	5.78	3.42797[11]	5.78	3.42150[11]	5.78	6.47328[8]	9.15
64	3.70310[12]	10.56	4.97148[11]		5.75	4.97436[11]	5.75	4.96269[11]	5.74	1.16734[9]	9.11
70	9.58288[12]	10.69	8.30599[11]	8.2975[11][a]	5.70	8.31297[11]	5.70	8.28657[11]	5.69	2.63989[9]	9.09
80	4.05532[13]	10.96	1.76726[12]	1.7655[12][a]	5.59	1.76988[12]	5.60	1.76102[12]	5.58	8.85741[9]	9.04
90	1.50037[14]	11.35	3.39348[12]	3.3899[12][a]	5.46	3.40186[12]	5.47	3.37619[12]	5.44	2.56687[10]	9.03
92	1.92408[14]	11.44	3.82557[12]	3.8216[12][a]	5.43	3.83600[12]	5.44	3.80469[12]	5.41	3.13168[10]	9.00
100	5.04074[14]	11.79	5.98484[12]	5.9782[12][a]	5.28	6.00879[12]	5.30	5.94218[12]	5.25	6.66209[10]	9.15
110	1.58119[15]	12.45	9.80101[12]		5.04	9.86357[12]	5.07	9.70146[12]	4.99	1.62121[11]	9.87
118	3.81066[15]	12.81	1.38978[13]		5.02	1.40264[13]	5.02	1.36779[13]	4.80	3.48470[11]	13.40
120	4.72890[15]	12.94	1.51115[13]		5.06	1.52650[13]	5.12	1.48250[13]	4.80	4.39623[11]	15.44

Two-photon transition probabilities for muon

TABLE IV. Transition probabilities $W^{(\mu)}$ (in s^{-1}) for the one- and two-photon decay of the $2s$ state of one-muon ions. The values $W_0^{(\mu)}$ are calculated with the pointlike nucleus. The values $W^{(\mu)}$ are calculated with the Fermi distribution of the nuclear charge density (the nuclear recoil and the vacuum polarization corrections are also taken into account). The notation is the same as in Table II.

Nucleus	$M1$: $2s \to 1s$		$E1$: $2s \to 2p_{1/2}$	$E1$: $2s \to 2p_{3/2}$	$2s \to 1s$, total 2γ	
Z	$W_0^{(\mu)}$	$W^{(\mu)}$	$W^{(\mu)}$	$W^{(\mu)}$	$W_0^{(\mu)}$	$W^{(\mu)}$
1	5.16078[−4]	4.66233[−4]	2.25265	5.09542	1.70151[3]	1.53071[3]
2	5.28554[−1]	5.18875[−1]	1.53770[2]	4.16282[2]	1.08885[5]	1.06174[5]
5	5.04671[3]	4.99412[3]	4.66810[3]	1.88840[2]	2.65636[7]	2.64451[7]
		5[3][a]	1[4][a]			3[7][a]
10	5.18997[6]	4.72237[6]	2.17920[8]	2.20546[8]	1.69563[9]	2.12691[9]
		5[6][a]	1[9][a]			2[9][a]
20	5.40686[9]	3.44581[9]	3.62043[11]	4.17798[11]	1.07420[11]	8.84592[11]
30	3.21013[11]	1.17226[11]	2.02420[13]	2.42670[13]	1.20366[12]	4.56417[13]
		1[11][a]	5[13][a]			4[11][a]
40	5.94395[12]	1.14696[12]	2.65386[14]	3.21458[14]	6.61441[12]	5.92612[14]
50	5.85222[13]	5.56180[12]	1.69154[15]	2.09944[15]	2.45416[13]	3.80941[15]
60	3.89006[14]	1.82667[13]	6.50114[15]	8.25557[15]	7.08851[13]	1.47996[16]
70	1.98575[15]	4.33285[13]	1.85809[16]	2.45839[16]	1.71910[14]	4.32417[16]
80	8.42309[15]	9.37768[13]	4.05345[16]	5.46345[16]	3.66026[14]	9.52912[16]
90	3.13278[16]	1.66535[14]	7.67106[16]	1.06633[17]	7.03501[14]	1.83509[17]
92	4.02531[16]	1.72935[14]	8.64745[16]	1.22282[17]	7.93361[14]	2.08923[17]
100	1.06466[17]	2.80172[14]	1.27813[17]	1.81081[17]	1.24176[15]	3.09106[17]
110	3.43275[17]	4.13532[14]	1.95286[17]	2.83513[17]	2.03054[15]	4.79044[17]
118	8.68675[17]	5.49249[14]	2.60176[17]	3.83097[17]	2.84567[15]	6.43540[17]
120	1.09894[18]	5.85407[14]	2.77829[17]	4.10504[17]	3.06912[15]	6.88604[17]

Ration one/two - photon transition probability



The ratio between the one-photon and two-photon transition probabilities for electron (black solid curve) and muon (red dashed line) ions presented as function of the nuclear charge Z: $\log_{10}\left(W_{1\mathrm{ph}}^{(e,\mu)}/W_{2\mathrm{ph}}^{(e,\mu)}\right)$.

Bound-bound transitions in muonic ions

The bound-bound transitions in the muonic ion, initialy being in a state $i = 2s$ (or $i = 2p$) to the final state $f = 1s$, with the emission of "some particles" can be described as

$$\mu_i \quad \rightarrow \quad \mu_f + \Delta\varepsilon^{(\mu)}_{i\rightarrow f}\Big(\ldots\text{some particles}\ldots\Big)\,.$$

The energy release during the transition can be estimated by the (nonrelativistic) Bohr formula for the energy levels of H-like ions

$$\begin{aligned} E^{(\mathrm{Bohr})}_{\mu_n} &= -\,m_\mu c^2\frac{(\alpha Z)^2}{2n^2}\,, \\ \Delta\varepsilon^{(\mu)}_{i\rightarrow f} &\approx E^{(\mathrm{Bohr})}_{\mu_i} - E^{(\mathrm{Bohr})}_{\mu_f}\,, \end{aligned}$$

where m_μ is the mass of the muon ($m_\mu \approx 207 m_e$, where m_e is the electron mass), c is the speed of light, $\alpha \approx \frac{1}{137}$ is the fine-structure constant.

Bound-bound EPP transitions in muonic ions

Accordingly, the estimate of the energy release during the $2s \to 1s$ transition is

$$\begin{aligned} \Delta\varepsilon^{(\mu)}_{2s\to 1s} &\approx E^{(\mathrm{Bohr})}_{\mu_{2s}} - E^{(\mathrm{Bohr})}_{\mu_{1s}} \\ &= m_\mu c^2 (\alpha Z)^2 \frac{3}{8} > 2m_e c^2 \,. \end{aligned}$$

This shows that if the atomic number Z is larger or equal to 22 (the more precise calculation predicts $Z \geq 24$), then the energy release becomes greater than $2m_e c^2$. This energy is enough to produce an electron-positron pair (EPP): an electron with energy $\varepsilon > m_e c^2$ and its antiparticle – a positron with energy $\varepsilon' > m_e c^2$.

This process was first discussed by Wheeler [J.A. Wheeler, *"Some consequences of the electromagnetic interaction between μ^--mesons and nuclei"*, Rev. Mod. Phys. **21**, 133 (1949).].

Bound-bound EPP transitions in muonic ions

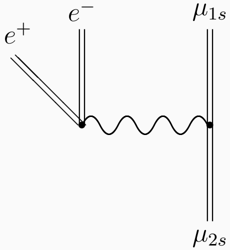
The nonradiative transition, in which an electron-positron pair is produced, can be described as

$$\mu_i \quad \rightarrow \quad \mu_f + e^-(\varepsilon, \boldsymbol{p}, \mu) + e^+(\varepsilon', \boldsymbol{p}', \mu')\,.$$

The produced electron is described by its energy ($\varepsilon = \sqrt{m_e^2 c^4 + c^2 \boldsymbol{p}^2}$), momentum ($\boldsymbol{p}$) and polarization ($\mu$) (the spin projection on the $\boldsymbol{p}$ direction). The positron is described by the energy ($\varepsilon' = \sqrt{m_e^2 c^4 + c^2 \boldsymbol{p}'^2}$), momentum ($\boldsymbol{p}'$) and polarization ($\mu'$) (the spin projection on the direction $\boldsymbol{p}'$). The energy conservation law reads

$$\Delta\varepsilon^{(\mu)}_{2s\rightarrow 1s} = \varepsilon^{(\mu)}_{2s} - \varepsilon^{(\mu)}_{1s} \quad = \quad \varepsilon + \varepsilon' > 2 m_e c^2\,.$$

The amplitude



$$A = e^2 \int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \psi^{(-)+}_{\varepsilon,\boldsymbol{p}\mu}(\boldsymbol{r}_1)\psi^{(\mu)+}_f(\boldsymbol{r}_2)(1-\boldsymbol{\alpha}^{(1)}\boldsymbol{\alpha}^{(2)}) \times \frac{e^{i\frac{\omega r_{12}}{c}}}{r_{12}}\psi^{(-)}_{-\varepsilon',-\boldsymbol{p}'\mu'}(\boldsymbol{r}_1)\psi^{(\mu)}_i(\boldsymbol{r}_2)\,,$$

$$\psi^{(\pm)}_{\varepsilon,\boldsymbol{p}\mu}(\boldsymbol{r}) = \frac{(2\pi\hbar)^{3/2}c}{\sqrt{p|\varepsilon|}}\sum_{jlm}[\Omega^+_{jlm}(\hat{\boldsymbol{p}})v_\mu(\hat{\boldsymbol{p}})]e^{i\phi^{(\pm)}_{\varepsilon jl}}i^l\psi_{\varepsilon jlm}(\boldsymbol{r})\,.$$

Differential EPP transition probability

The transition probability can be written as

$$dw^{(e^+e^-)} = \frac{2\pi}{\hbar}\delta(\varepsilon_{2s} - \varepsilon_{1s} - \varepsilon - \varepsilon')|A|^2 \times \frac{d^3\boldsymbol{p}}{(2\pi\hbar)^3}\frac{d^3\boldsymbol{p}'}{(2\pi\hbar)^3}\,.$$

The integration over the positron momentum angles and the azimuthal angle of the electron momentum gives a factor $8\pi^2$. Averaging over the projection of the total angular momentum of the initial state of muon ($j_i^{(\mu)}$), the differential transition probability can be written as

$$\frac{d^2w^{(e^+e^-)}}{d\varepsilon\,\sin\theta d\theta} = \frac{pp'\varepsilon\varepsilon'}{4\pi^3(2j_i^{(\mu)}+1)\hbar^7c^4}|A|^2\,,$$

where θ is the angle between the electron and positron momenta.

Transition probabilities

Transition probabilities (in s^{-1}) for one-muon ions. The digits in square brackets indicate powers of 10. The first row lists the initial (excited) states (i). The total energies of the $1s$-muon states ($\varepsilon_{1s}^{(\mu)}$), including the rest mass of muon, are given explicitly for each Z value. The rows labeled $\Delta\varepsilon_{i\to 1s}^{(\mu)}$ show the energy differences $\Delta\varepsilon_{i\to 1s}^{(\mu)} = \varepsilon_i^{(\mu)} - \varepsilon_{1s}^{(\mu)}$ (in $m_e c^2$), where $\varepsilon_i^{(\mu)}$ represent the total energies of the initial (i) muon states. The rows W_i represent the total radiative transition probability for transitions from state i to all lower states. The rows $w_{i\to 1s}^{(e^+e^-)}$ provide the transition probabilities for the nonradiative electron-positron pair production transitions (from excited state i to the $1s$ muon state), while $w_{i\to 1s}^{(e^+e^-)\mathrm{C}}$ gives these transition probabilities considering only the Coulomb interaction. The rows $w_{i\to n\to 1s}^{(\gamma,e^+e^-)}$ present the transition probabilities for radiative cascade transitions $i \to n \to 1s$ involving electron-positron pair production, where i refers to the states listed in the first row.

	$2p_{1/2}$	$2p_{3/2}$	$2s_{1/2}$	$3p_{1/2}$	$3d_{3/2}$	$3p_{3/2}$	$3d_{5/2}$	$3s_{1/2}$
			$Z = 36$,	$\varepsilon_{1s}^{(\mu)} = 200.7596$				
$\Delta\varepsilon_{i\to 1s}^{(\mu)}$	4.1839	4.2119	4.3563	5.2018	5.2093	5.2099	5.2124	5.2536

Transition probabilities

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	$2p_{1/2}$	$2p_{3/2}$	$2s_{1/2}$	$3p_{1/2}$	$3d_{3/2}$	$3p_{3/2}$	$3d_{5/2}$	$3s_{1/2}$
$Z = 36$, $\varepsilon_{1s}^{(\mu)} = 200.7596$								
$\Delta\varepsilon_{i\to 1s}^{(\mu)}$	4.1839	4.2119	4.3563	5.2018	5.2093	5.2099	5.2124	5.2536
W_i	1.75[17]	1.74[17]	2.46[14]	4.96[16]	2.34[16]	5.04[16]	2.29[16]	6.91[14]
$w_{i\to 1s}^{(e^+e^-)}$	1.20[14]	1.23[14]	1.62[11]	4.08[13]	1.53[11]	4.28[13]	1.52[11]	7.07[10]

Transition probabilities

Transition probabilities (in s^{-1}) for one-muon ions. The digits in square brackets indicate powers of 10. The first row lists the initial (excited) states (i). The total energies of the $1s$-muon states ($\varepsilon_{1s}^{(\mu)}$), including the rest mass of muon, are given explicitly for each Z value. The rows labeled $\Delta\varepsilon_{i\to 1s}^{(\mu)}$ show the energy differences $\Delta\varepsilon_{i\to 1s}^{(\mu)} = \varepsilon_{i}^{(\mu)} - \varepsilon_{1s}^{(\mu)}$ (in $m_e c^2$), where $\varepsilon_{i}^{(\mu)}$ represent the total energies of the initial (i) muon states. The rows W_i represent the total radiative transition probability for transitions from state i to all lower states. The rows $w_{i\to 1s}^{(e^+e^-)}$ provide the transition probabilities for the nonradiative electron-positron pair production transitions (from excited state i to the $1s$ muon state), while $w_{i\to 1s}^{(e^+e^-)\mathrm{C}}$ gives these transition probabilities considering only the Coulomb interaction. The rows $w_{i\to n\to 1s}^{(\gamma,e^+e^-)}$ present the transition probabilities for radiative cascade transitions $i \to n \to 1s$ involving electron-positron pair production, where i refers to the states listed in the first row.

	$2p_{1/2}$	$2p_{3/2}$	$2s_{1/2}$	$3p_{1/2}$	$3d_{3/2}$	$3p_{3/2}$	$3d_{5/2}$	$3s_{1/2}$
				$Z = 36$, $\varepsilon_{1s}^{(\mu)} = 200.7596$				
$\Delta\varepsilon_{i\to 1s}^{(\mu)}$	4.1839	4.2119	4.3563	5.2018	5.2093	5.2099	5.2124	5.2536
W_i	1.75[17]	1.74[17]	2.46[14]	4.96[16]	2.34[16]	5.04[16]	2.29[16]	6.91[14]
$w_{i\to 1s}^{(e^+e^-)}$	**1.20[14]**	**1.23[14]**	**1.62[11]**	4.08[13]	1.53[11]	4.28[13]	1.52[11]	7.07[10]
$w_{i\to 1s}^{(e^+e^-)\mathrm{C}}$	**2.47[13]**	**2.55[13]**	**1.61[11]**	7.27[12]	2.96[10	7.74[12]	3.00[10]	7.04[10]

Transition probabilities

Transition probabilities (in s^{-1}) for one-muon ions. The digits in square brackets indicate powers of 10. The first row lists the initial (excited) states (i). The total energies of the $1s$-muon states ($\varepsilon_{1s}^{(\mu)}$), including the rest mass of muon, are given explicitly for each Z value. The rows labeled $\Delta\varepsilon_{i\to 1s}^{(\mu)}$ show the energy differences $\Delta\varepsilon_{i\to 1s}^{(\mu)} = \varepsilon_{i}^{(\mu)} - \varepsilon_{1s}^{(\mu)}$ (in $m_e c^2$), where $\varepsilon_{i}^{(\mu)}$ represent the total energies of the initial (i) muon states. The rows W_i represent the total radiative transition probability for transitions from state i to all lower states. The rows $w_{i\to 1s}^{(e^+e^-)}$ provide the transition probabilities for the nonradiative electron-positron pair production transitions (from excited state i to the $1s$ muon state), while $w_{i\to 1s}^{(e^+e^-)\mathrm{C}}$ gives these transition probabilities considering only the Coulomb interaction. The rows $w_{i\to n\to 1s}^{(\gamma,e^+e^-)}$ present the transition probabilities for radiative cascade transitions $i \to n \to 1s$ involving electron-positron pair production, where i refers to the states listed in the first row.

	$2p_{1/2}$	$2p_{3/2}$	$2s_{1/2}$	$3p_{1/2}$	$3d_{3/2}$	$3p_{3/2}$	$3d_{5/2}$	$3s_{1/2}$
			$Z = 36$,	$\varepsilon_{1s}^{(\mu)} = 200.7596$				
$\Delta\varepsilon_{i\to 1s}^{(\mu)}$	4.1839	4.2119	4.3563	5.2018	5.2093	5.2099	5.2124	5.2536
W_i	1.75[17]	1.74[17]	2.46[14]	4.96[16]	2.34[16]	5.04[16]	2.29[16]	6.91[14]
$w_{i\to 1s}^{(e^+e^-)}$	1.20[14]	1.23[14]	**1.62[11]**	4.08[13]	1.53[11]	4.28[13]	1.52[11]	7.07[10]
$w_{i\to 2p_{1/2}\to 1s}^{(\gamma,e^+e^-)}$		2.20[5]	**7.60[10]**	2.78[5]	1.33[13]	3.84[9]	2.32[7]	1.23[11]
$w_{i\to 2p_{3/2}\to 1s}^{(\gamma,e^+e^-)}$			**9.59[10]**	7.79[9]	2.67[12]	3.85[9]	1.60[13]	3.35[11]

Transition probabilities

Transition probabilities (in s^{-1}) for one-muon ions. The digits in square brackets indicate powers of 10. The first row lists the initial (excited) states (i). The total energies of the 1s-muon states ($\varepsilon_{1s}^{(\mu)}$), including the rest mass of muon, are given explicitly for each Z value. The rows labeled $\Delta\varepsilon_{i\to 1s}^{(\mu)}$ show the energy differences $\Delta\varepsilon_{i\to 1s}^{(\mu)} = \varepsilon_i^{(\mu)} - \varepsilon_{1s}^{(\mu)}$ (in $m_e c^2$), where $\varepsilon_i^{(\mu)}$ represent the total energies of the initial (i) muon states. The rows W_i represent the total radiative transition probability for transitions from state i to all lower states. The rows $w_{i\to 1s}^{(e^+e^-)}$ provide the transition probabilities for the nonradiative electron-positron pair production transitions (from excited state i to the 1s muon state), while $w_{i\to 1s}^{(e^+e^-)\mathrm{C}}$ gives these transition probabilities considering only the Coulomb interaction. The rows $w_{i\to n\to 1s}^{(\gamma,e^+e^-)}$ present the transition probabilities for radiative cascade transitions $i \to n \to 1s$ involving electron-positron pair production, where i refers to the states listed in the first row.

	$2p_{1/2}$	$2p_{3/2}$	$2s_{1/2}$	$3p_{1/2}$	$3d_{3/2}$	$3p_{3/2}$	$3d_{5/2}$	$3s_{1/2}$
$Z = 36$, $\varepsilon_{1s}^{(\mu)} = 200.7596$								
$\Delta\varepsilon_{i\to 1s}^{(\mu)}$	4.1839	4.2119	4.3563	5.2018	5.2093	5.2099	5.2124	5.2536
W_i	1.75[17]	1.74[17]	2.46[14]	4.96[16]	2.34[16]	5.04[16]	2.29[16]	6.91[14]
$w_{i\to 1s}^{(e^+e^-)}$	1.20[14]	1.23[14]	1.62[11]	4.08[13]	1.53[11]	4.28[13]	1.52[11]	7.07[10]
$w_{i\to 1s}^{(e^+e^-)\mathrm{C}}$	2.47[13]	2.55[13]	1.61[11]	7.27[12]	2.96[10	7.74[12]	3.00[10]	7.04[10]
$w_{i\to 2p_{1/2}\to 1s}^{(\gamma,e^+e^-)}$		2.20[5]	7.60[10]	2.78[5]	1.33[13]	3.84[9]	2.32[7]	1.23[11]
$w_{i\to 2p_{3/2}\to 1s}^{(\gamma,e^+e^-)}$			9.59[10]	7.79[9]	2.67[12]	3.85[9]	1.60[13]	3.35[11]
$w_{i\to 2s\to 1s}^{(\gamma,e^+e^-)}$				5.75[12]	9.50[9]	5.57[12]	9.66[9]	4.74[5]

O.Yu. Andreev, D. Yu, K.N. Lyashchenko, D.M. Vasileva, Chinese Physics Letters **42**, 090301 (2025)

Transition probabilities

	$2p_{1/2}$	$2p_{3/2}$	$2s_{1/2}$	$3p_{1/2}$	$3d_{3/2}$	$3p_{3/2}$	$3d_{5/2}$	$3s_{1/2}$
$Z = 24\,, \quad \varepsilon_{1s}^{(\mu)} = 203.8292$								
$\Delta\varepsilon_{i\to 1s}^{(\mu)}$	2.1362	2.1421	2.1717	2.5833	2.5852	2.5851	2.5858	2.5939
W_i	3.97[16]	3.94[16]	4.95[12]	1.17[16]	4.54[15]	1.17[16]	4.49[15]	2.78[14]
$w_{i\to 1s}^{(e^+e^-)}$	2.14[11]	2.42[11]	6.23[7]	1.19[12]	6.73[8]	1.22[12]	6.77[8]	5.71[8]
$w_{1s}^{(e^+e^-)\mathrm{C}}$	7.65[10]	8.55[10]	6.22[7]	3.35[11]	2.21[8]	3.45[11]	2.23[8]	5.71[8]
$w_{i\to 2p_{1/2}\to 1s}^{(\gamma,e^+e^-)}$		1.67[1]	1.22[7]	3.71[1]	2.04[10]	2.61[6]	6.79[3]	4.70[8]
$w_{i\to 2p_{3/2}\to 1s}^{(\gamma,e^+e^-)}$			1.63[7]	5.90[6]	4.57[9]	2.93[6]	2.75[10]	1.17[9]
$w_{i\to 2s\to 1s}^{(\gamma,e^+e^-)}$				2.10[10]	2.10[7]	2.05[10]	2.11[7]	2.27[2]
$Z = 36\,, \quad \varepsilon_{1s}^{(\mu)} = 200.7596$								
$\Delta\varepsilon_{i\to 1s}^{(\mu)}$	4.1839	4.2119	4.3563	5.2018	5.2093	5.2099	5.2124	5.2536
W_i	1.75[17]	1.74[17]	2.46[14]	4.96[16]	2.34[16]	5.04[16]	2.29[16]	6.91[14]
$w_{i\to 1s}^{(e^+e^-)}$	1.20[14]	1.23[14]	1.62[11]	4.08[13]	1.53[11]	4.28[13]	1.52[11]	7.07[10]
$w_{i\to 1s}^{(e^+e^-)\mathrm{C}}$	2.47[13]	2.55[13]	1.61[11]	7.27[12]	2.96[10	7.74[12]	3.00[10]	7.04[10]
$w_{i\to 2p_{1/2}\to 1s}^{(\gamma,e^+e^-)}$		2.20[5]	7.60[10]	2.78[5]	1.33[13]	3.84[9]	2.32[7]	1.23[11]
$w_{i\to 2p_{3/2}\to 1s}^{(\gamma,e^+e^-)}$			9.59[10]	7.79[9]	2.67[12]	3.85[9]	1.60[13]	3.35[11]
$w_{i\to 2s\to 1s}^{(\gamma,e^+e^-)}$				5.75[12]	9.50[9]	5.57[12]	9.66[9]	4.74[5]

Transition probabilities for $2s$-muon state

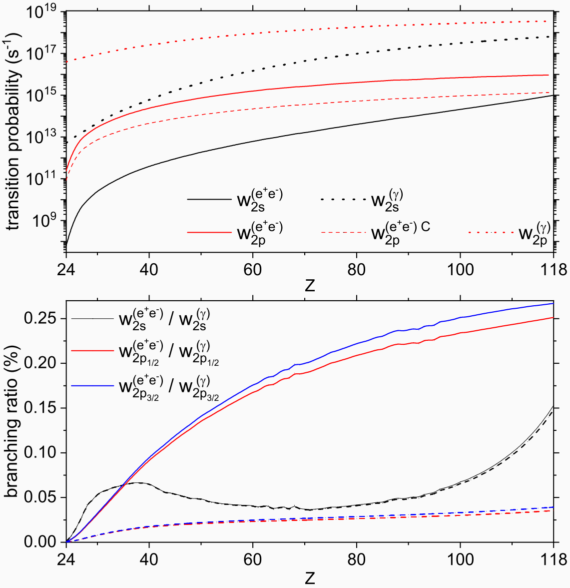
	$2p_{1/2}$	$2p_{3/2}$	$2s_{1/2}$	$3p_{1/2}$	$3d_{3/2}$	$3p_{3/2}$	$3d_{5/2}$	$3s_{1/2}$
$Z = 54\,, \quad \varepsilon^{(\mu)}_{1s} = 195.2742$								
$\Delta\varepsilon^{(\mu)}_{i\to 1s}$	7.3448	7.4593	8.0501	9.6624	9.6805	9.6941	9.6964	9.8765
W_i	6.47[17]	6.54[17]	7.08[15]	1.69[17]	1.21[17]	1.79[17]	1.15[17]	1.44[15]
$w^{(e^+e^-)}_{i\to 1s}$	9.60[14]	1.01[15]	3.04[12]	2.41[14]	3.75[12]	2.71[14]	3.66[12]	1.15[12]
$w^{(e^+e^-)\mathrm{C}}_{i\to 1s}$	1.42[14]	1.52[14]	3.01[12]	3.05[13]	4.72[11]	3.55[13]	4.84[11]	1.13[12]
$w^{(\gamma,e^+e^-)}_{i\to 2p_{1/2}\to 1s}$		3.22[7]	4.64[12]	3.15[7]	1.48[14]	9.54[10]	1.34[9]	4.54[9]
$w^{(\gamma,e^+e^-)}_{i\to 2p_{3/2}\to 1s}$			6.13[12]	1.91[11]	2.90[13]	9.42[10]	1.75[14]	3.19[11]
$w^{(\gamma,e^+e^-)}_{i\to 2s\to 1s}$				1.81[13]	4.06[10]	1.76[13]	4.26[10]	9.72[6]
$Z = 92\,, \quad \varepsilon^{(\mu)}_{1s} = 183.183$								
$\Delta\varepsilon^{(\mu)}_{i\to 1s}$	12.160	12.607	15.314	18.579	18.392	18.692	18.521	19.561
W_i	2.13[18]	2.32[18]	2.09[17]	4.88[17]	9.50[17]	5.71[17]	8.85[17]	6.21[16]
$w^{(e^+e^-)}_{i\to 1s}$	4.72[15]	5.51[15]	1.05[14]	7.28[14]	1.02[14]	9.96[14]	9.84[13]	4.10[13]
$w^{(e^+e^-)\mathrm{C}}_{i\to 1s}$	6.04[14]	7.22[14]	1.02[14]	8.43[13]	1.02[14]	9.66[12]	1.20[14]	3.95[13]
$w^{(\gamma,e^+e^-)}_{i\to 2p_{1/2}\to 1s}$		2.78[9]	1.92[14]	3.70[9]	1.71[15]	3.08[12]	1.10[11]	2.56[13]
$w^{(\gamma,e^+e^-)}_{i\to 2p_{3/2}\to 1s}$			2.91[14]	6.09[12]	3.24[14]	3.03[12]	2.00[15]	2.71[13]
$w^{(\gamma,e^+e^-)}_{i\to 2s\to 1s}$				1.15[14]	2.16[11]	1.20[14]	2.72[11]	6.74[8]

Transition probabilities involving electron-positron pair production for $2s \to 1s$

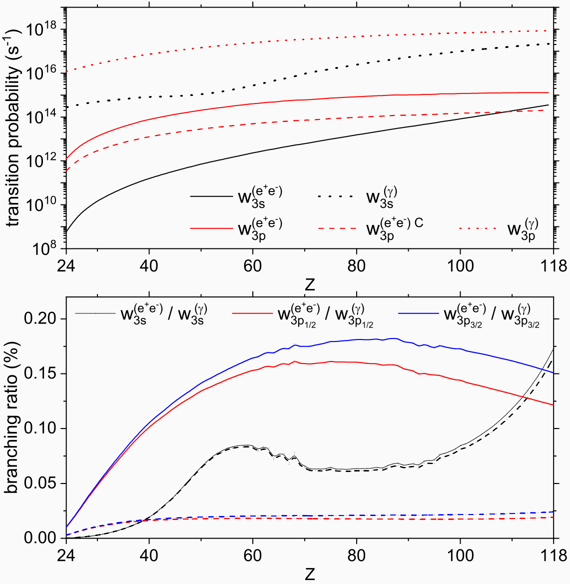
Z	25	30	45	40
J.A. Wheeler	2[8]	5[10]	8[11]	5[12]
Our work	5.54[8]	2.46[10]	1.25[11]	3.87[11]

J. A. Wheeler, "*Some consequences of the electromagnetic interaction between μ^--mesons and nuclei*", Reviews of Modern Physics **21**, 133 (1949).

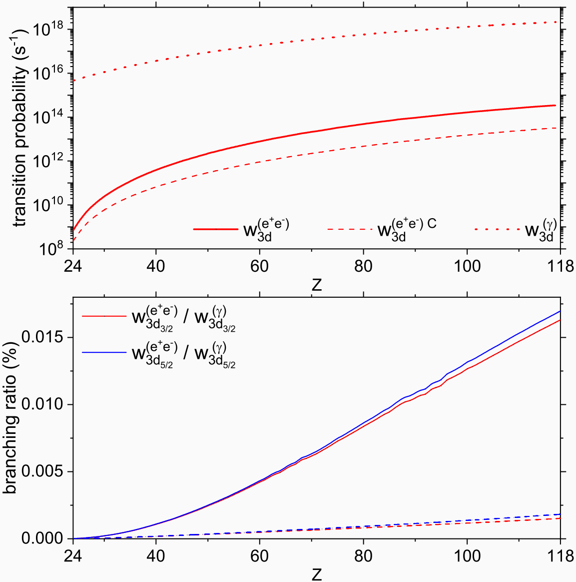
EPP transition probability for $2s$ and $2p$ states



EPP transition probability for $3s$ and $3p$ states



EPP transition probability for *d* states



The energy sharing parameter

It is convenient to express the energy of the emitted electron via the energy sharing parameter

$$x(\varepsilon) \;=\; \frac{\varepsilon - \varepsilon'}{\varepsilon^{(\mu)}_{2s} - \varepsilon^{(\mu)}_{1s} - 2m_e c^2}\,,$$

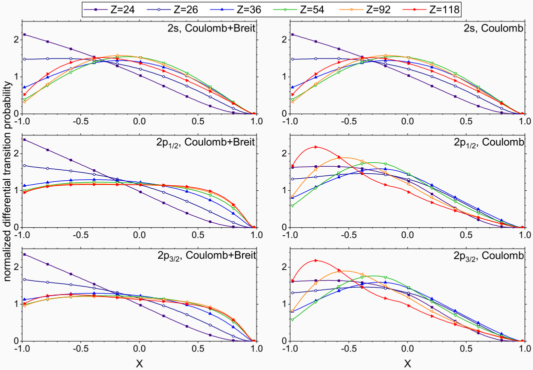
where ε is the energy of the emitted electron, the energy of the emitted positron (ε') is determined by

$$\varepsilon + \varepsilon' \;=\; \varepsilon^{(\mu)}_{2s} - \varepsilon^{(\mu)}_{1s}\,.$$

The values for the excess energy $\Delta\varepsilon^{(\mu)}_{2s\to 1s} = \varepsilon^{(\mu)}_{2s} - \varepsilon^{(\mu)}_{1s}$ are presented in the table.

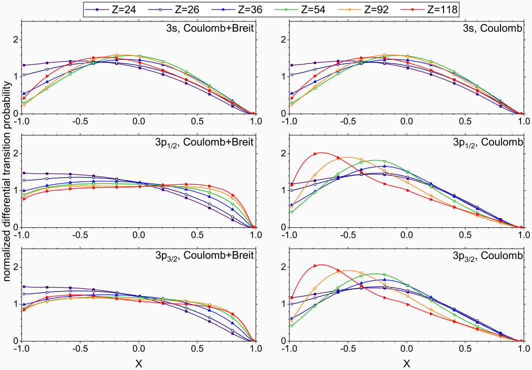
The energy sharing parameter varies in the region $-1 \le x \le 1$ with $\mathbf{x} = \mathbf{1}$ corresponding to the **electron** with the highest possible energy, $\mathbf{x} = -\mathbf{1}$ corresponding to the most energetic **positron**, and $x = 0$ corresponding to the electron end positron with equal energies.

The normalized differential EPP transition probability over the energy



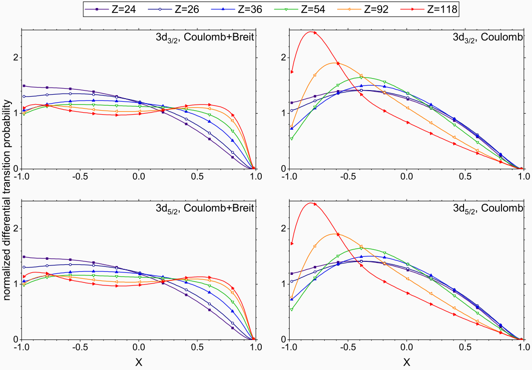
$$\frac{1}{w^{(e^+e^-)}}\frac{dw^{(e^+e^-)}}{dx} \text{ for } 2s, 2p \to 1s$$

The normalized differential EPP transition probability over the energy



$$\frac{1}{w^{(e^+e^-)}}\frac{dw^{(e^+e^-)}}{dx} \text{ for } 3s, 3p \to 1s$$

The normalized differential EPP transition probability over the energy



$$\frac{1}{w^{(e^+e^-)}}\frac{dw^{(e^+e^-)}}{dx} \text{ for } 3d \to 1s$$

The angular distributon

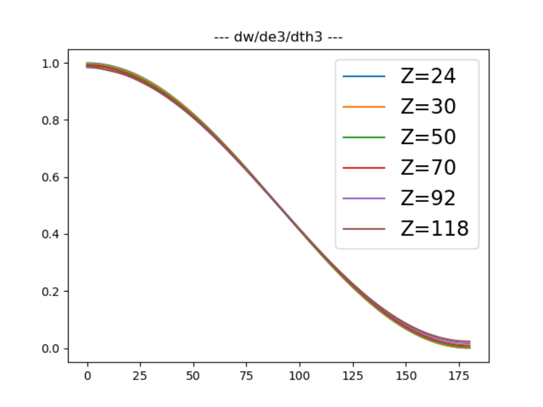


Рис. 1: The normalized angular distributon of the electron-positron pair for $2s \to 1s$.

The angular distributon

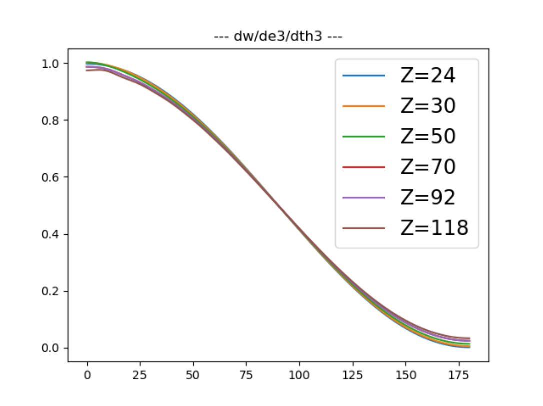


Рис. 2: The normalized angular distributon of the electron-positron pair for $3s \to 1s$.

The angular distributon

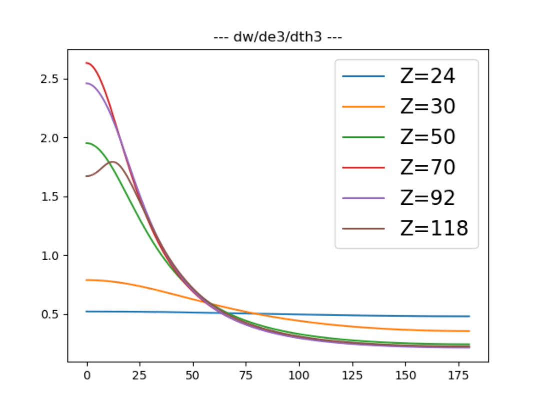


Рис. 3: The normalized angular distributon of the electron-positron pair for $2p_{1/2} \to 1s$.

The angular distributon

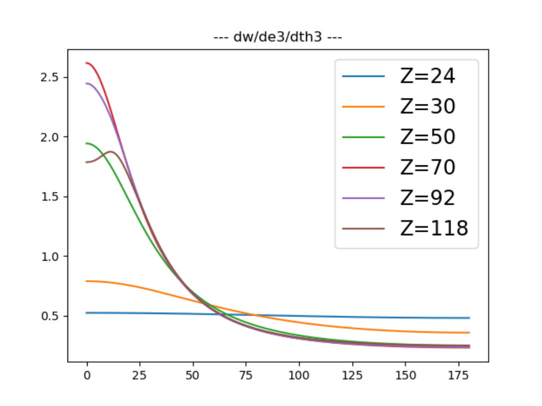


Рис. 4: The normalized angular distributon of the electron-positron pair for $2p_{3/2} \to 1s$.

The angular distributon

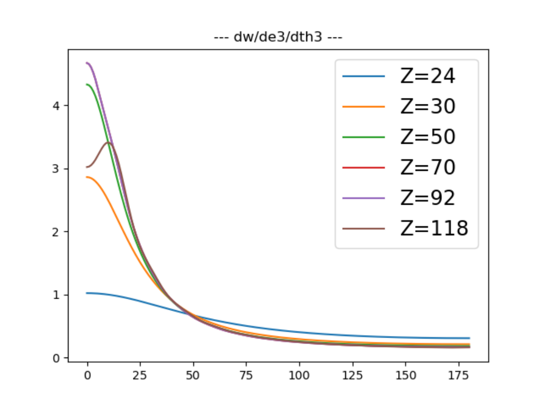


Рис. 5: The normalized angular distributon of the electron-positron pair for $3p_{1/2} \to 1s$.

The angular distributon

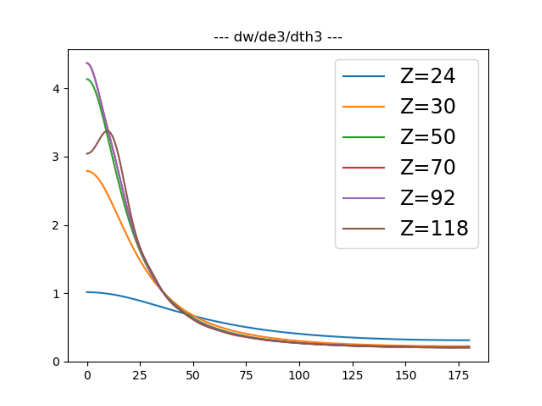


Рис. 6: The normalized angular distributon of the electron-positron pair for $3p_{1/2} \to 1s$.

The angular distributon

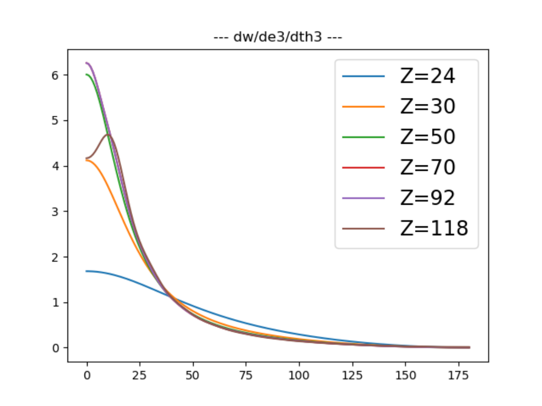


Рис. 7: The normalized angular distributon of the electron-positron pair for $3d_{1/2} \to 1s$.

The angular distributon

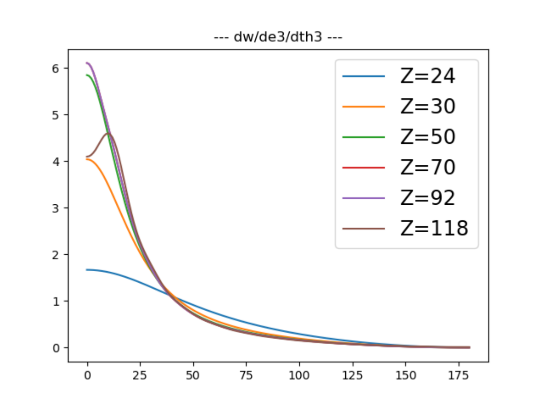


Рис. 8: The normalized angular distributon of the electron-positron pair for $3d_{5/2} \to 1s$.

The doubly differential transition probability for $Z = 30$

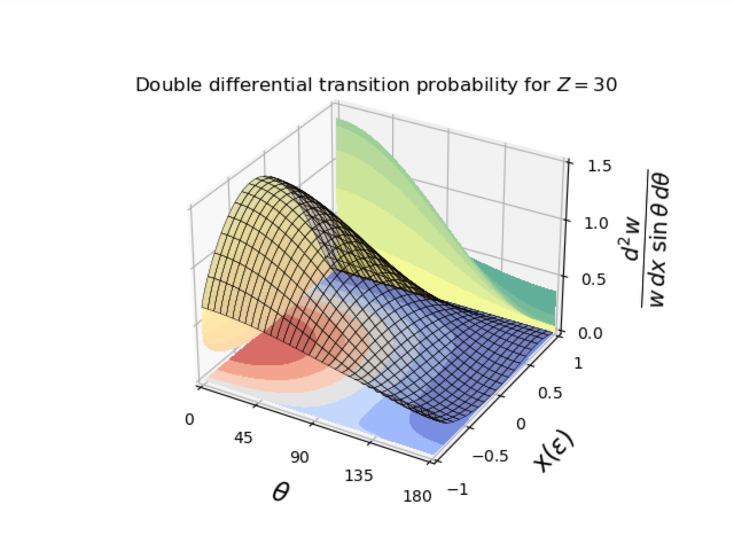


Рис. 9: $2s \to 1s$

The doubly differential transition probability for $Z = 30$

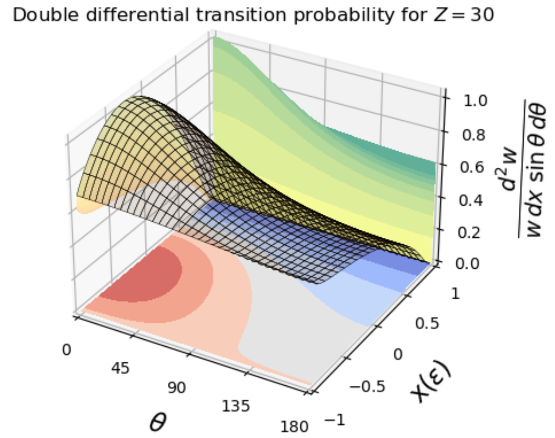


Рис. 10: $2p_{1/2} \to 1s$

The doubly differential transition probability for $Z = 30$

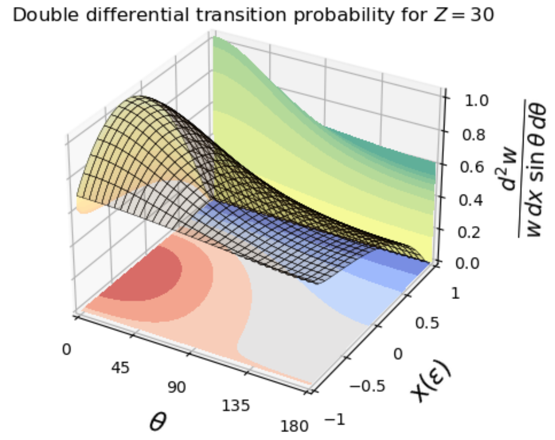


Рис. 11: $2p_{3/2} \rightarrow 1s$

Summary

- We have explored a mechanism for matter creation through the deexcitation of muonic ions, where the excess energy is emitted as an electron-positron pair. For the first time, we calculated the probabilities of these transitions (beyond the non-relativistic limit) and investigated the emission spectrum.
- The Breit interaction plays a crucial role in decays of states with orbital angular momentum $l \geq 1$, increasing the corresponding transition probabilities by a factor of 3 to 7.
- Numerical analysis shows that the dominant contributions to pair production come from the decay of $2p$ states.
- In this process the energy conservation law is fulfilled: $\varepsilon^{(\mathrm{el})} + \varepsilon^{(\mathrm{pos})} = \varepsilon_i^{(\mu)} - \varepsilon_f^{(\mu)}$.

Thank you for your attention!