Exclusive Electroproduction of ρ^0, ϕ, and ω Mesons at hermes

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Seminar of High Energy Physics Division

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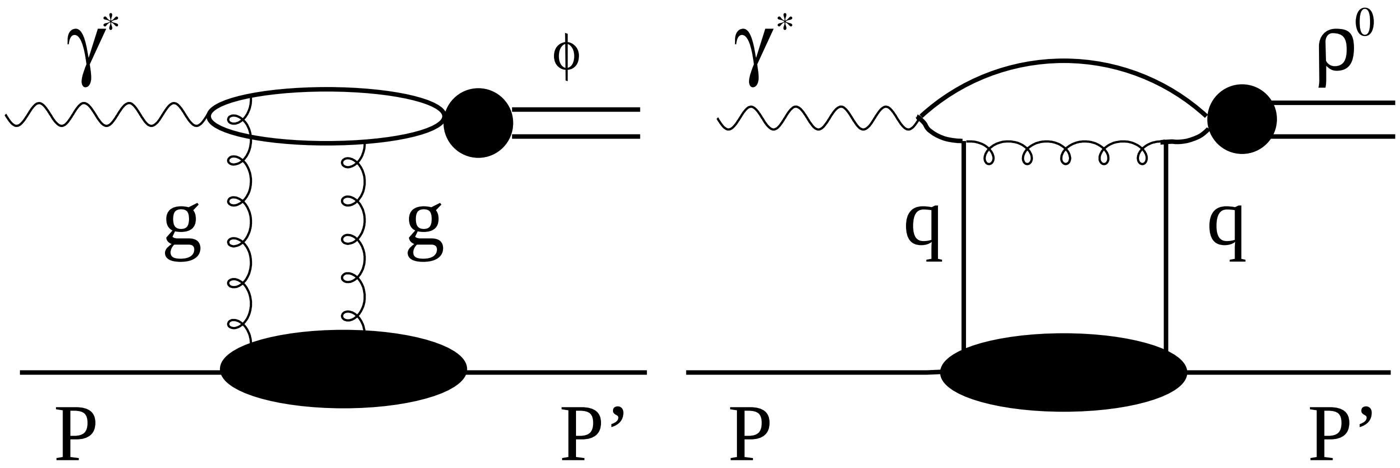
- Physics Motivation
- Data Processing
- ρ^0 and ϕ Meson Spin Density Matrix Elements (SDMEs) on Unpolarized Targets
  - Validity of S-Channel Helicity Conservation Approximation
- SDMEs and Asymmetries of ρ^0 and ω Mesons for Transversely Polarized Proton
- Direct Extraction of Helicity Amplitude Ratios
- Comparison of Amplitude and SDME Methods
- Summary and Outlook

Physics Motivation



- $\gamma^* + N \rightarrow V + N$ is a perfect reaction to study vector-meson ($V = \rho^0,\ \phi,\ \ \omega$) production mechanism: spin state of γ^* is known from QED ; decays $\rho^0 \rightarrow \pi^+ + \pi^-$, $\phi \rightarrow K^+ + K^-$ and $\omega \rightarrow \pi^+ + \pi^- + \pi^0$ provide information on Spin Density Matrix Elements (SDMEs). Generalized Parton Distributions (GPD) of the nucleon can be extracted from the data.
- Vector-meson production mechanisms can be tested both by comparing experimental SDMEs with theoretical calculations and via comparison of ρ^0, ω and ϕ production. Two gluon (Pomeron) and quark-antiquark (secondary reggeons) exchanges in the t-channel contribute to ρ^0 and ω production while the dominant contribution to ϕ production is two gluon exchange.
- Measurement of SDMEs provides a possibility to distinguish between contributions of Natural Parity Exchange (NPE) amplitudes $T_{\lambda_V \lambda_\gamma}$ and Unnatural Parity Exchange (UPE) amplitudes $U_{\lambda_V \lambda_\gamma}$. In Regge phenomenology, NPE corresponds to Pomeron, ρ, ω, a_2, ... exchanges ($J^P = 0^+,\ 1^-,\ 2^+,\ \ldots$) and exchange with π, a_1, b_1,...($J^P = 0^-,\ 1^+,\ 2^-,\ \ldots$) is UPE.

- Difference between proton and deuteron SDMEs points out to contribution of $q\bar{q}$-exchange with isospin $I = 1$ and natural parity ($P = (-1)^J$).

- Direct extraction of helicity amplitude ratios of the reaction under study is possible.

- Hierarchy of amplitudes can be established from both the analysis of SDME values and extracted amplitude ratios.

- Measurement of s-channel helicity violation (SCHV) proves existence of spin-flip amplitudes $T_{01},\ T_{10},\ T_{-11}$. In the absence of quark Fermi-motion in vector mesons, $T_{01} = T_{10} = 0$.

Kinematics of Exclusive ρ^0, ω and ϕ Production at HERMES

- $W = 3.0 \div 6.5$ GeV, $< W >= 4.9$ GeV
- $Q^2 = 1.0 \div 7.0$ GeV^2, $< Q^2 >= 2.3$ GeV^2
- $x_B = 0.01 \div 0.35$, $< x_B >= 0.07$

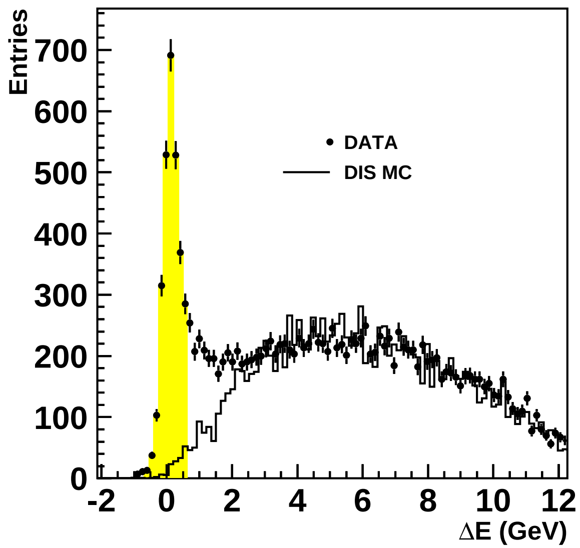
Total number of events

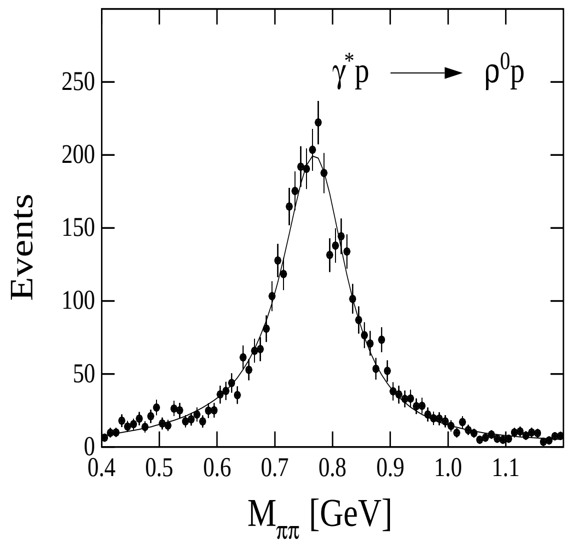
Deuteron: ρ^0 - 16388, ϕ - 1038

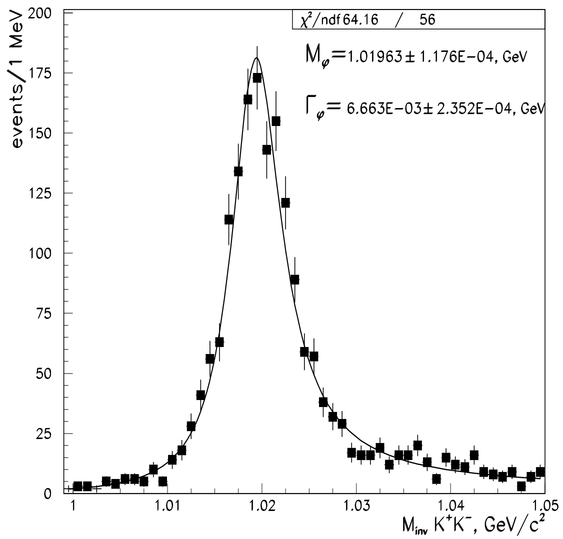
Hydrogen: ρ^0 - 9860, 7390 ($\Uparrow$), ω - 429, ϕ - 711

- $0 \leq -t' \leq 0.4$ GeV^2, $< -t' >= 0.13$ GeV^2 with $t' = t - t_{min}$

$\Delta E = \frac{M_X^2 - M_p^2}{2M_p}$ with $M_X^2 = (p + q - v)^2$ and M_X being mass of recoil hadronic system



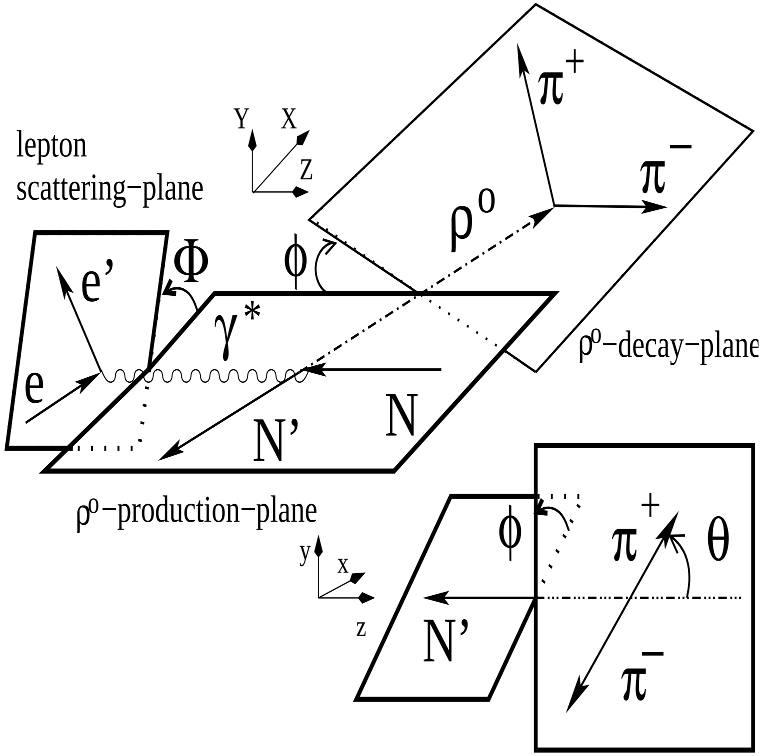


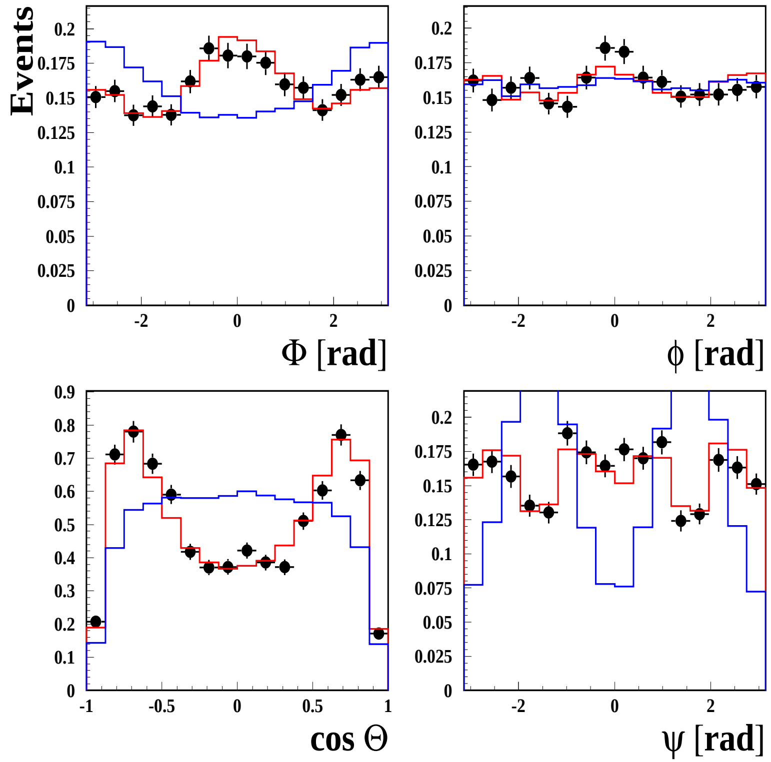


$-1.0 < \Delta E < 0.6$ GeV, $0.6 < M_{\pi\pi} < 1$ GeV, $1.01 < M_{KK} < 1.03$ GeV

SIDIS background is subtracted with the help of MC (PYTHIA)

Data Processing using Maximum Likelihood Method in MINUIT





$\Psi = \phi - \Phi$ (in S-Channel Helicity Conservation (SCHC) approximation)

- Monte Carlo Events: 3-dimensional matrix of fully reconstructed MC events at initial uniform angular distribution.

- Binned Maximum Likelihood (BML) Method: $8 \times 8 \times 8$ bins of $\cos(\Theta), \phi, \Phi$. Simultaneous fit of 23 SDMEs for data with negative and positive beam helicity ($< P_b >= \pm 53.5\%$).
  Agreement of fitted angular distributions with the HERMES data

Amplitudes and Spin Density Matrices of Reaction $e + N \to e + V + N$

- First: $e \to e + \gamma^*$ (QED)
  Spin-density matrix of the virtual photon $\rho(\Phi, \epsilon)$
- Second: $\gamma^* + N \to V + N$ (QCD)
  Helicity amplitudes in CMS of $\gamma^* N$
  $F_{\lambda_V \lambda'_N; \lambda_\gamma \lambda_N}(W, Q^2, t') = F_{\lambda_V \lambda_\gamma}$,
  Vector-meson spin-density matrix
  $r = \frac{1}{2N} \mathrm{tr}_{\lambda_N \lambda'_N} \{\mathrm{F}\ \rho\ \mathrm{F}^+\}$.
  Free parameters $r^{\alpha}_{\lambda_V \lambda'_V}$ where
  $r^{\alpha} = \frac{1}{2N} \mathrm{tr}_{\lambda_N \lambda'_N} \{\mathrm{F}\ \Sigma^{\alpha}\ \mathrm{F}^+\}$
  with $\alpha = 0,\ 1, ...,\ 8$. If contributions of transverse and longitudinal photons are not separated, then
  $r^0_{\lambda_V \lambda'_V} + \epsilon r^4_{\lambda_V \lambda'_V} \Rightarrow r^{04}_{\lambda_V \lambda'_V}$.
- Third: $\rho^0 \ \Rightarrow\ \pi^+ \pi^-$ (conservation of $\vec{J}$)
  $|\rho^0; 1m > \to\ |\pi^+ \pi^-; 1m > \Rightarrow\ Y_{1m}(\theta, \phi)$

Hierarchy of amplitudes: $|T_{00}|^2 \sim |T_{11}|^2 \gg |T_{01}|^2 \gg |T_{10}|^2 \sim |T_{1-1}|^2$

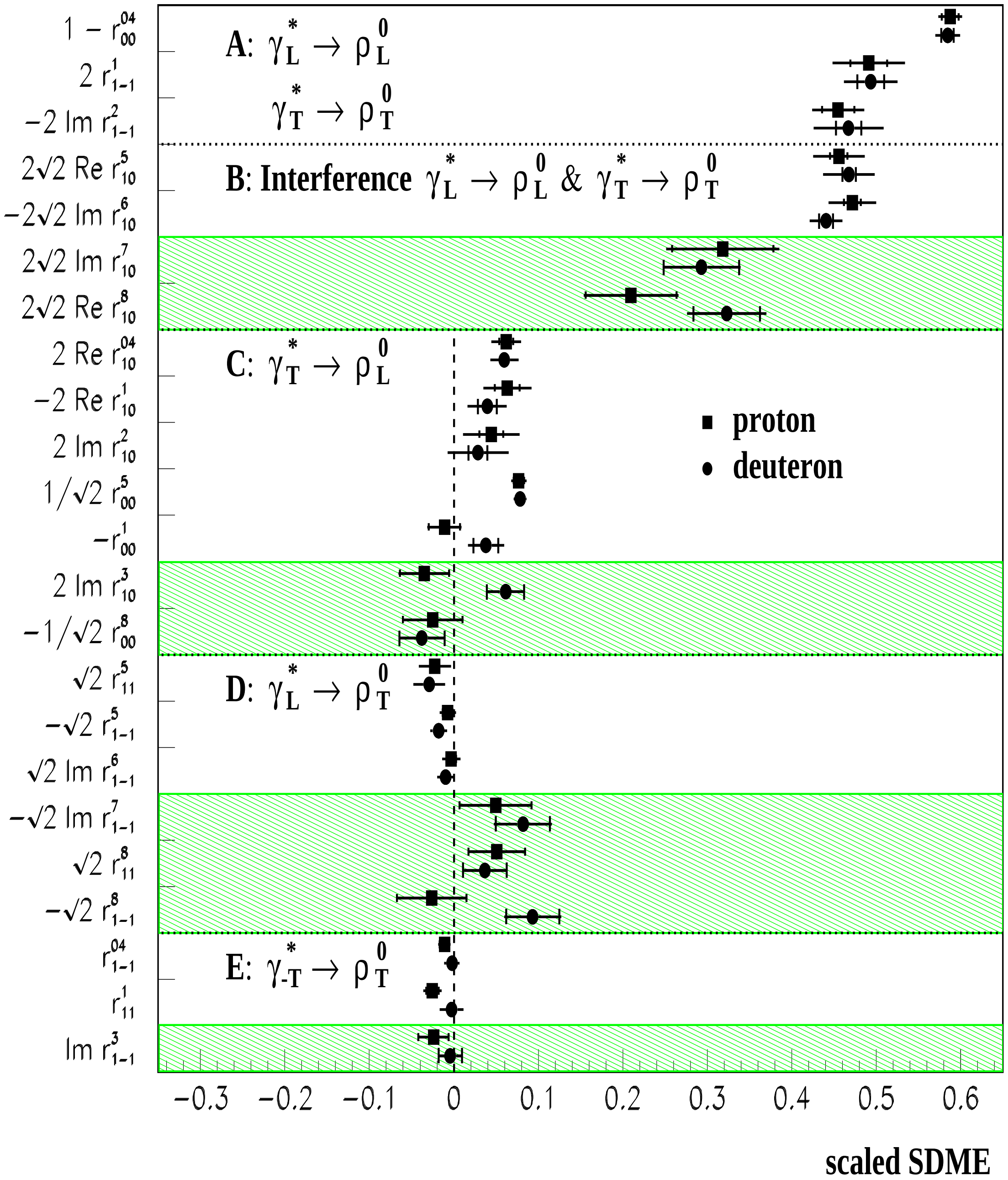
- Class A. Main terms: $\propto\ |T_{00}|^2$ or $|T_{11}|^2$.
- Class B. $\mathrm{Re}\{\mathrm{T}_{11} \mathrm{T}^*_{00}\}$, $\mathrm{Im}\{\mathrm{T}_{11} \mathrm{T}^*_{00}\}$.
- Class C. $\mathrm{Re}\{\mathrm{T}_{01} \mathrm{T}^*_{00}\}$, $\mathrm{Im}\{\mathrm{T}_{01} \mathrm{T}^*_{00}\}$,

$$\mathrm{Re}\{\mathrm{T}_{01} \mathrm{T}^*_{11}\}, \quad \mathrm{Im}\{\mathrm{T}_{01} \mathrm{T}^*_{11}\}$$

.

- Class D. $\mathrm{Re}\{\mathrm{T}_{10} \mathrm{T}^*_{11}\}$, $\mathrm{Im}\{\mathrm{T}_{10} \mathrm{T}^*_{11}\}$
- Class E. $\mathrm{Re}\{\mathrm{T}_{-11} \mathrm{T}^*_{11}\}, \mathrm{Im}\{\mathrm{T}_{-11} \mathrm{T}^*_{11}\}$
- Class C. $T_{01} \sim (z - 1/2)$
  where z is momentum fraction carried by quark in vector meson.

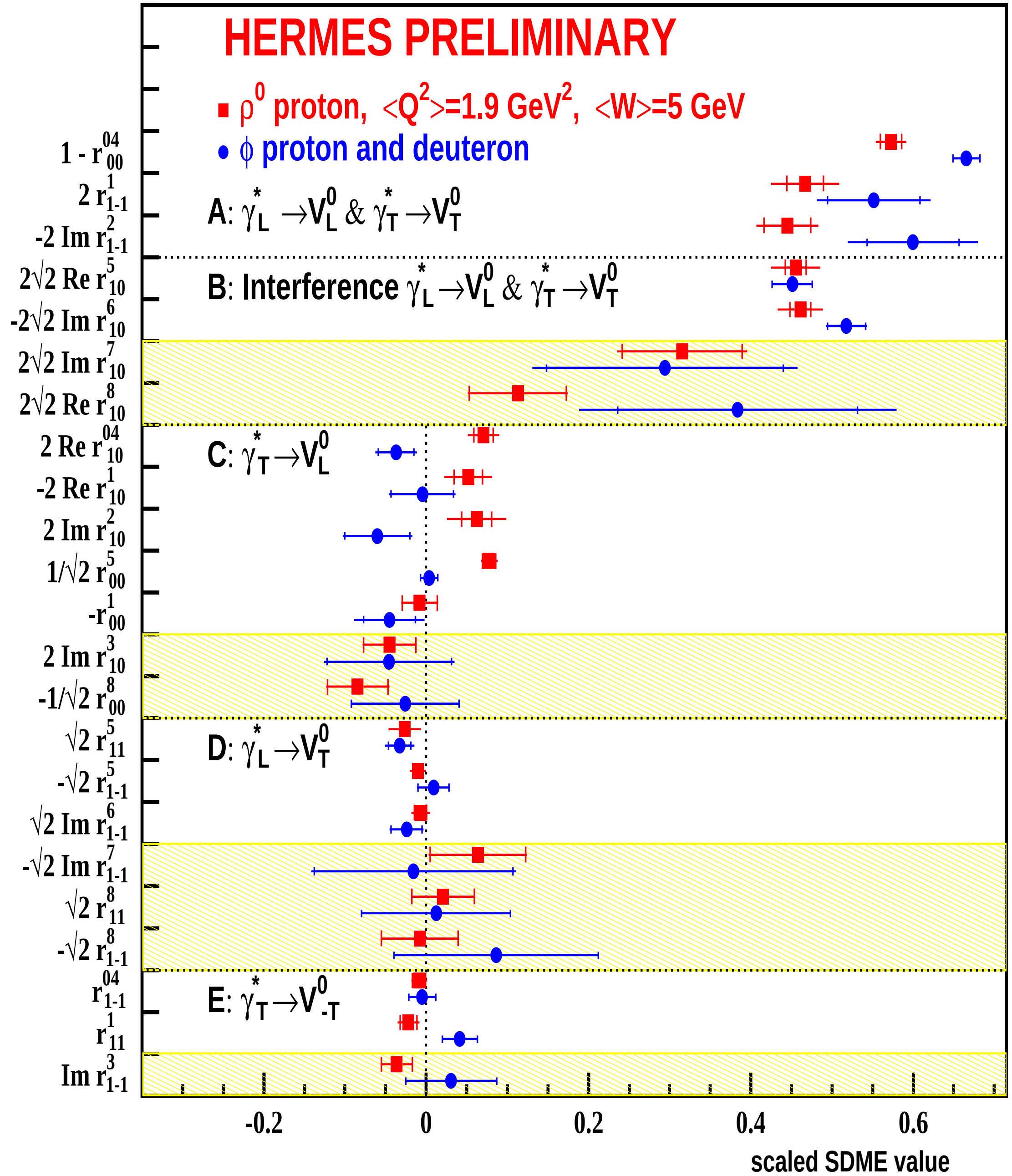
Spin Density Matrix Elements $r^{\alpha}_{\lambda_\rho \lambda'_\rho}$ of ρ^0 meson



Longitudinally polarized beam (53.5%), Unpolarized target (LU)

- Polarized LU SDMEs have been measured by HERMES for the first time
- No statistically significant difference between proton and deuteron.
- S-Channel Helicity Conservation. Non-zero amplitudes: NPE $T_{00} \equiv F_{00}$, $T_{11} = (F_{11} + F_{-1-1})/2$, UPE $U_{11} = (F_{11} - F_{-1-1})/2$.
- Violation of S-Channel Helicity: significance of $3 \div 10\ \sigma_{tot}$. **Linear** contribution of NPE spin-flip amplitudes T_{01}, T_{10}, T_{-11}.
- Hierarchy of amplitudes at HERMES kinematics: $|T_{00}|^2 \sim |T_{11}|^2 \gg |U_{11}|^2 > |T_{01}|^2 \gg |T_{10}|^2 \sim |T_{-11}|^2$

Comparison of SDMEs $r^{\alpha}_{\lambda_\rho \lambda'_\rho}$ for ρ^0 and ϕ mesons

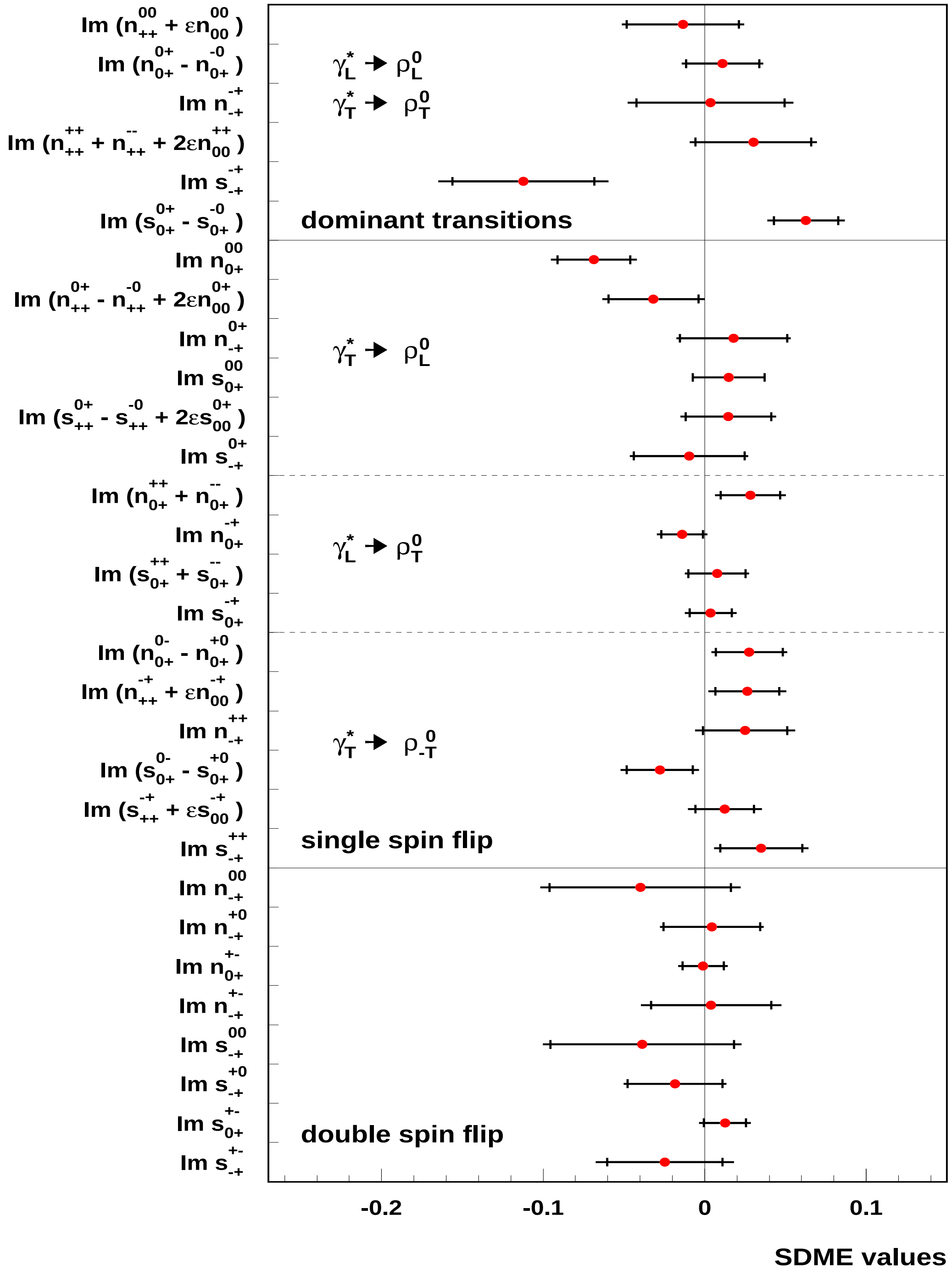


ϕ meson:

Longitudinally polarized beam (53.5%),
Unpolarized target (LU)

- Polarized LU SDMEs have been measured by HERMES for the first time
- No statistically significant difference between proton and deuteron. $\Rightarrow$ No noticeable contribution of reggeon exchanges with natural parity and isospin $I = 1$ ($q\bar{q}$-pair exchanges)
- No violation of S-Channel Helicity: Non-zero NPE amplitudes: T_{00}, T_{11}.
- Hierarchy of amplitudes at HERMES kinematics: $|T_{00}|^2 \sim |T_{11}|^2 \gg |T_{01}|^2 \approx |T_{10}|^2 \approx |T_{-11}|^2 \approx |U_{11}|^2 \approx 0$

ρ^0-Meson SDMEs on Transversely Polarized Proton



- 7390 UT events in 2002-2005 years
  $0.023 < x_B < 0.4$, $W^2 > 10$ GeV^2
  $P_T = 0.724 \pm 0.059$ with
  scale uncertainty 8.1%
  Unbinned Maximum Likelihood (UML) Method
- Diehl's Formalism for SDMEs

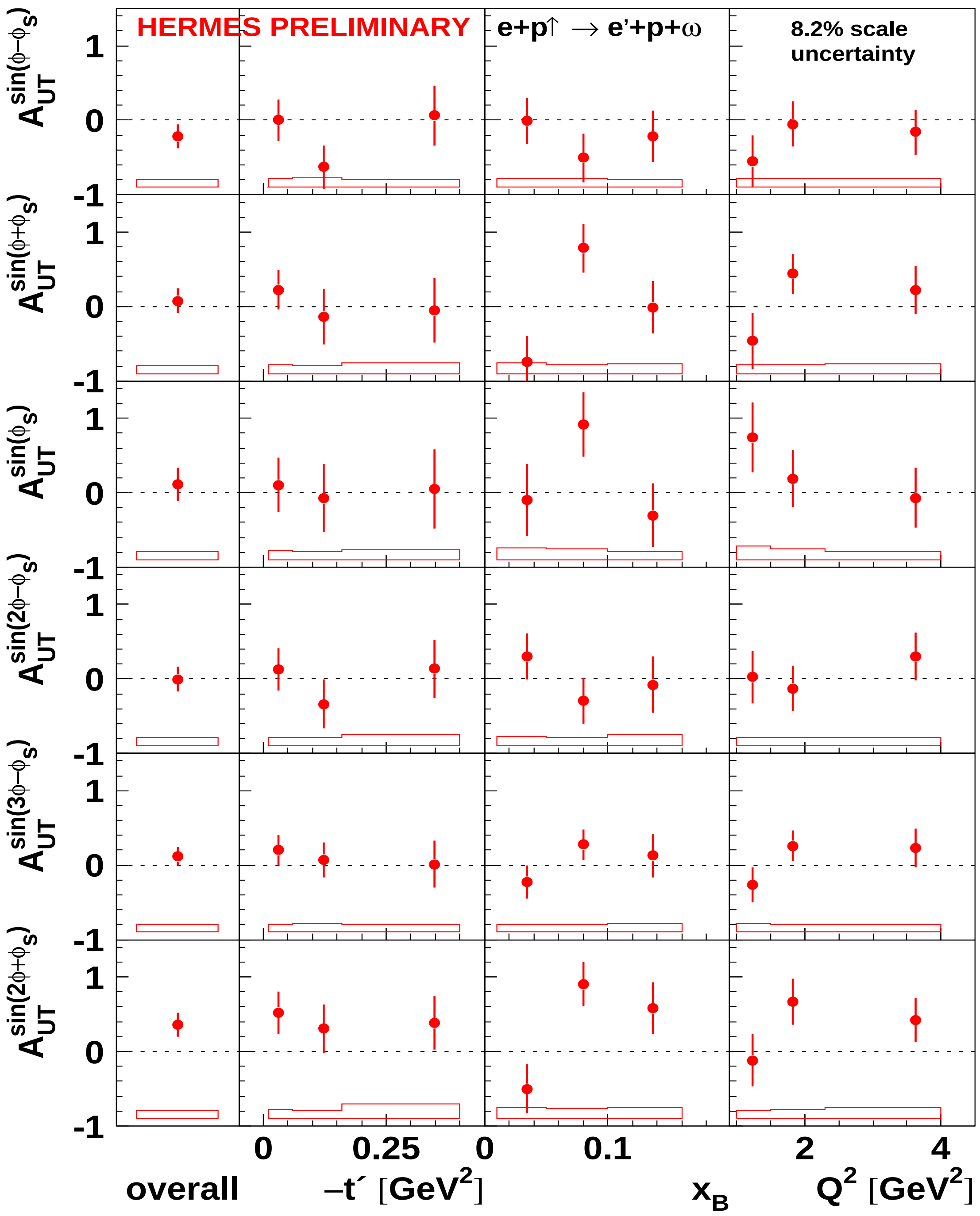
$$u^{\nu\nu'}_{\mu\mu'} = \sum_\sigma \frac{[T_{\nu\sigma\mu+}T^*_{\nu'\sigma\mu'+} + U_{\nu\sigma\mu+}U^*_{\nu'\sigma\mu'+}]}{2(\mathcal{N}_T+\epsilon\mathcal{N}_L)},$$

$$s^{\nu\nu'}_{\mu\mu'} = \sum_\sigma \frac{[T_{\nu\sigma\mu+}U^*_{\nu'\sigma\mu'-} + U_{\nu\sigma\mu+}T^*_{\nu'\sigma\mu'-}]}{2(\mathcal{N}_T+\epsilon\mathcal{N}_L)},$$

$$n^{\nu\nu'}_{\mu\mu'} = \sum_\sigma \frac{[T_{\nu\sigma\mu+}T^*_{\nu'\sigma\mu'-} + U_{\nu\sigma\mu+}U^*_{\nu'\sigma\mu'-}]}{2(\mathcal{N}_T+\epsilon\mathcal{N}_L)}.$$

- Nonzero SDMEs ($\mathcal{N} = \mathcal{N}_T + \epsilon\mathcal{N}_L$)
  $\mathrm{Im}(\mathrm{s}^{-+}_{-+}) \approx 2\mathrm{Im}(\mathrm{T}_{1\frac{1}{2}1\frac{1}{2}}\mathrm{U}^*_{1\frac{1}{2}1-\frac{1}{2}})/\mathcal{N}$,
  $\mathrm{Im}(\mathrm{s}^{0+}_{0+}) \approx \mathrm{Im}(\mathrm{T}_{0\frac{1}{2}0\frac{1}{2}}\mathrm{U}^*_{1\frac{1}{2}1-\frac{1}{2}})/\mathcal{N}$,
  $\mathrm{Im}(\mathrm{n}^{00}_{0+}) \approx \mathrm{Im}\sum_\sigma(\mathrm{T}_{0\sigma0\frac{1}{2}}\mathrm{T}^*_{0\sigma1-\frac{1}{2}})/\mathcal{N}$.
- $A_{UT}^{(\gamma^*_L p \to \rho^0_L p)} = \frac{\mathrm{Im}(\mathrm{n}^{00}_{00})}{u^{00}_{00}} \sin(\phi - \phi_s)$
  $\approx \frac{\mathrm{Im}(\mathrm{n}^{00}_{++} + \epsilon \mathrm{n}^{00}_{00})}{(u^{00}_{++} + \epsilon u^{00}_{00})} \sin(\phi - \phi_s)$

Asymmetries of ω Meson Produced on Transversely Polarized Proton



- 429 UT events in 2002-2005 years of $e + p \Uparrow \rightarrow e' + \omega(\rightarrow \pi^+\pi^-\pi^0) + p'$ BML and UML Methods

- $A_{UT} = \frac{[\sigma(\phi,\phi_s)-\sigma(\phi,\phi_s+\pi)]/P_T}{\int[\sigma(\phi,\phi_s)+\sigma(\phi,\phi_s+\pi)]d\phi/(2\pi)}$

  $= \sum_{m,n} A_{UT}^{\sin(m\phi+n\phi_s)} \sin(m\phi + n\phi_s)$

  $\phi = 2\pi - \Phi$ - angle between lepton scattering and vector-meson production planes.
  ϕ_s - angle between lepton scattering plane and the plane of vectors $\vec{P}_T$ and $\vec{q}$.

- $A_{UT}^{\sin(\phi-\phi_s)}(\rho^0) \propto \frac{\sqrt{-t'}}{M}\mathrm{Im}\left\{\frac{2\mathrm{E^u}+\mathrm{E^d}}{2\mathrm{H^u}+\mathrm{H^d}}\right\}$

  $A_{UT}^{\sin(\phi-\phi_s)}(\omega) \propto \frac{\sqrt{-t'}}{M}\mathrm{Im}\left\{\frac{2\mathrm{E^u}-\mathrm{E^d}}{2\mathrm{H^u}-\mathrm{H^d}}\right\}$

- Ji's sum rule

  $J^a = \frac{1}{2}\int dx x[H^a(x,0,0)+E^a(x,0,0)]$
  $a = u,\ d,\ s,\ g$

Direct Extraction of Amplitude Ratios from Angular Distributions

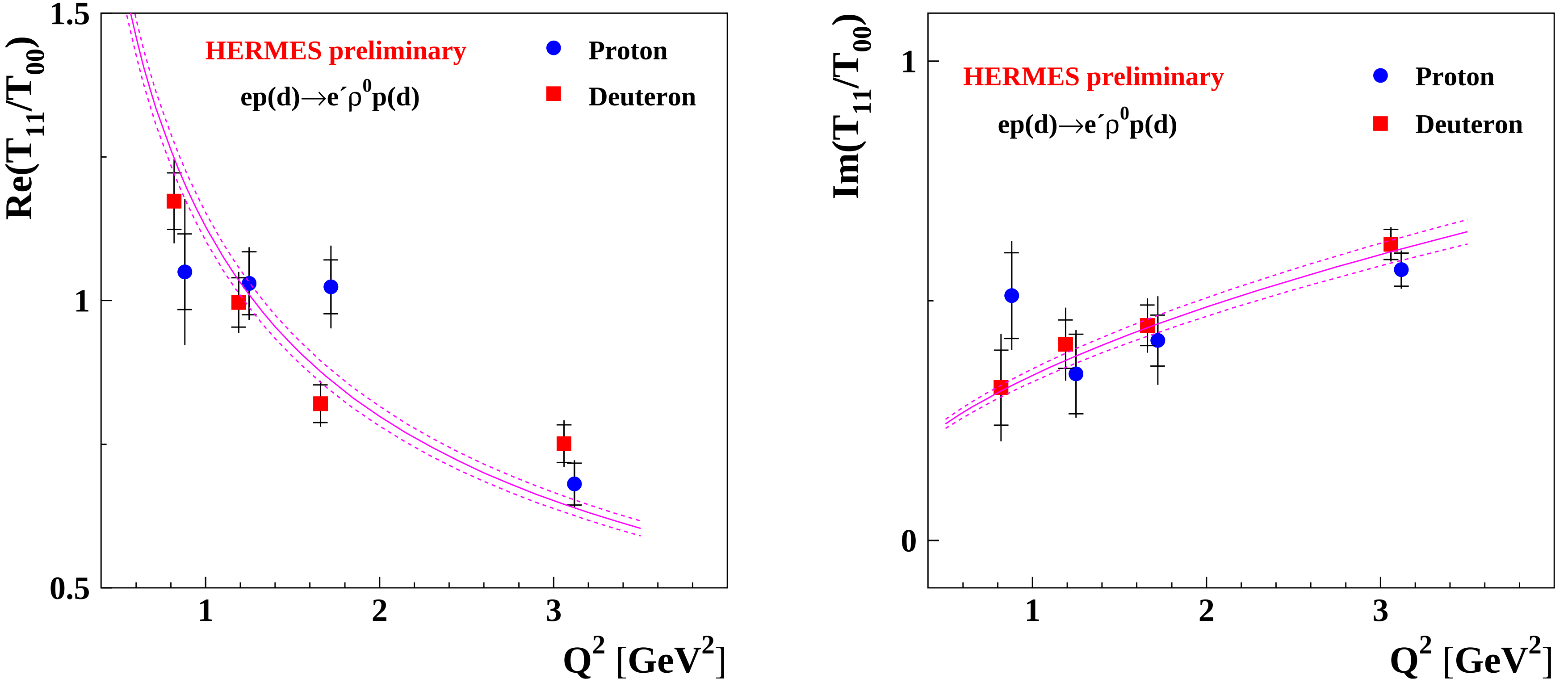
- Extraction of 9 ratios $\mathrm{Re}(\mathrm{T}_{11}/\mathrm{T}_{00})$, $\mathrm{Im}(\mathrm{T}_{11}/\mathrm{T}_{00})$, $\mathrm{Re}(\mathrm{T}_{01}/\mathrm{T}_{00})$, $\mathrm{Im}(\mathrm{T}_{01}/\mathrm{T}_{00})$, $\mathrm{Re}(\mathrm{T}_{10}/\mathrm{T}_{00})$, $\mathrm{Im}(\mathrm{T}_{10}/\mathrm{T}_{00})$, $\mathrm{Re}(\mathrm{T}_{1-1}/\mathrm{T}_{00})$, $\mathrm{Im}(\mathrm{T}_{1-1}/\mathrm{T}_{00})$, $|U_{11}/T_{00}|$ in 16 bins.

- Fit of Q^2 and t dependences of the amplitude ratios

Mean values of kinematic variables for 16 bins. The q1, q2, q3, and q4 bins for Q^2 are 0.5; 1.0; 1.4; 2.0; 7.0 GeV^2 while the t1, t2, t3, and t4 bins for $-t'$ are the following: 0.0; 0.04; 0.10; 0.20; 0.40 GeV^2.

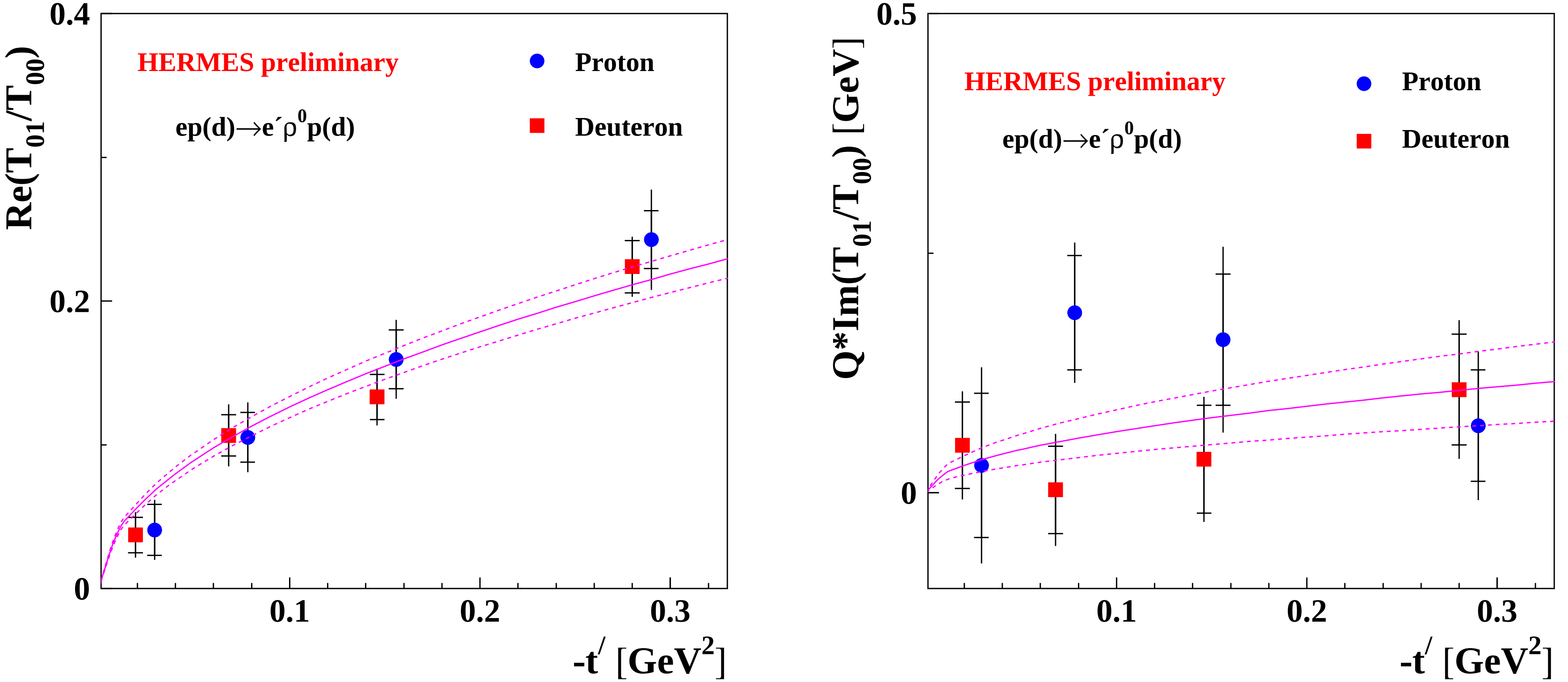
bin	$\langle Q^2 \rangle$, GeV^2	$\langle -t' \rangle$, GeV^2	bin	$\langle Q^2 \rangle$, GeV^2	$\langle -t' \rangle$, GeV^2
q1t1	0.817	0.019	q3t1	1.658	0.019
q1t2	0.823	0.068	q3t2	1.660	0.068
q1t3	0.821	0.146	q3t3	1.663	0.146
q1t4	0.815	0.280	q3t3	1.663	0.282
q2t1	1.184	0.019	q4t1	2.996	0.019
q2t2	1.188	0.068	q4t2	3.056	0.068
q2t3	1.189	0.145	q4t3	3.076	0.146
q2t4	1.188	0.282	q4t3	3.134	0.284

Direct Extraction of Amplitude Ratios from Angular Distributions



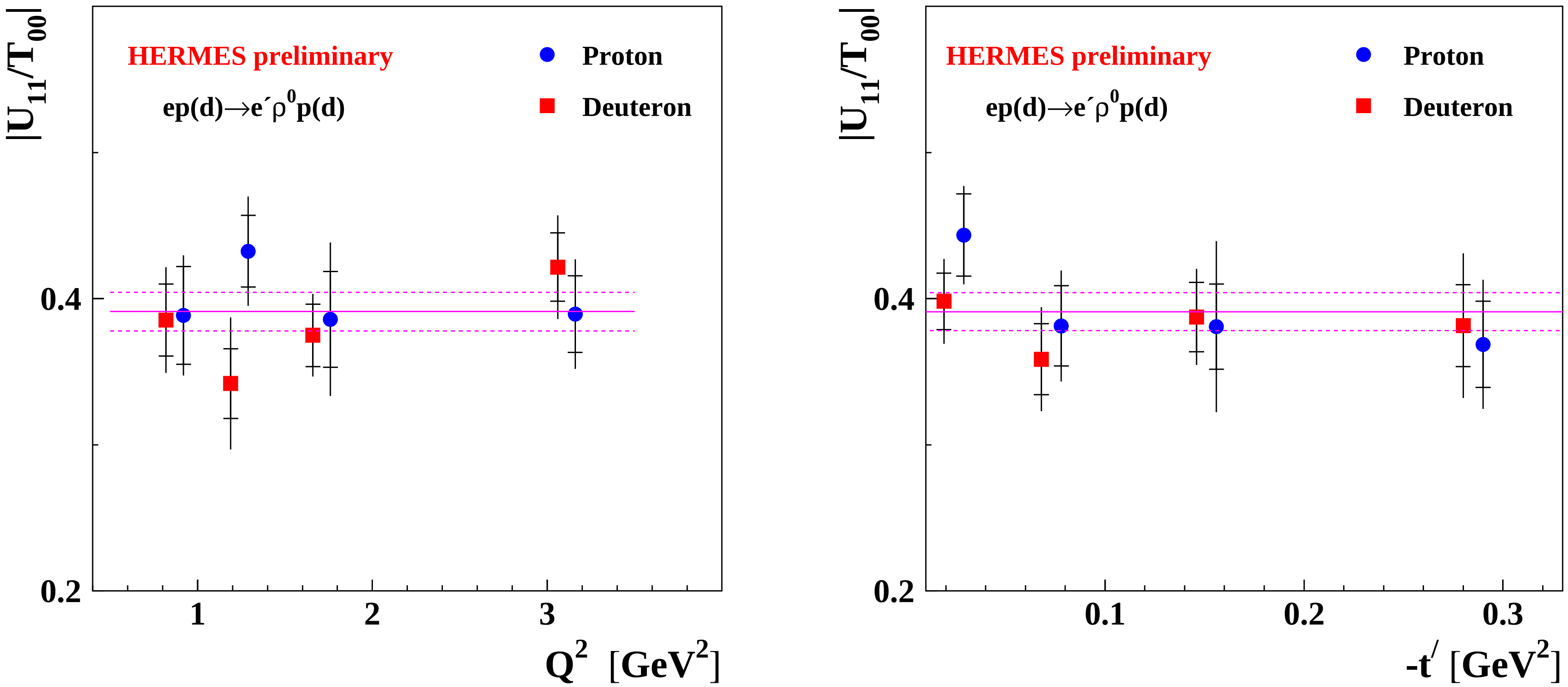
- pQCD prediction (Ivanov, Kirshner; Kuraev, Nikolaev,Zakharov): $T_{11}/T_{00} \propto M_V/Q$
- Fit of Q dependence: $\mathrm{Re}\{\mathrm{T}_{11}/\mathrm{T}_{00}\} = \mathrm{a}/\mathrm{Q}$, $\mathrm{Im}\{\mathrm{T}_{11}/\mathrm{T}_{00}\} = \mathrm{b} \cdot \mathrm{Q}$.
  Combined $p + d$ data: $a = 1.129 \pm 0.024$ GeV, $\chi^2/N_{df} = 1.02$;
  $b = 0.344 \pm 0.014$ GeV^{-1}, $\chi^2/N_{df} = 0.87$.
  Behaviour of $\mathrm{Im}\{\mathrm{T}_{11}/\mathrm{T}_{00}\}$ is in a contradiction with high-Q asymptotics.
  Phase difference $\delta_{11} \sim 30^\circ$ and grows with Q^2 in disagreement with GK model.
- No t dependence: differences of slopes $\beta_L - \beta_T = -0.6 \pm 0.4$ GeV^{-2}

Direct Extraction of Amplitude Ratios from Angular Distributions



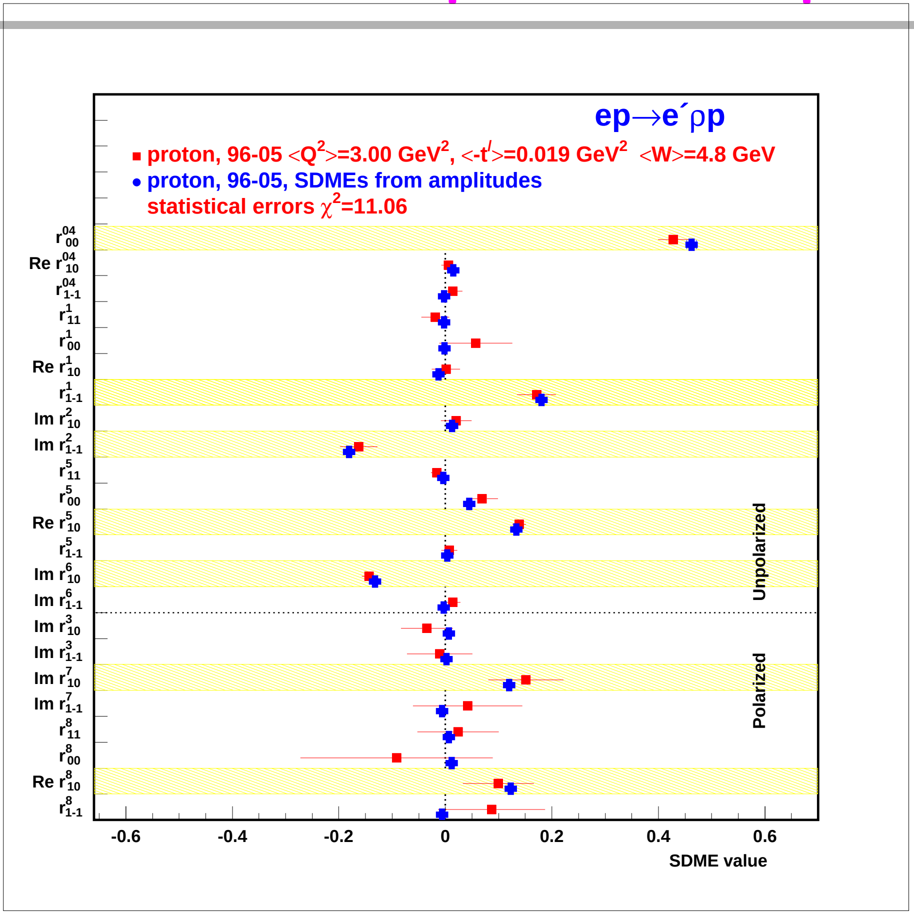
- pQCD prediction (Ivanov, Kirshner; Kuraev, Nikolaev,Zakharov): $T_{01}/T_{00} \propto \sqrt{-t'}/Q$
- Fit of t' dependence: $\mathrm{Re}\{\mathrm{T}_{01}/\mathrm{T}_{00}\} = \mathrm{a}\sqrt{-\mathrm{t}'}$, $\mathrm{Im}\{\mathrm{T}_{01}/\mathrm{T}_{00}\} = \mathrm{b}\sqrt{-\mathrm{t}'}/\mathrm{Q}$
  Combined $p + d$ data: $a = 0.399 \pm 0.023$ GeV^{-1}, $\chi^2/N_{df} = 0.72$;
  $b = 0.20 \pm 0.07$, $\chi^2/N_{df} = 1.09$.
  Behaviour of $\mathrm{Re}\{\mathrm{T}_{01}/\mathrm{T}_{00}\}$ is probably in a contradiction with high-Q asymptotics.
  Phase difference δ_{01} decreases with Q^2 and $\delta_{01} = (29 \pm 9)^\circ$ at $Q^2 = 0.8$ GeV^2.

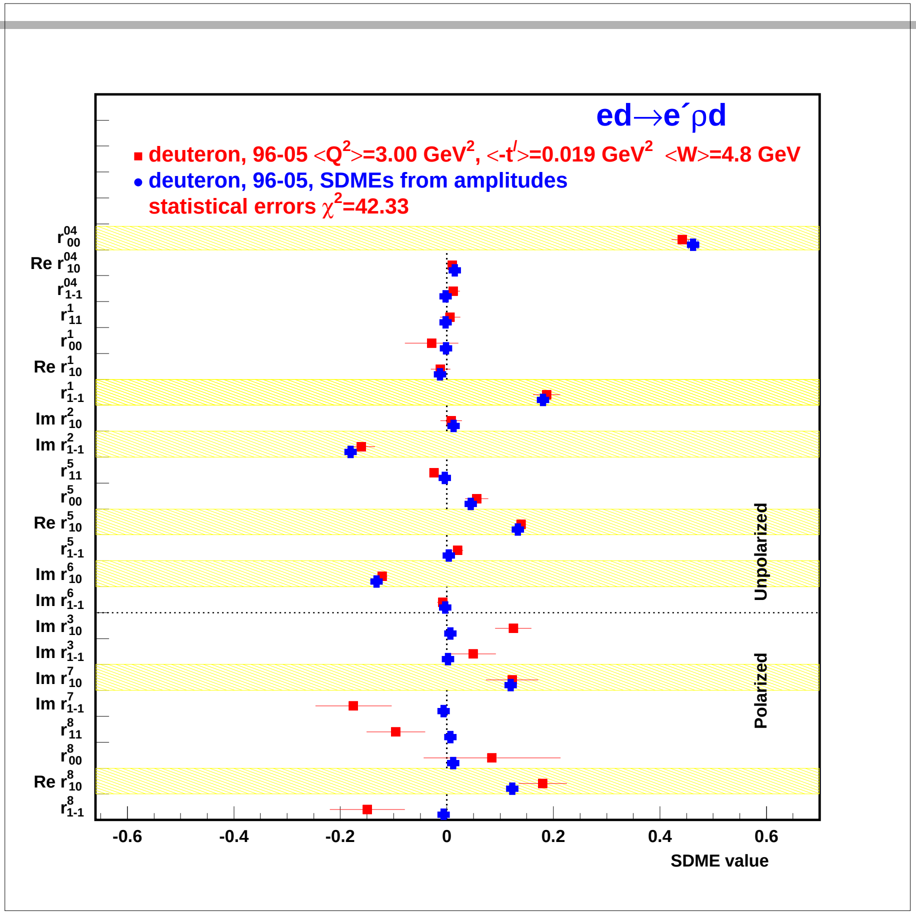
Direct Extraction of Amplitude Ratios from Angular Distributions



- pQCD prediction: $U_{11}/T_{00} \propto M_V/Q$
- Neither Q^2 nor t' dependence: $|U_{11}/T_{00}| = a$, $a = 0.391 \pm 0.013$, $\chi^2/N_{df} = 0.44$
- Unnatural Parity Exchange (UPE) is seen much better than in SDME method
- Contradiction both with high-Q asymptotics and one pion exchange dominance for UPE amplitude

Comparison of Amplitude and SDME Methods

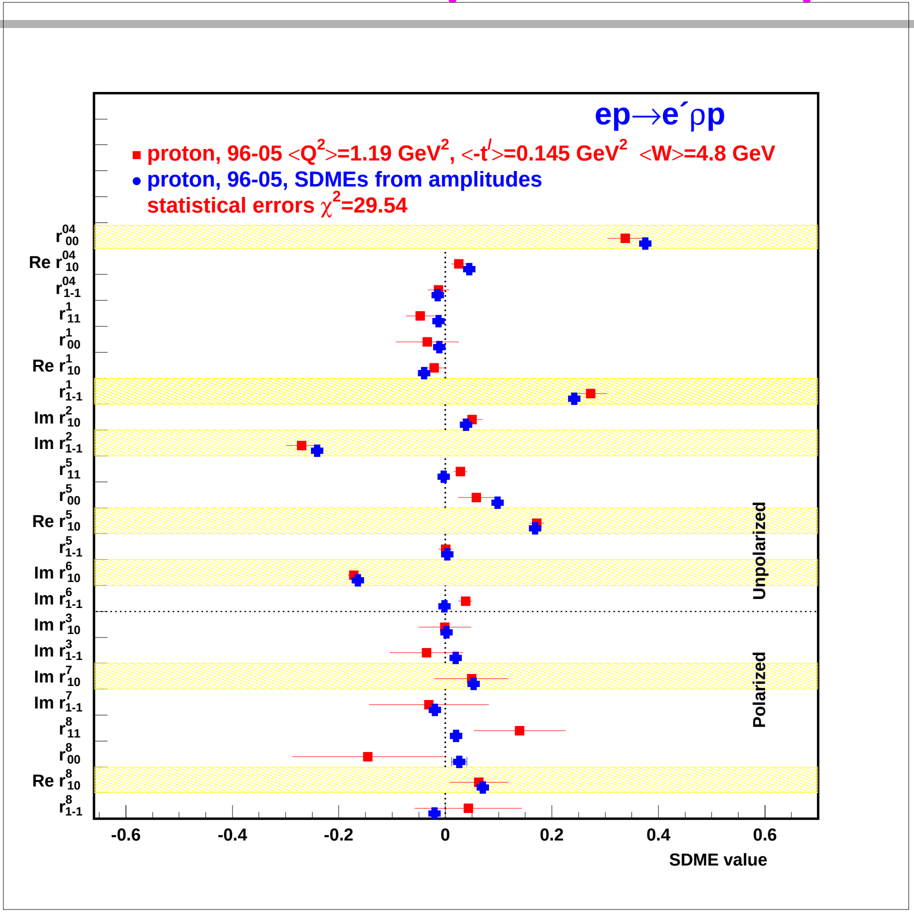


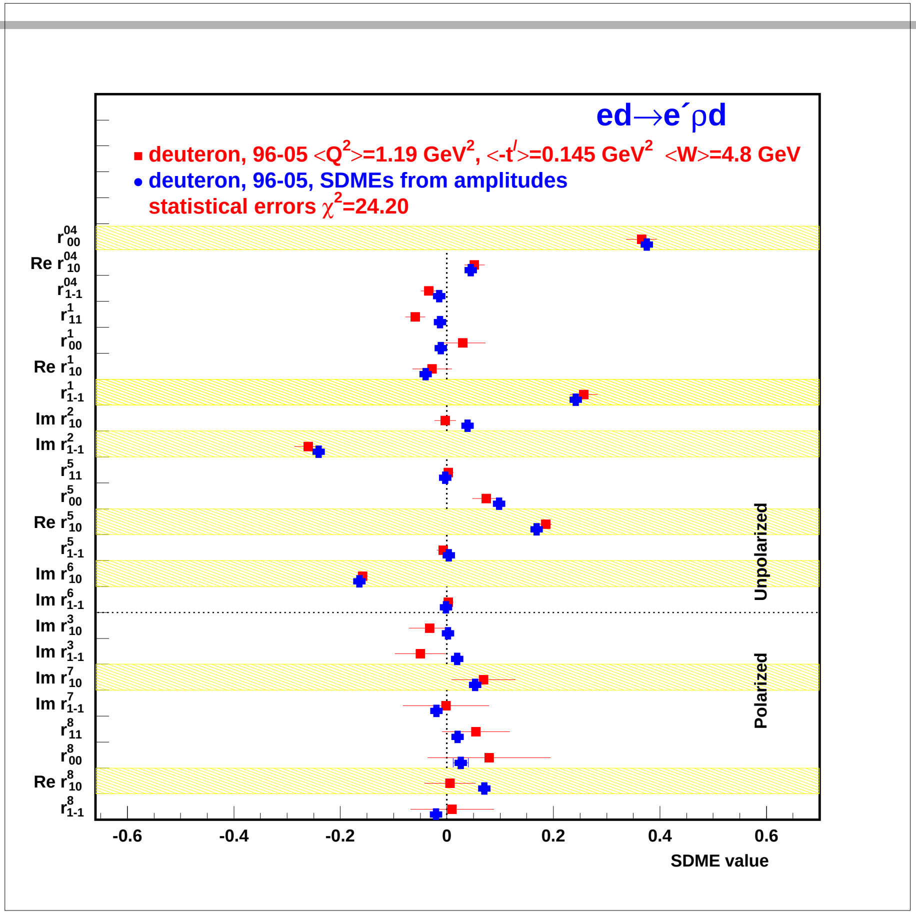


- Number of free parameters in amplitude method is equal to 9 while SDME approach needs 23x16=368 free parameters

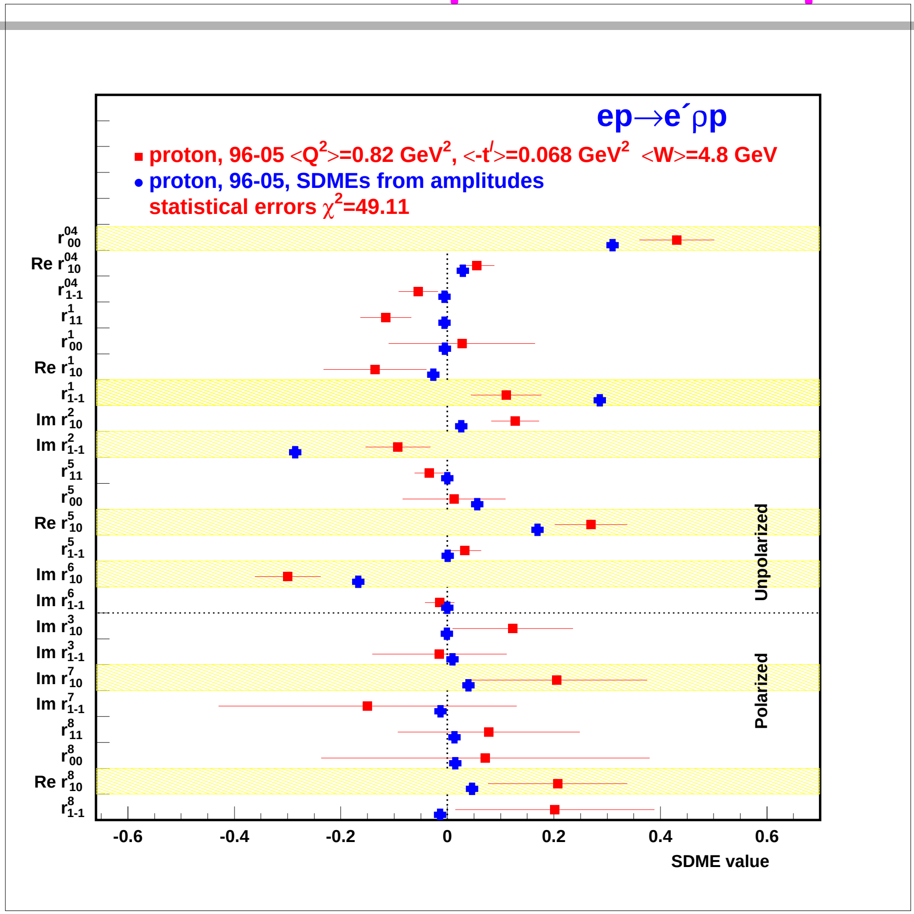
- Accuracy of SDMEs calculated through the amplitude ratios are higher than those in direct extraction of SDMEs (SDME method).

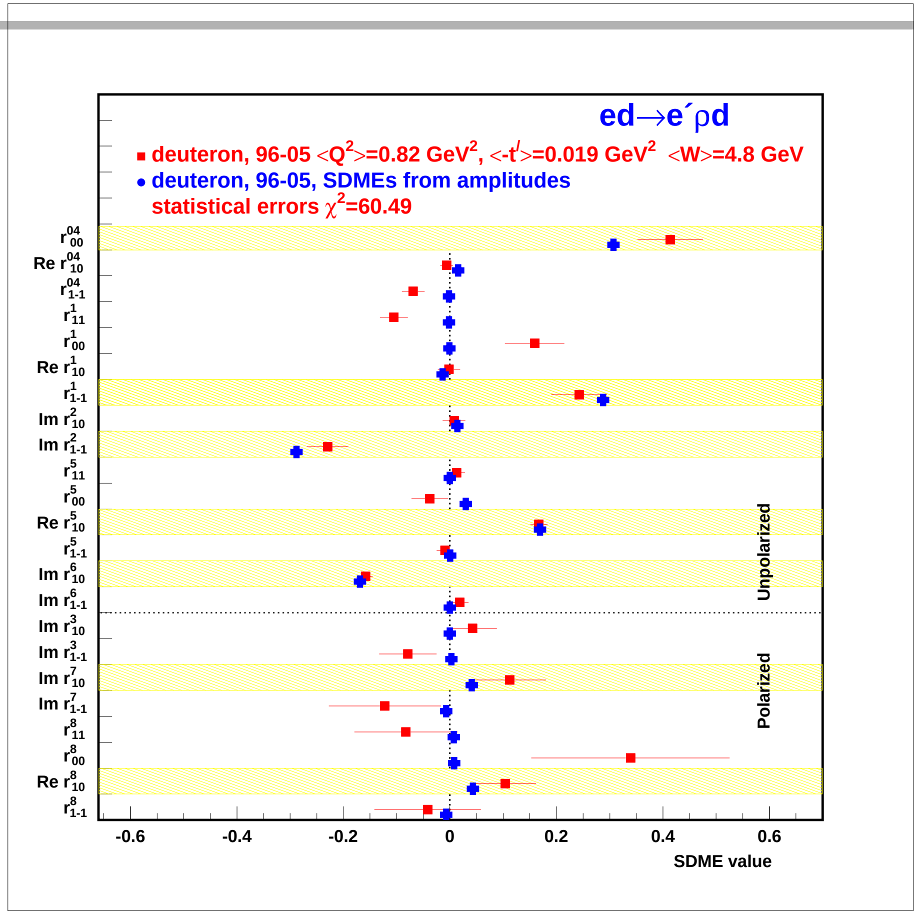
Comparison of Amplitude and SDME Methods





Comparison of Amplitude and SDME Methods





- Number of free parameters in amplitude method is equal to 9 while SDME approach needs 23x16=368 free parameters

- Kinematic dependence of SDMEs is reproduced better in the amplitude method rather than in SDME method

Summary

- 15 unpolarized and, for the first time, 8 polarized LU SDMEs are obtained for ρ^0 and ϕ produced by longitudinally polarized electron/positron beam on unpolarized proton and deuteron. For the first time, HERMES measured 30 SDMEs for ρ^0 and single spin asymmetry for ω in experiment with transversely polarized proton.

- Violation of S-channel helicity is observed for several SDMEs in ρ^0 production both on proton and deuteron with $3 \div 10\ \sigma_{tot}$. No signal of S-channel helicity violation is found for ϕ meson.

- No statistically significant difference between proton and deuteron data on electroproduction of ρ^0 and ϕ mesons is observed.

- Ratios of amplitudes T_{11}/T_{00}, T_{01}/T_{00}, T_{10}/T_{00}, T_{1-1}/T_{00}, and $|U_{11}/T_{00}|$ are extracted in 16 bins from angular distributions of $\pi^+\pi^-$ in ρ^0-meson production. Their Q^2 and t' dependences have been studied and compared with pQCD prediction of asymptotic behaviour.

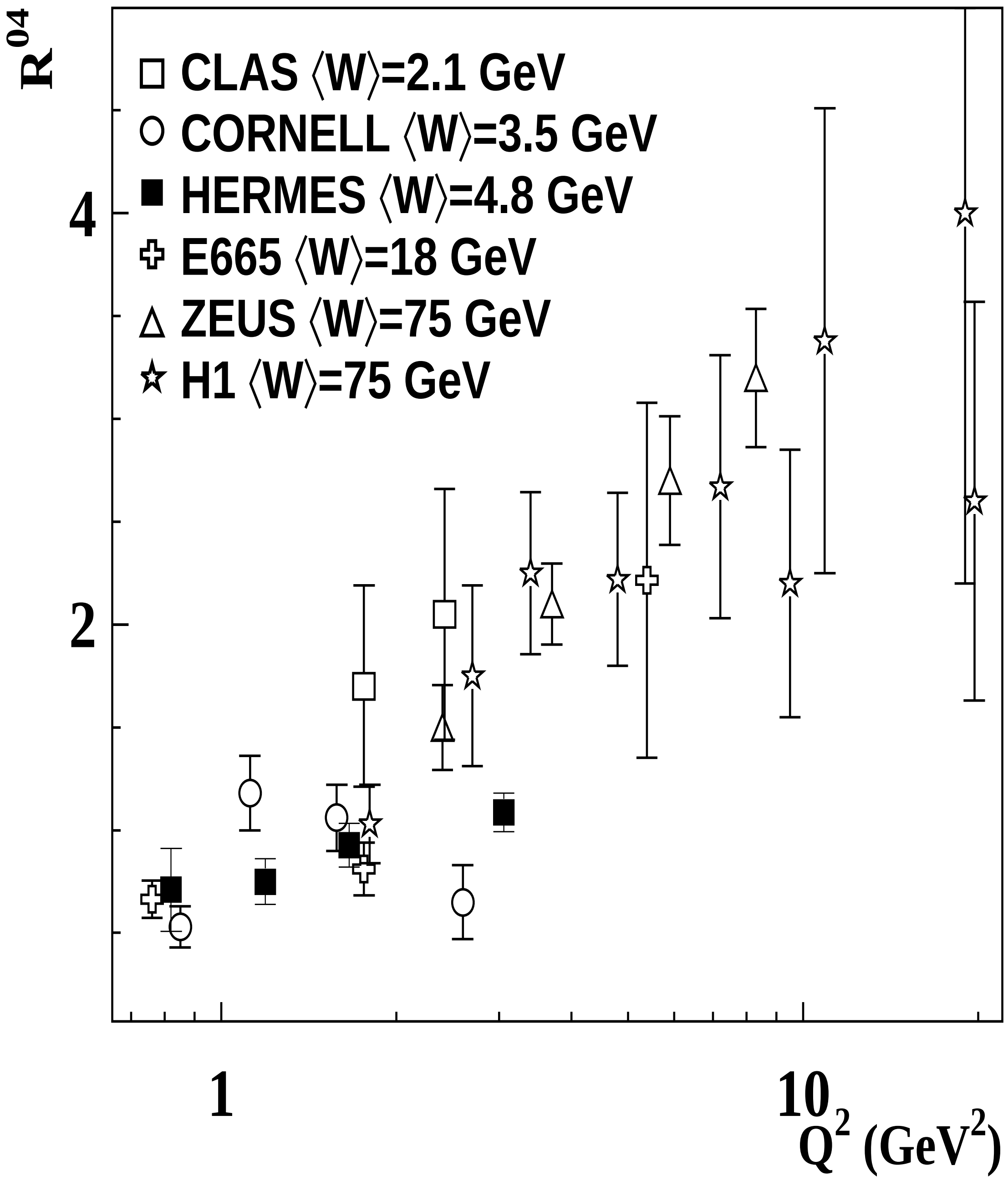
- Contribution of unnatural parity exchange amplitude in ρ^0-meson production is found with much higher accuracy than in SDME analysis.

- Amplitude method permits to describe the HERMES data with 9 parameters which is much less than the free parameter number (23x16=368) in the SDME method.

Outlook

- To decrease background contribution by measuring recoil nucleon.
- Include data on transversely polarized target into amplitude analysis.

Longitudinal-to-Transverse Cross-section Ratio for ρ^0 Meson



$$R = \sigma_L/\sigma_T = N_L/N_T,$$

$$N_L = \frac{1}{2}\sum_{\lambda_N \lambda'_N}\Big(|T_{00}|^2 + 2|T_{10}|^2 + 2|U_{10}|$$

$$N_T = \frac{1}{2}\sum_{\lambda_N \lambda'_N}\Big(|T_{11}|^2 + |T_{01}|^2 + |T_{-11}|$$

$$+|U_{11}|^2 + |U_{01}|^2 + |U_{-11}|^2\Big).$$

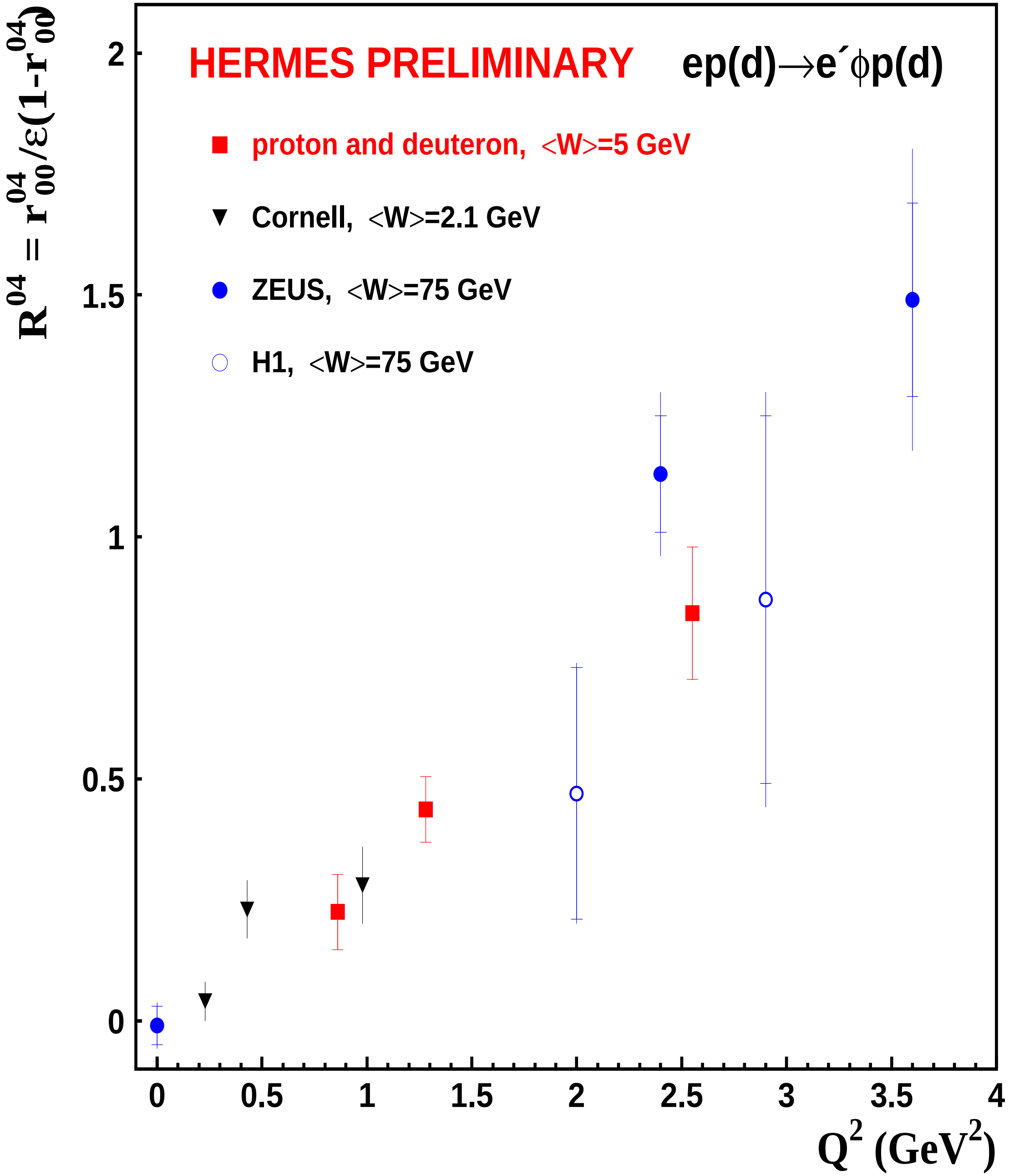
Second order contribution of spin-flip amplitudes (violating SCHC) to R.

SCHC approximation

$$R^{SCHC} = \frac{|T_{00}|^2}{|T_{11}|^2 + |U_{11}|^2} \approx R^{04},$$

$$R^{04} = \frac{r^{04}_{00}}{\epsilon(1 - r^{04}_{00})}.$$

Longitudinal-to-Transverse Cross-Section Ratio for ϕ Mesons

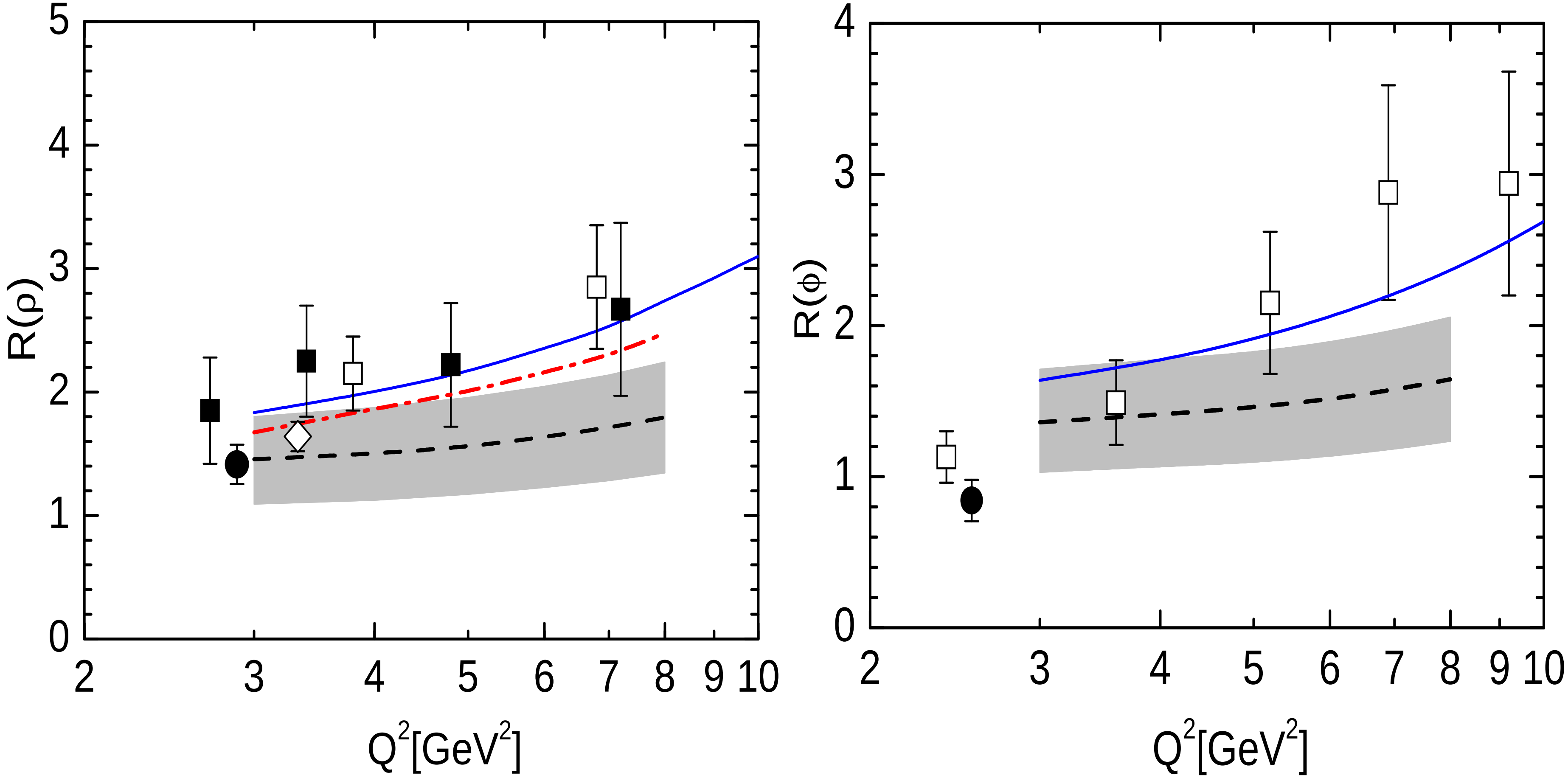


- SCHC approximation for $R = \sigma_L/\sigma_T$

$$R \approx R^{04} = \frac{r^{04}_{00}}{\epsilon(1 - r^{04}_{00})}$$

- Linear dependence of R^{04} on Q^2
- Agreement with world data
- Two-gluon (Pomeron) exchange dominance

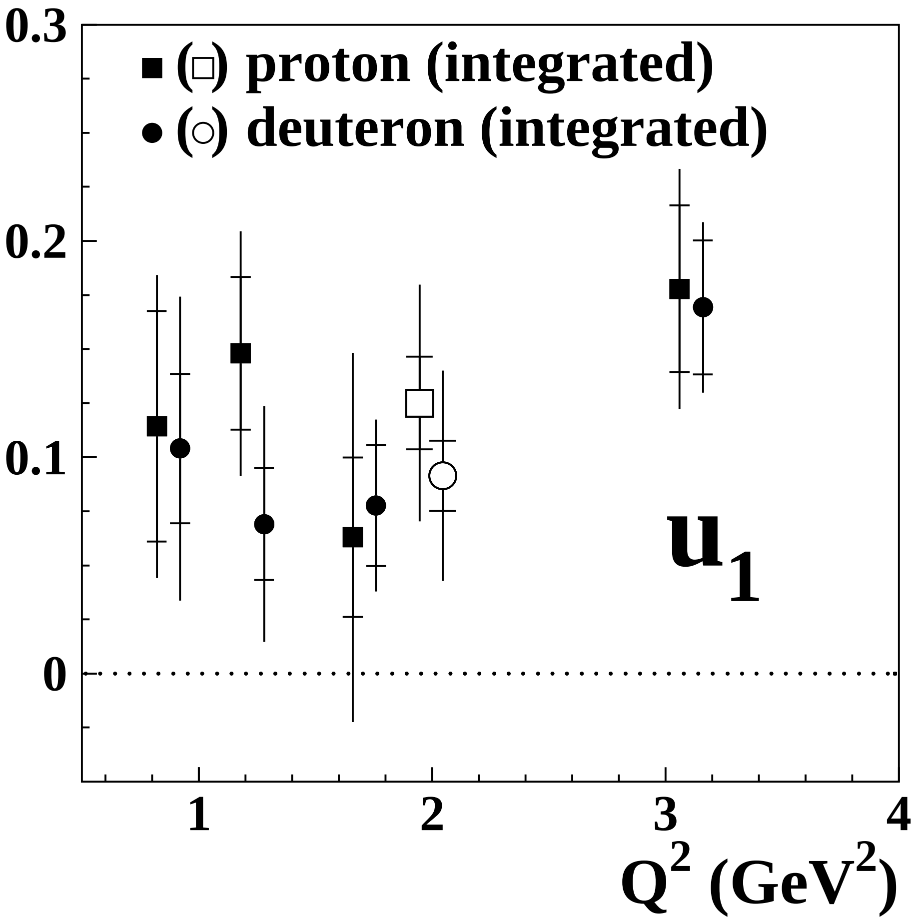
Comparison of R^{04} for ρ^0 and ϕ mesons with Calculations in GK Model



Calculations in the Goloskokov-Kroll (GK) model after tuning GPD parameters.
Blue line W=90 GeV, red line W=10 GeV, black line W=5 GeV.
Data: solid squares - H1, open squares - ZEUS, diamond - COMPASS,
circles - HERMES (without UPE contribution).

$\Longrightarrow$ **W-dependence of R^{04} is confirmed by calculations.**

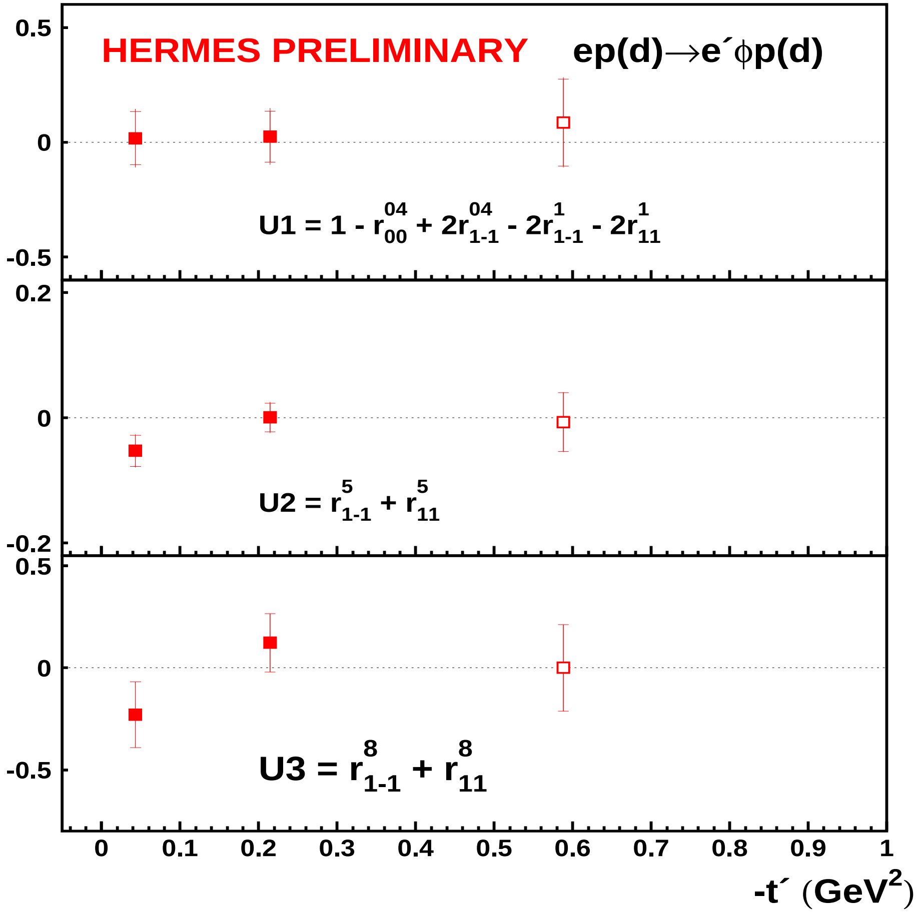
Test of Unnatural-Parity Exchange for ρ^0 meson

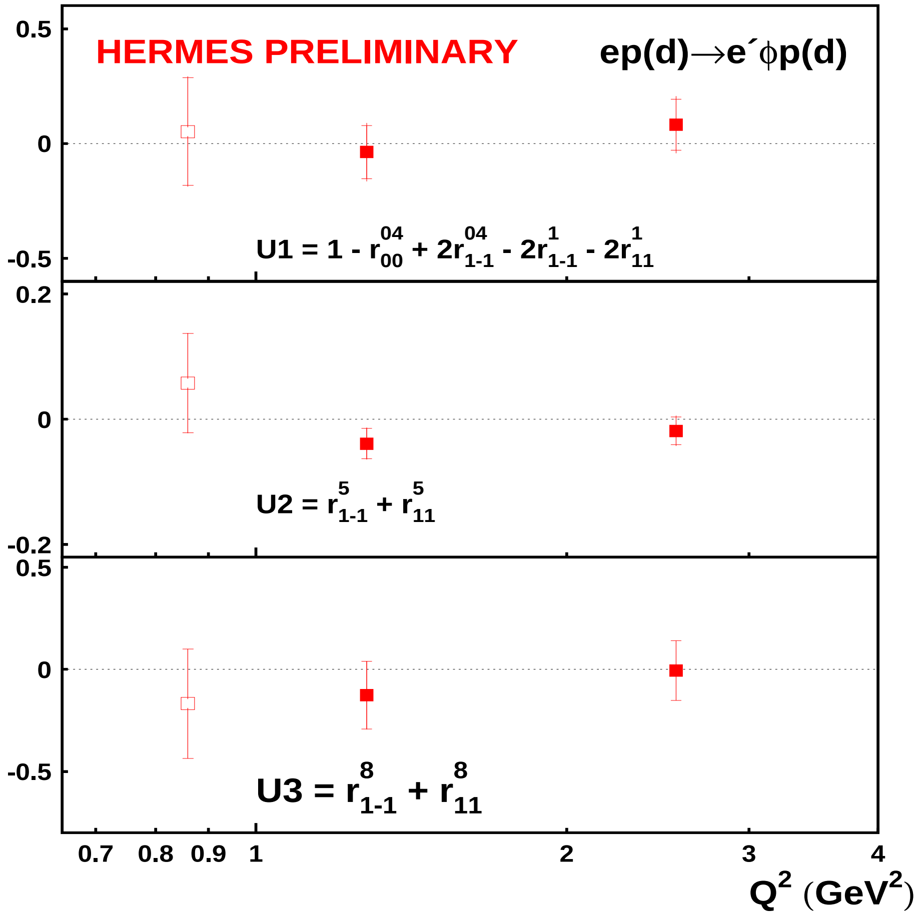


$u_1 = 0.125 \pm 0.021_{stat} \pm 0.050_{syst}$ (H),
$u_1 = 0.091 \pm 0.016_{stat} \pm 0.046_{syst}$ (D)
$u_1 = 0.106 \pm 0.036_{tot}$ (H+D)

- Natural and Unnatural Parity Exchanges in the t-channel
  NPE: GPD H, E ; $T_{\lambda_\rho \lambda_\gamma}$
  UPE: GPD $\tilde{H}$, $\tilde{E}$; $U_{\lambda_\rho \lambda_\gamma}$
  NPE (Pomeron, ρ, ω, f_2, a_2, ...) dominate and UPE (π, a_1, b_1...) are suppressed at high energies
- $F_{\lambda_\rho \lambda_\gamma} = T_{\lambda_\rho \lambda_\gamma} + U_{\lambda_\rho \lambda_\gamma}$
  where $T_{\lambda_\rho \lambda_\gamma} =$
  $(F_{\lambda_\rho \lambda_\gamma} + (-1)^{\lambda_\rho - \lambda_\gamma} F_{-\lambda_\rho - \lambda_\gamma})/2$
  No interference between NPE and UPE contributions to SDMEs $r^\alpha_{\lambda_\rho \lambda'_\rho}$ for **unpolarized** target
- $u_1 = 1 - r^{04}_{00} + 2r^{04}_{1-1} - 2r^1_{11} - 2r^1_{1-1}$,
  $u_1 = \sum_{\lambda_N \lambda'_N} (2\epsilon |U_{10}|^2$
  $+ |U_{11} + U_{-11}|^2)/(N_T + \epsilon N_L)$

Test of Unnatural-Parity Exchange for ϕ meson





$$u_1 = 0.02 \pm 0.07_{stat} \pm 0.16_{syst},$$
$$u_2 = -0.03 \pm 0.01_{stat} \pm 0.03_{syst},$$
$$u_3 = -0.05 \pm 0.12_{stat} \pm 0.07_{syst}.$$

No signal of unnatural parity exchange (as expected since Pomeron has natural parity).

Angular Distributions in ρ^0 Electroproduction

$$\mathcal{W}^{U+L}(\Phi,\phi,\cos\Theta) = \mathcal{W}^{U}(\Phi,\phi,\cos\Theta) + P_B \mathcal{W}^{L}(\Phi,\phi,\cos\Theta),$$

$$\mathcal{W}^{U}(\Phi,\phi,\cos\Theta) = \frac{3}{8\pi^2}\Bigg\{\frac{1}{2}(1-r^{04}_{00}) + \frac{1}{2}(3r^{04}_{00}-1)\cos^2\Theta$$

$$-\sqrt{2}\mathrm{Re}\{r^{04}_{10}\}\sin 2\Theta\cos\phi - r^{04}_{1-1}\sin^2\Theta\cos 2\phi$$

$$-\epsilon\Bigg[\sin 2\Phi\Big(\sqrt{2}\mathrm{Im}\{r^{2}_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^{2}_{1-1}\}\sin^2\Theta\sin 2\phi\Big)$$

$$+\cos 2\Phi\Big(r^{1}_{11}\sin^2\Theta + r^{1}_{00}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^{1}_{10}\}\sin 2\Theta\cos\phi - r^{1}_{1-1}\sin^2\Theta\cos 2\phi\Big)\Bigg]$$

$$+\sqrt{2\epsilon(1+\epsilon)}\Bigg[\sin\Phi\Big(\sqrt{2}\mathrm{Im}\{r^{6}_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^{6}_{1-1}\}\sin^2\Theta\sin 2\phi\Big)$$

$$+\cos\Phi\Big(r^{5}_{11}\sin^2\Theta + r^{5}_{00}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^{5}_{10}\}\sin 2\Theta\cos\phi - r^{5}_{1-1}\sin^2\Theta cos 2\phi\Big)\Bigg]\Bigg\},$$

Angular Distributions in ρ^0 Electroproduction

$$\mathcal{W}^L(\Phi,\phi,\cos\Theta) = \frac{3}{8\pi^2}\Bigg\{\sqrt{1-\epsilon^2}\Big(\sqrt{2}\mathrm{Im}\{r^3_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^3_{1-1}\}\sin^2\Theta\sin 2\phi\Big)$$

$$+\sqrt{2\epsilon(1-\epsilon)}\Bigg[\cos\Phi\Big(\sqrt{2}\mathrm{Im}\{r^7_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^7_{1-1}\}\sin^2\Theta\sin 2\phi\Big)$$

$$+\sin\Phi\Big(r^8_{11}\sin^2\Theta + r^8_{00}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^8_{10}\}\sin 2\Theta\cos\phi - r^8_{1-1}\sin^2\Theta\cos 2\phi\Big)\Bigg]\Bigg\}.$$

23 SDMEs Expressed in Terms of Helicity Amplitudes

$$r^{04}_{00} = \widetilde{\sum}\{\epsilon|T_{00}|^2 + |T_{01}|^2 + |U_{01}|^2/\}/\mathcal{N}\,, \qquad (1)$$

$$r^{1}_{1-1} = \frac{1}{2}\widetilde{\sum}\{|T_{11}|^2 + |T_{1-1}|^2 - |U_{11}|^2 - |U_{1-1}|^2\}/\mathcal{N}\,, \qquad (2)$$

$$\mathrm{Im}\{r^{2}_{1-1}\} = \frac{1}{2}\widetilde{\sum}\{-|T_{11}|^2 + |T_{1-1}|^2 + |U_{11}|^2 - |U_{1-1}|^2\}/\mathcal{N}\,, \qquad (3)$$

$$\mathrm{Re}\{r^{5}_{10}\} = \frac{1}{\sqrt{8}}\widetilde{\sum}\mathrm{Re}\{2T_{10}T^{*}_{01} + (T_{11} - T_{1-1})T^{*}_{00}\}/\mathcal{N}\,, \qquad (4)$$

$$\mathrm{Im}\{r^{6}_{10}\} = \frac{1}{\sqrt{8}}\widetilde{\sum}\mathrm{Re}\{2U_{10}U^{*}_{01} - (T_{11} + T_{1-1})T^{*}_{00}\}/\mathcal{N}\,, \qquad (5)$$

$$\mathrm{Im}\{r^{7}_{10}\} = \frac{1}{\sqrt{8}}\widetilde{\sum}\mathrm{Im}\{2U_{10}U^{*}_{01} + (T_{11} + T_{1-1})T^{*}_{00}\}/\mathcal{N}\,, \qquad (6)$$

$$\mathrm{Re}\{r^{8}_{10}\} = \frac{1}{\sqrt{8}}\widetilde{\sum}\mathrm{Im}\{-2T_{10}T^{*}_{01} + (T_{11} - T_{1-1})T^{*}_{00}\}/\mathcal{N}\,, \qquad (7)$$

$$\mathrm{Re}\{r^{04}_{10}\} = \widetilde{\sum}\mathrm{Re}\{\epsilon T_{10}T^{*}_{00} + \frac{1}{2}T_{01}(T_{11} - T_{1-1})^{*} + \frac{1}{2}U_{01}(U_{11} + U_{1-1})^{*}\}/\mathcal{N}\,, \qquad (8)$$

$$\mathrm{Re}\{r^{1}_{10}\} = \frac{1}{2}\widetilde{\sum}\mathrm{Re}\{-T_{01}(T_{11} - T_{1-1})^{*} + U_{01}(U_{11} + U_{1-1})^{*}\}/\mathcal{N}\,, \qquad (9)$$

$$\mathrm{Im}\{r^2_{10}\} = \frac{1}{2}\widetilde{\sum}\mathrm{Re}\{T_{01}(T_{11}+T_{1-1})^* - U_{01}(U_{11}-U_{1-1})^*\}/\mathcal{N}\,, \tag{10}$$

$$r^5_{00} = \sqrt{2}\widetilde{\sum}\mathrm{Re}\{T_{01}T^*_{00}\}/\mathcal{N}\,, \tag{11}$$

$$r^1_{00} = \widetilde{\sum}\{-|T_{01}|^2 + |U_{01}|^2\}/\mathcal{N}\,, \tag{12}$$

$$\mathrm{Im}\{r^3_{10}\} = -\frac{1}{2}\widetilde{\sum}\mathrm{Im}\{T_{01}(T_{11}+T_{1-1})^* + U_{01}(U_{11}-U_{1-1})^*\}/\mathcal{N}\,, \tag{13}$$

$$r^8_{00} = \sqrt{2}\widetilde{\sum}\mathrm{Im}\{T_{01}T^*_{00}\}/\mathcal{N}\,, \tag{14}$$

$$r^5_{11} = \frac{1}{\sqrt{2}}\widetilde{\sum}\mathrm{Re}\{T_{10}(T_{11}-T_{1-1})^* + U_{10}(U_{11}-U_{1-1})^*\}/\mathcal{N}\,, \tag{15}$$

$$r^5_{1-1} = \frac{1}{\sqrt{2}}\widetilde{\sum}\mathrm{Re}\{-T_{10}(T_{11}-T_{1-1})^* + U_{10}(U_{11}-U_{1-1})^*\}/\mathcal{N}\,, \tag{16}$$

$$\mathrm{Im}\{r^6_{1-1}\} = \frac{1}{\sqrt{2}}\widetilde{\sum}\mathrm{Re}\{T_{10}(T_{11}+T_{1-1})^* - U_{10}(U_{11}+U_{1-1})^*\}/\mathcal{N}\,, \tag{17}$$

$$\mathrm{Im}\{r^{7}_{1-1}\} = \frac{1}{\sqrt{2}}\widetilde{\sum}\mathrm{Im}\{T_{10}(T_{11}+T_{1-1})^{*} - U_{10}(U_{11}+U_{1-1})^{*}\}/\mathcal{N}, \tag{18}$$

$$r^{8}_{11} = -\frac{1}{\sqrt{2}}\widetilde{\sum}\mathrm{Im}\{T_{10}(T_{11}-T_{1-1})^{*} + U_{10}(U_{11}-U_{1-1})^{*}\}/\mathcal{N}, \tag{19}$$

$$r^{8}_{1-1} = \frac{1}{\sqrt{2}}\widetilde{\sum}\mathrm{Im}\{T_{10}(T_{11}-T_{1-1})^{*} - U_{10}(U_{11}-U_{1-1})^{*}\}/\mathcal{N}, \tag{20}$$

$$r^{04}_{1-1} = \widetilde{\sum}\mathrm{Re}\{-\epsilon|T_{10}|^{2} + \epsilon|U_{10}|^{2} + T_{1-1}T^{*}_{11} - U_{1-1}U^{*}_{11}\}/\mathcal{N}, \tag{21}$$

$$r^{1}_{11} = \widetilde{\sum}\mathrm{Re}\{T_{1-1}T^{*}_{11} + U_{1-1}U^{*}_{11}\}/\mathcal{N}, \tag{22}$$

$$\mathrm{Im}\{r^{3}_{1-1}\} = -\widetilde{\sum}\mathrm{Im}\{T_{1-1}T^{*}_{11} - U_{1-1}U^{*}_{11}\}/\mathcal{N}, \tag{23}$$

where the normalization factor $\mathcal{N} = \mathcal{N}_T + \epsilon\mathcal{N}_L$

$$\mathcal{N}_T = \widetilde{\sum}(|T_{11}|^2 + |T_{01}|^2 + |T_{-11}|^2 + |U_{11}|^2 + |U_{01}|^2 + |U_{-11}|^2)$$

$$\mathcal{N}_L = \widetilde{\sum}(|T_{00}|^2 + 2|T_{10}|^2 + 2|U_{10}|^2)$$