Nucleon and Deuteron Structure in Quark-diquark Model

S.I. Manaenkov, PNPI of NRC <<KURCHATOV INSTITUTE>>,

Seminar at Petersburg Nuclear Physics Institute,

Gatchina, 16.06.2026

Contents

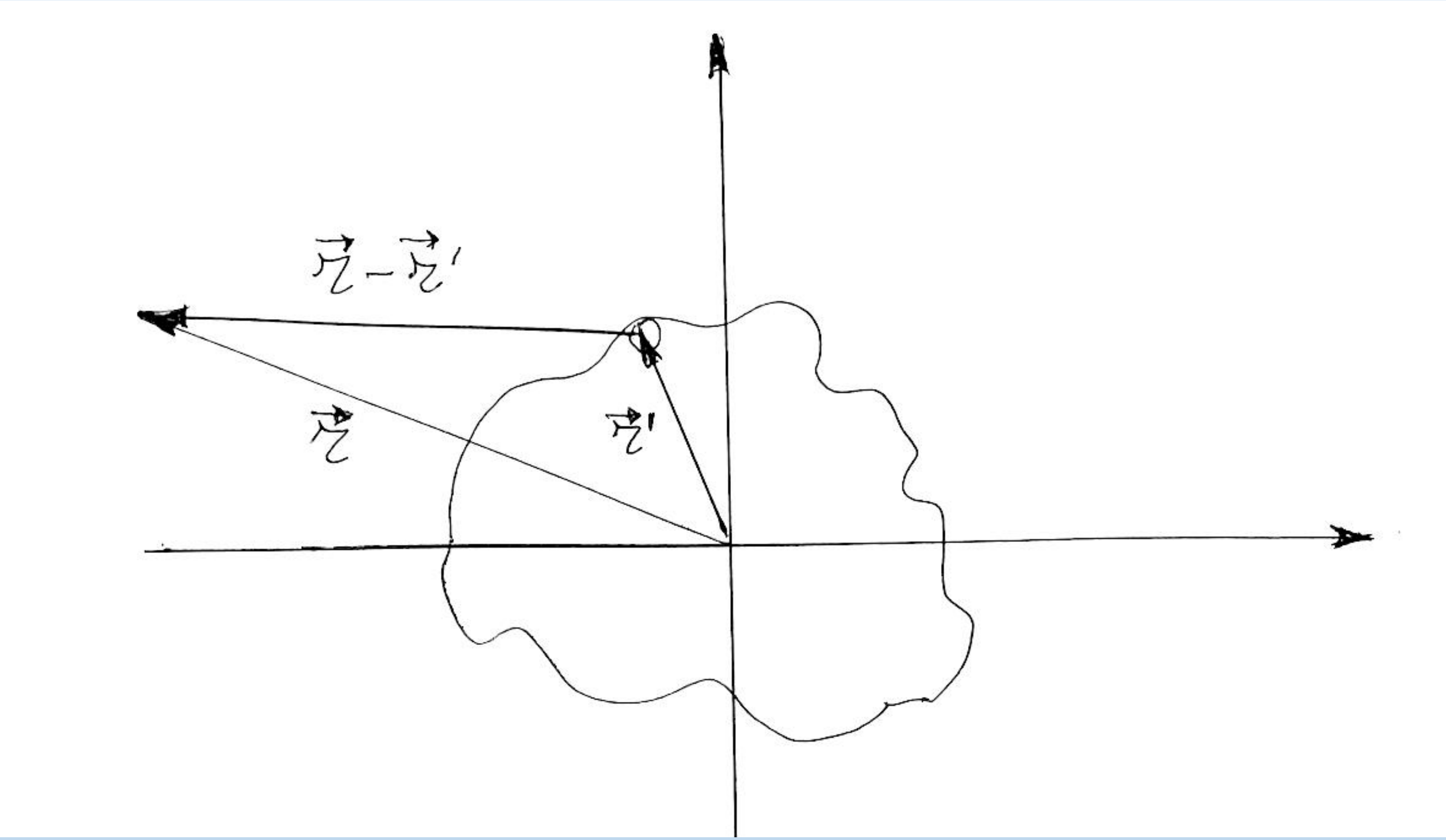
Introduction

- Nucleon and deuteron are studied for many years in electromagnetic and strong interaction. Many efforts have been done to take into account relativistic effects. In the present work we shall consider the manufestations of the nucleon quark structure in scattering amplitudes of electrons and protons on protons and deuterons.
- Electron scattering on proton in Born approximation

T($\vec{q}$) = - -$\frac{M_{ep}}{2\pi\hbar^2}\int U(\vec{r}\,)\, e^{-i\vec{q}\,\vec{r}} d^3r$, $\vec{q} \;=\; \overrightarrow{p_2}$ - $\overrightarrow{p_1}$, $\overrightarrow{p_2}$, $\overrightarrow{p_1}$ -- proton momenta.

- Charged drop (nucleus): zeρ($\vec{r}$) is a charge density $\int \rho\,(\vec{r}) d^3$r = 1, e > 0.
- Electric potential φ
- φ ($\vec{r}$) = $\int \frac{ze}{|\vec{r}-\vec{r}'|}\, \rho(\vec{r}'\,) d^3\, r'$, $U(\vec{r}) =- e\varphi(\vec{r})$, α =e^2/(4π$\hbar$).

Electric potential of charged body



Introduction

- Electron scattering on nucleon in Born approximation
- T($\vec{q}$)= $\frac{2\alpha z M_{ep}}{\hbar} \int e^{-i\vec{q}(\vec{r}-\vec{r}')} \frac{1}{|\vec{r}-\vec{r}'|} e^{-i\vec{q}\ \vec{r}'} \rho(\vec{r}')\ d^3r\ d^3r'$;

$$\vec{R} = \vec{r} - \vec{r}';$$

- $A(\vec{q}) = \int e^{-i\vec{q}\vec{R}} \ \frac{1}{R} d^3R = \frac{4\pi}{q^2}$,
- $F(\vec{q}) = \int e^{-i\vec{q}\vec{r}} \rho(\vec{r})\ d^3r$,
- T($\vec{q}$) = $\frac{2\alpha z M_{ep}}{\hbar} A(\vec{q})\ F(\vec{q})$.

Introduction

- High energy electron-proton scattering
- $T_{fi} = \frac{4\pi e^2}{q^2}\{\overline{u}(k_2)\gamma^\mu u(k_1)\}\{\overline{U}(p_2)\Lambda_\mu, U(p_1)\}$, $k_1+p_1=k_2+p_2$, $q=k_1-k_2=p_2-p_1$.
- k_1, p_1 are initial 4-momenta of electron and nucleon, k_2, p_2 are final momenta.
- $\Lambda_\mu.=2M(G_E - \frac{q^2}{4M^2}G_M)\frac{P_\mu}{P^2} - \frac{G_M}{2M}\sigma_{\mu\tau}q^\tau$, where $\sigma_{\mu\tau} = \frac{1}{2}(\gamma_\mu\gamma_\tau - \gamma_\tau\gamma_\mu)$, $Q^2=-q^2 >0$, $P= p_1+p_2$, $G_E(Q^2)$ is charge form factor and $G_M(Q^2)$ is magnetic form factor, u and $\overline{u}$ are electron bispinors while U and $\overline{U}$ are for nucleon.
- Two independent structures: $\overline{U}(p_2)\{\frac{1}{2M}\sigma_{\mu\tau}q^\tau + \gamma^\mu - \frac{P_\mu}{2M}\} U(p_1) =0$.
- $q^2= q_0^2 - q_x^2 - q_y^2 - q_z^2$, $q=(0,\vec{q})$ only in Breit systems ($\vec{p}_1+\vec{p}_2=0$) of frame, but the systems are different for various $\vec{q}$: $\vec{p}_2=-\vec{p}_1=\vec{q}/2$, $q^2 =-Q^2$.

Electron-proton scattering

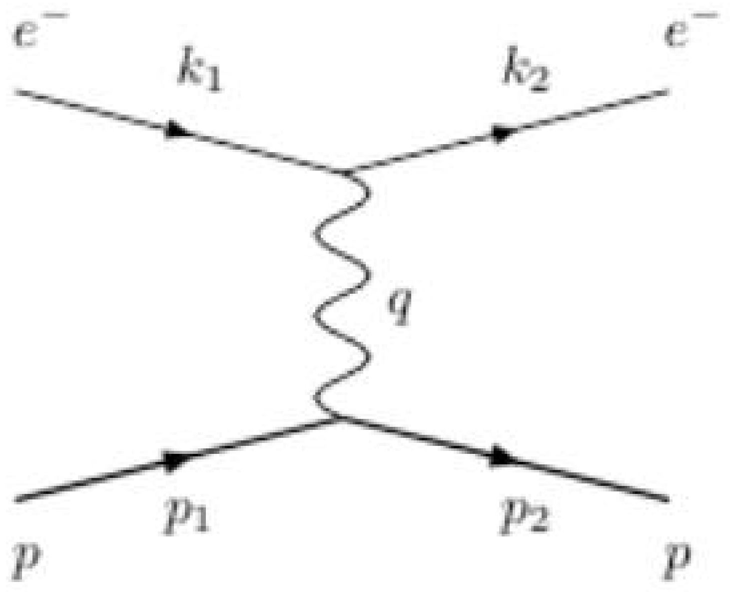


Рис. 2: Диаграмма низшего порядка, описывающая рассеяние электрона на протоне.

Dipole Formula

- $G_E(Q^2) = (1+\frac{Q^2}{b^2})^{-2}$, b^2=0,71 GeV^2 $->$ r_P^2= $(0{,}81)^2\ fm^2$, r_n^2 = -$(0{,}34)^2$ =-0,115 fm^2.
- It is impossible to explain in naive quark model (Karl, Isgur) negative sign of neutron's r_n^2.
- It is easy to check, that $\frac{b^3}{8\pi}\int_0^\infty e^{-i\vec{q}\vec{r}-br}\,d^3$ r = $\frac{1}{(1+q^2/b^2)^2}$. Coulomb wave function?

Charge radii in quark-diquark model: r_P^2= $(0{,}84)^2\ fm^2$, r_n^2 = -$(0{,}34)^2$, Q_u=$\frac{2}{3}$, Q_d= -$\frac{1}{3}$, Q_{ud}=$\frac{1}{3}$.

- m_1 is constituent quark mass, m_2 is diquark mas. $(m_1$+$m_2)\vec{R}$=$m_1\vec{r_1}$+$m_2\vec{r_2}$ =0 -->r_2^2 = $\frac{m_1^2}{m_2^2}\,r_1^2$.

Proton u(ud), r_P^2 = $Q_u< r_1^2 >$+ $Q_{ud}< r_2^2 >$. *Neutron d(ud)* r_n^2 $=Q_d< r_1^2 >$ + $Q_{ud}< r_2^2 >$,

$r_P^2 - r_n^2$ = $(Q_u - Q_d)< r_1^2 >$=0,8212 fm^2, r_P^2 +2r_n^2 = 3 Q_{ud} $<r_2^2>$ = $<r_2^2>$ = 0,4744 fm^2,

x=r_1^2/r_2^2=m_2^2/m_1^2=1,731, if m_1= 350 MeV, then m_2= 460 MeV.

Dipole formula quality

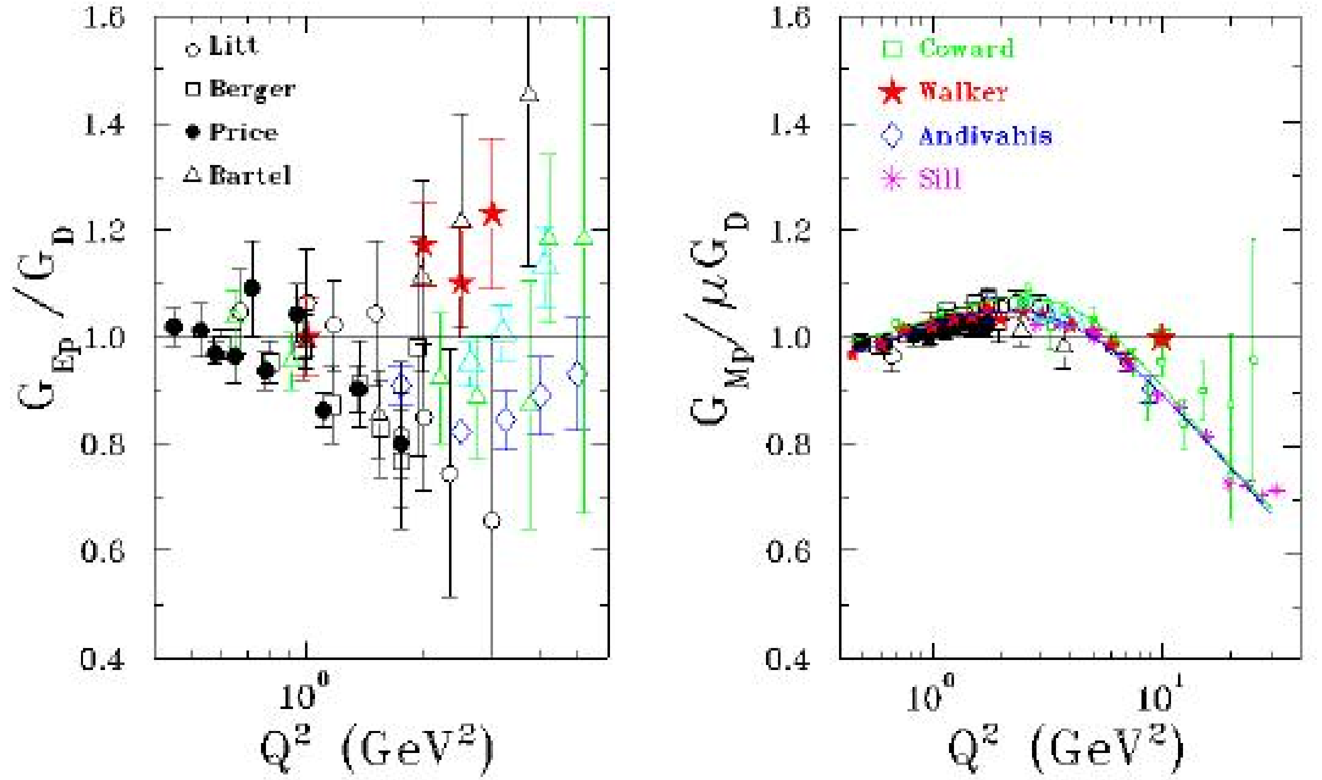


Рис. 3: Электрический и магнитный формфакторы нормированные на дипольное приближение

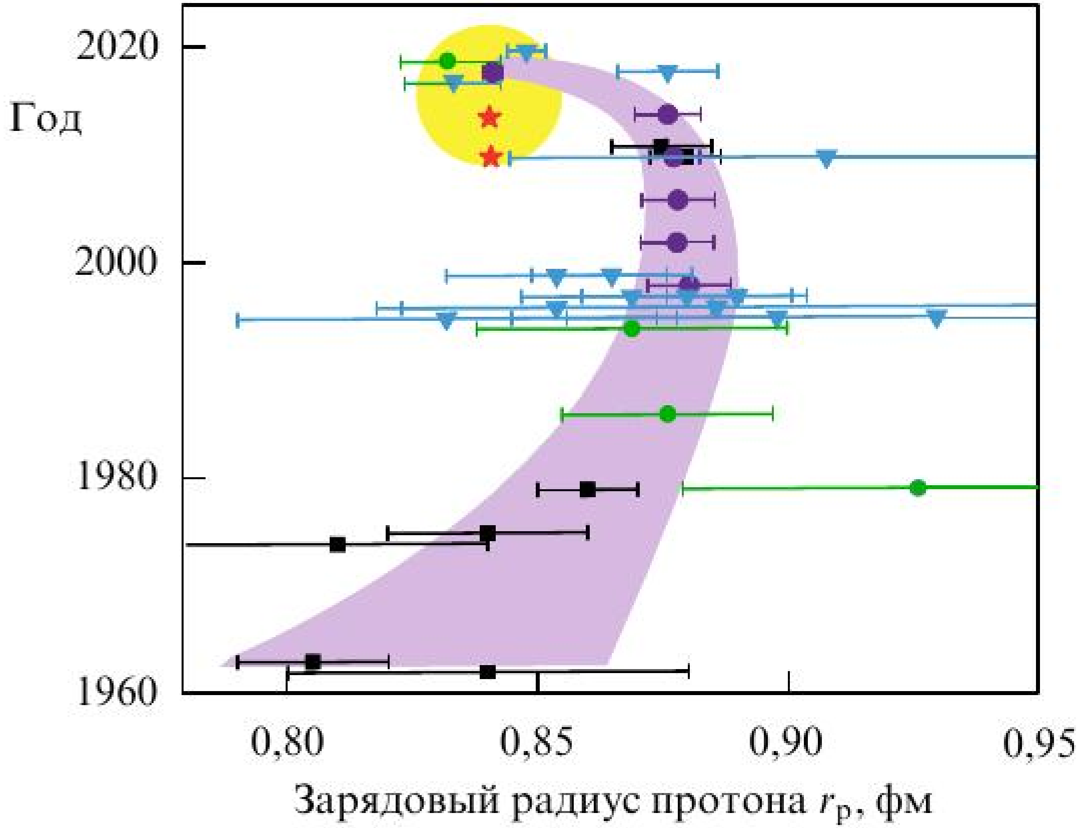


Рис. 8. (В цвете онлайн.) Ретроспектива результатов по измерению зарядового радиуса протона различными методами. Чёрные квадраты — результаты экспериментов по упругому e–p-рассеянию, зелёные кружки — радиочастотные измерения лэмбовского сдвига в водороде, голубые треугольники — результаты спектроскопии переходов в водороде, красные звёзды — результаты спектроскопии мюонного водорода, фиолетовые кружки — рекомендованные CODATA величины радиуса протона. Значения взяты из обзора [96]. Сиреневая полоса демонстрирует изменение со временем общепринятого значения радиуса протона и уменьшение погрешности его определения. Жёлтая область обозначает группу экспериментов, результаты которых привели к сдвигу рекомендованного значения зарядового радиуса протона в меньшую сторону.

Cross section of electron-nucleon scattering

Formula by Rosenbluth

$$\frac{d\,\sigma}{d\cos\theta} = \frac{\alpha^2 cos^2\frac{\theta}{2}}{(4E^2 sin^2\frac{\theta}{2})\varepsilon(1+\tau)(1+\frac{2E}{M}sin^2\frac{\theta}{2})}[\varepsilon G_E^2 + \tau G_M^2],$$

where M is nucleon mass, θ is scattering angle in laboratory system, α =1/137 denotes fine structure constant, $G_E(Q^2)$ and $G_M(Q^2)$ are charge and magnetic form factors, respectively. Four-momentum squared transferred to nucleon, Q^2 is equal to 2M(E-E') with E and E' being laboratory energies of electron before and after scattering, respectively,

while $\tau = \frac{Q^2}{4M^2}$ and $\varepsilon = (1 + 2(1+\tau)tan^2\frac{\theta}{2})^{-1}$.

There is no interference between charge and magnetic form factors.

Comparison of magnetic form factors

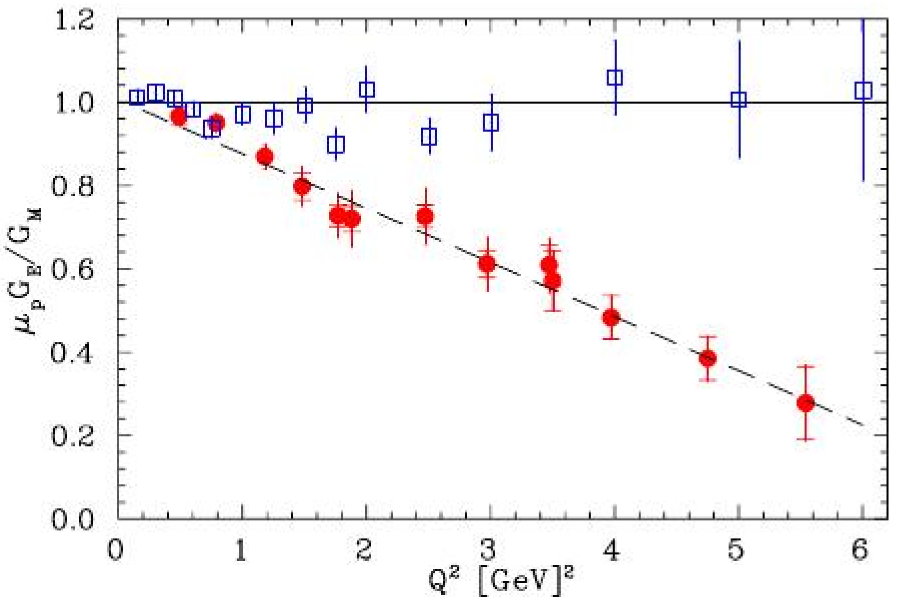


Рис. 5: Отношения формфакторов, полученные методом Розенблюта (квадраты) и поляризационным методом (кружки).

Magnetic form factor

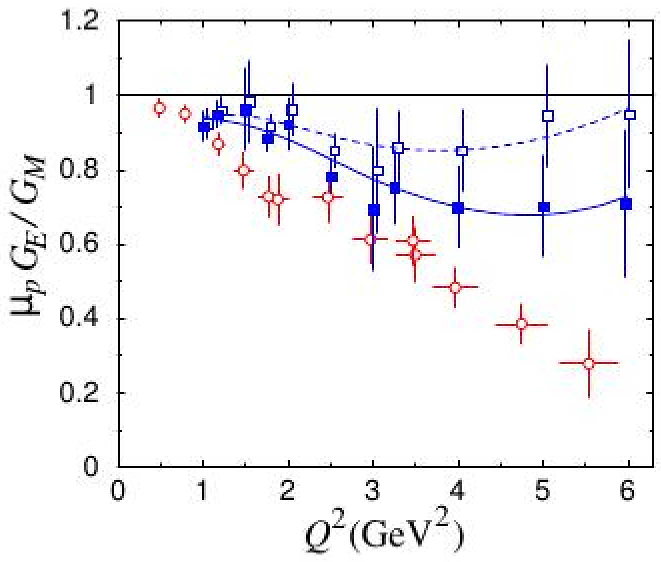


Рис. 6: Отношение формфакторов протона до (пустые квадраты) и после (закрашенные квадраты) корректировки связанной с двуфотонноым обменом, а так же данные полученные поляризационным методом (пустые кружки).

Radiative corrections to form factors

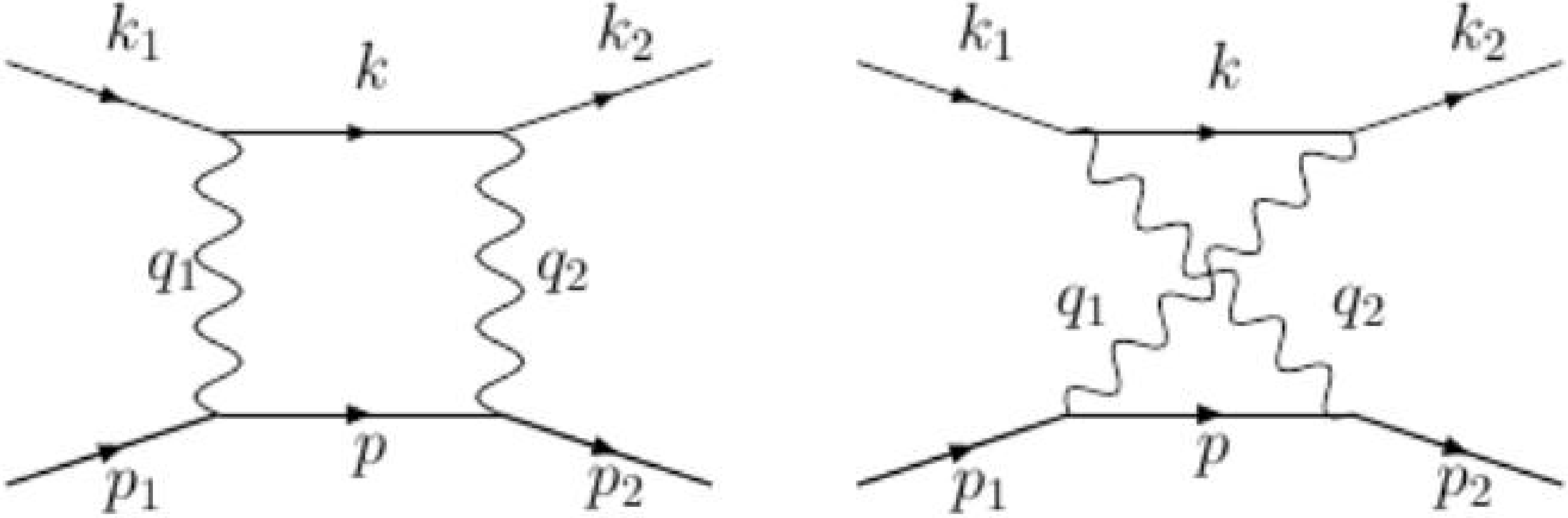


Рис. 8: Box и x-Box диаграммы

Quark wave function of diquark

Fermi-Dirac statistics, color etc.

q^i, i=1, 2, 3 =red, yellow, blue; 3 --> $\vec{q}$ = (q^1, q^2, q^3). (red, green, blue)

$\tilde{q}^i$, i = 1, 2, 3 anti-red, anti-yellow, anti-blue; $\overline{3}$.

$\bar{q}_i q^i$ = $\bar{q}_1 q^1$ + $\bar{q}_2 q^2$ +$\bar{q}_3 q^3$ -- colorless for mesons.

$q_1^i\ q_2^j$ --> 3 x 3 = $\overline{3}$ + 6 ; $\overline{3}$ quarks are attracted: f_i=$\varepsilon_{ijk} q_1^j q_2^k$, $\vec{f}$=[$\overrightarrow{q_1}$,$\overrightarrow{q_2}$].

Quarks are repulsed for state 6.

Baryons are colorless: B=$\varepsilon_{ijk} q_1^i q_2^j q_3^k$: B=($\overrightarrow{q_1}$[$\overrightarrow{q_2}$, $\overrightarrow{q_3}$]).

Diquark wave function: Ψ(1,2) = R(1,2) $\chi^c(1,2)\varphi^s(1,2)\xi^I(1,2)$.

Fermi-Dirac statistics: Ψ(2,1) = - Ψ(1,2).

Quark wave function of diquark

χ^c: *Color force for* q_1q_2 *states:* $\bar{3}$ *(antisymmetric)- attraction, 6 - repulsion.*

$\xi^I(1,2)$: I=0, uu -->(u_1u_2-u_2u_1=0), ud --> $(u_1d_2 - d_1u_2)/\sqrt{2} \neq 0$, antisymmetric.

I=1, $\xi^I(1,2) = \xi^I(2,1)$, symmetric.

R(1,2) space part of wave function for S-waves R(1,2)=R(2,1), symmetric.

Spin wave function $\varphi^s(1,2)$. Spin-spin interaction V_{ss} = 2a $\overrightarrow{s_1}\,\overrightarrow{s_2}$, $\overrightarrow{s_j}$=$\overrightarrow{\sigma_j}$/2, a>0.

2 $\overrightarrow{s_1}\,\overrightarrow{s_2}$ =$(\overrightarrow{s_1}+\overrightarrow{s_2})^2$-$(\overrightarrow{s_1})^2$ - $(\overrightarrow{s_1})^2$=$\vec{J}^2$-s_1^2-s_2^2=J(J+1) $-\frac{3}{2}$, s^2=$\frac{1}{2}(\frac{1}{2}$+1)=$\frac{3}{4}$.

$V_{ss} = a[J(J+1) - 3/2]$. J=0 V_{ss}=$-\frac{3}{2}$a --> attraction, J=1 V_{ss}=$\frac{a}{2}$ --> repulsion.

Diquark wave function: Ψ(1,2) = R(1,2) $\chi^c(1,2)\varphi^s(1,2)\xi^I(1,2)$.

The lowest energy for J=0 φ^santisymmetric, I=0 ξ^Iantisymmetric ud-diquark only.

Calculation of nucleon's charge form factors

- $\vec{\xi}=\frac{m_1}{m}\vec{r}_1+\frac{m_2}{m}\vec{r}_2, \vec{\eta} = -\frac{m_2}{m}\vec{r}_1 + \frac{m_1}{m}\vec{r}_2, m^2 = m_1^2 + m_2^2$.

$\vec{r}_1 = \frac{m_1}{m}\vec{\xi} - \frac{m_2}{m}\vec{\eta}, \vec{r}_2 = \frac{m_2}{m}\vec{\xi} + \frac{m_1}{m}\vec{\eta}.$ $\rho(\vec{\eta}) = |\Psi|^2(\vec{\eta}) = e^{-b\eta}, \ \eta^2 = \vec{\eta}^2.$

$$G_E(q^2) = \int \rho(\vec{\eta})[Q_u exp\{-i\,\vec{q}\vec{r}_1\} + Q_{ud} exp\{-i\,\vec{q}\vec{r}_2\}]\delta(\vec{\xi}) d^3\xi\, d^3\eta ,$$

$Proton$: $Q_u = \frac{2}{3}, Q_{ud} = \frac{1}{3}$. $G_E^p = \frac{2}{3}\frac{1}{[1+\frac{q^2}{b^2}\frac{m_2^2}{m^2}]^2} + \frac{1}{3}\frac{1}{[1+\frac{q^2}{b^2}\frac{m_1^2}{m^2}]^2}$

$Neutron$: $Q_d = -\frac{1}{3}, Q_{ud} = \frac{1}{3}, G_E^n = -\frac{1}{3}\frac{1}{[1+\frac{q^2}{b^2}\frac{m_2^2}{m^2}]^2} + \frac{1}{3}\frac{1}{[1+\frac{q^2}{b^2}\frac{m_1^2}{m^2}]^2}$,

Deuteron structure

- $Temptation\ to\ announce\ that\ deuteron\ consists\ of\ three$ scalar ud-diquarks. In this case it would have zero total angular momentum while deuteron is vector particle (J=1). So it is wrong idea. Why?

One-gluon exchange potential V contains color factor V^c. For colorles state of q$\bar{q}$-pair or quark-diquark system V^c = -4/3 while for qq-system in $\bar{3}$ state V^c = -2/3. General formula is $V^c = \frac{1}{2}\{C(R_0) - C(R_1) - C(R_2)\}$

with C(1)=0, C(3)=C($\bar{3}$)$=$4/3, where C(R) is Casimir invariant of su(3) group . *Here subsystems in states* R_1 *and* R_2 *are jointed in state* R_0.
Result: energy of three scalar ud-diquarks is large than energy of proton and neutron.

Deuteron structure

- Does nucleon loose its individuality in deuteron? It is the main question. It definitely does this due to Fermi-Dirac statistics of quarks but the effect is small due to numerical factor 1/27 (B.G. Zakharov).
- Has the short range correlation quantum numbers of deuteron (spin, parity etc.)
- Can we obtain any information on quark structure of deuteron from proton-deuteron elastic and inelastic scattering which is cannot be understood in terms of nucleon states only?

Conclusion

- Electron scattering on protons and nuclei provides information on quark structure of targets
- Can proton scattering on protons and nuclei give us new information on quark structure of targets complimentary to that obtained from lepton scattering on protons and nuclei?