Feasibility study for reaction $(p, p'\gamma)$ at Gatchina accelerator

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HEPD Scientific Council
Gatchina, 2022, December 22

Contents

Introduction

- Registration of gamma-quantum energy fixes the nucleus energy level with better accuracy than the magnetic spectrometer

- Study of the differential cross section at $t \to 0$ permits to establish the parity P of a natural parity level

- Investigation of the gamma-quantum angular distribution in coincidence with the scattered hadron allows to extract the Glauber amplitudes of nucleus excitation $F_M(q)$ and establish level's quantum numbers J^P $(M = J_z\ )$

- Knowledge of the amplitudes of nuclear level excitation offers a clearer understanding of nuclear structure than that of differential cross sections

- Measurements of γ-quantum energy spectrum permits to investigate also reactions ${}^AX(p,pp)^{(A-1)}Y^*$, ${}^AX(p,pn)^AX^*$ and ${}^AX(p,p\alpha)^{(A-4)}Z^*$ with excited nuclei X^*, Y^*, Z^* in the final state

Properties of Nucleus-Excitation Amplitudes in Glauber Theory

- Selection Rules for Nuclei with Ground State $J^P = 0^+$
  If the hadron-nucleon spin-dependent invariant amplitudes are much smaller than spin-independent amplitude, than the states J^P are exited with $M = J_z$ obeying the relation
  $$P(-1)^M = 1.$$
  Examples. Natural parity level 3^-: $M = \pm 3,\ \pm 1$.
  Unnatural parity level 3^+: $M = \pm 2,\ 0$. Natural parity level 2^+: $M = \pm 2,\ 0$.
  Even (odd) values of M correponds to $P = 1$ $(P = -1)$.

- Relation between $F_M(q)$ and $F_{-M}(q)$: $F_{-M}(q) = (-1)^J F_M(q)$
  Examples. Unnatural parity level 3^+: $F_{-2}(q) = -F_2(q)$,
  $F_{-0}(q) = (-1)^3 F_0(q) \Rightarrow F_0(q) \equiv 0$. It is general property of unnatural parity levels.
  Natural parity level 2^+: $F_{-0}(q) = (-1)^2 F_0(q)$, $F_0(q) \neq 0$.

- Asymptotic Behavier of Amplitudes $F_M(q)$ at $qR \to 0$: $F_M(q) \sim (qR)^{|M|}$.
  Examples. For 3^- level main terms: $F_{\pm 1}(q) \sim (qR)^1$.
  For 2^+ level main term: $F_0(q) \sim (qR)^0 = \mathrm{const}$.
  For 3^+ level main terms: $F_{\pm 2}(q) \sim (qR)^2$.
  1^+ level is not excited at high energies at all by spin-independent hadron-nucleon amplitudes since $F_0(q) \equiv 0$.

Photon Radiation with Excited Nuclei

- State of photon is described by vector spherical harmonics $\vec{Y}^{(\lambda)}_{JM}$
  $\lambda = 1$ electric type (E) or $\lambda = 0$ magnetic type (M). $\vec{Y}^{(1)}_{JM_1} \cdot \vec{Y}^{*(1)}_{JM_2} = \vec{Y}^{(0)}_{JM_1} \cdot \vec{Y}^{*(0)}_{JM_2}$
  For transition $J^P \to 0^+$ $\lambda = 1$ (electric type E), if $P = (-1)^J$ (natural parity level),
  while $\lambda = 0$ (magnetic type M), if $P = -(-1)^J$ (unnatural parity level).

- Angular distribution of photons emitted in the transition $J^P \to 0^+$
  $$\mathcal{W}(\theta, \varphi, \vec{q}) = |\vec{\mathcal{F}}|^2 = \left|\sum_M F_M(q)\vec{Y}^{(\lambda)}_{JM}(\theta,\varphi)\right|^2 = \sum \varrho_{M_1M_2}\vec{Y}^{(\lambda)}_{JM_1}(\theta,\varphi)\cdot\vec{Y}^{*(\lambda)}_{JM_2}(\theta,\varphi),$$
  where θ is polar and φ is azimuthal angle of photon momentum
  (Z-axis is along beam, X-axis is parallel to $\vec{q}$). $\vec{Y}^{(\lambda)}_{JM}(\theta,\varphi) = e^{iM\varphi}\vec{\xi}^{(\lambda)}_{JM}(\theta)$.
  Example: $3^- \to 0^+$. $F_{-3}(\vec{q}) = -F_3(\vec{q})$, $F_{-1}(\vec{q}) = -F_1(\vec{q})$,
  $\vec{\mathcal{F}}(\theta,\varphi,\vec{q}) = F_3(\vec{q})(\vec{Y}^{(E)}_{33}(\theta,\varphi) - \vec{Y}^{(E)}_{3-3}(\theta,\varphi)) + F_1(\vec{q})(\vec{Y}^{(E)}_{31}(\theta,\varphi) - \vec{Y}^{(E)}_{3-1}(\theta,\varphi))$.
  Two complex amplitudes $F_3(q)$ and $F_1(q) \equiv$ 4 real function of q.
  Common phase is unmeasurable, hence 3 function of q can be extracted from data.

- Which information contain amplitudes $F_M(q)$ at $qR \to 0$?
  $F_M(q) = i^{M+1}k\int_0^\infty J_M(qb)\langle\langle\epsilon_f, P, J, M|\Gamma(\vec{b}-\vec{s}_1, \vec{b}-\vec{s}_2, ..., \vec{b}-\vec{s}_A)|\epsilon_i, 0^+\rangle\rangle bdb$,
  where $\vec{s}_j$ is transverse part of radius-vector $\vec{r}_j$ of j-th nucleon in nucleus $(1 \le j \le A)$,
  while $\vec{b}$ is impact vector of beam hadron.
  Bessel functions $J_M(qb) \sim (qb)^{|M|}$ at $q \to 0$.
  The larger $|M|$ the more peripherical is contribution to $F_M(q)$ at small q.

Contributions of Spin-Dependent Parts of Hadron-Nucleon Amplitude

- Pion-nucleon amplitude: $f(\vec{q}) = a(q^2) + b(q^2)(\vec{\sigma} \cdot \vec{n})$,
  where $\vec{n} \propto (\vec{k} \times \vec{q})$ is the unit normal to the hadron-nucleon scattering plane.
  $F_M(q) = F_M^{(a)}(q) + F_M^{(b,a)}(q)$. Linear contributions of πN-amplitude $b(q^2)$ to $F_M^{(b,a)}$.
  Selection rules for $F_M^{(b,a)}(q)$: $P(-1)^M = -1$.
  Relations between $F_M^{(b,a)}(q)$ and $F_{-M}^{(b,a)}(q)$: $F_{-M}^{(b,a)}(q) = -(-1)^J F_M^{(b,a)}(q)$.
  Since different $F_M(q)$ are multiplied by different vector spherical harmonics $\vec{Y}_{JM}^{(X)}$ all amplitude moduli and phase differences of $F_M(q)$ can be extracted from data, besides the common phase shift of the amplitudes retains unknown.

- Example: $J^P = 3^-$. Nonzero $F_M^{(b,a)}(q)$ for $M = 0, \ \pm 2$.
  $F_{-2}^{(b,a)}(q) = F_2^{(b,a)}(q), \ F_0^{(b,a)}(q) \neq 0$.
  Big amplitudes $F_3^{(a)}(q), \ F_1^{(a)}(q)$; small amplitudes $F_2^{(b,a)}(q), \ F_0^{(b,a)}(q)$.
  Common phase is unmeasurable, hence 7 function of q can be extracted from data.

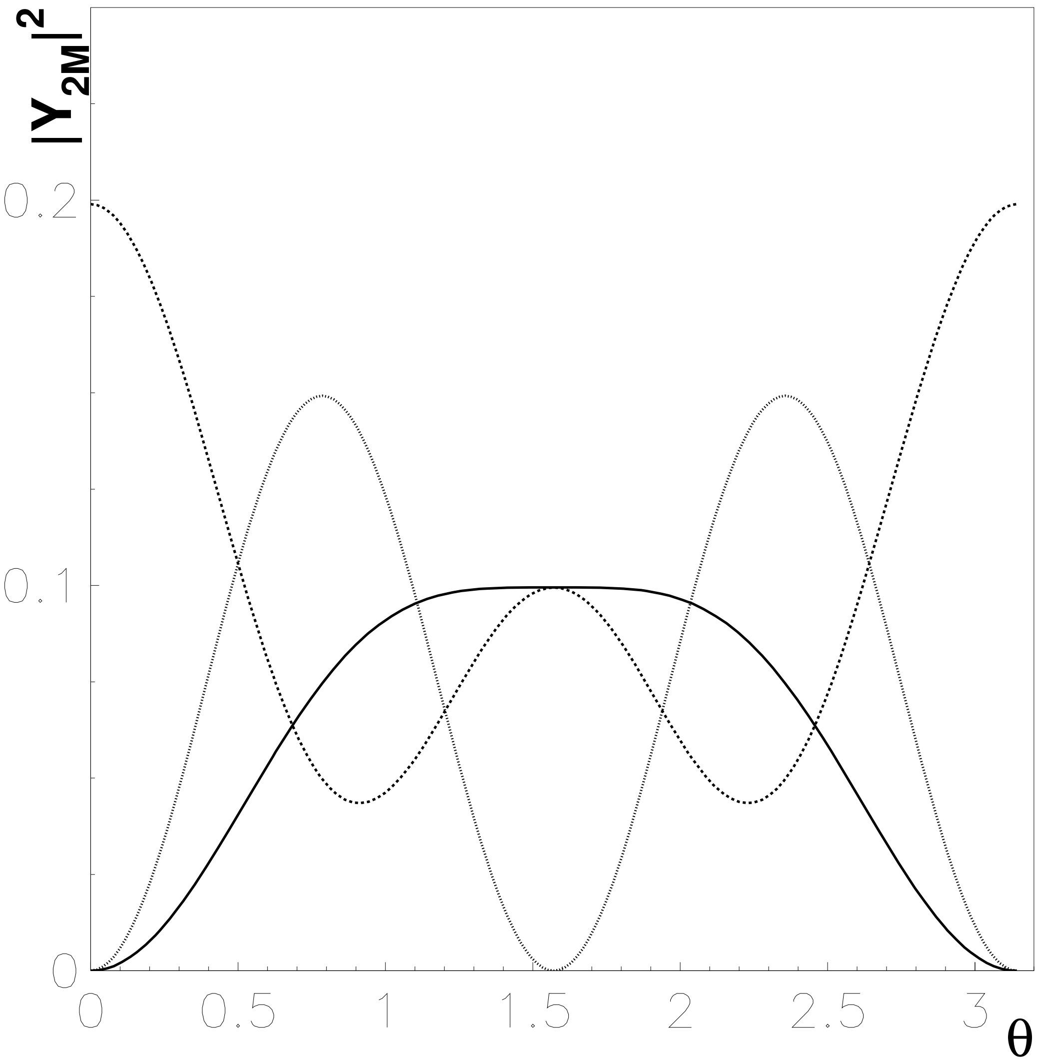
- Types of extracted form factors for pion scattering in impulse approximation

  $F_M^{(a)}(q) : A_M(q) = \langle \epsilon_f, P, J, M | \sum_{j=1}^{A} \exp\{i(\vec{q} \cdot \vec{r}_j)\} | \epsilon_i, 0^+ \rangle$.

  $F_M^{(b,a)}(q) : B_M(q) = \langle \epsilon_f, P, J, M | \sum_{j=1}^{A} (\vec{\sigma}_j \cdot \vec{n}) \exp\{i(\vec{q} \cdot \vec{r}_j)\} | \epsilon_i, 0^+ \rangle, \ \vec{n} \propto (\vec{k} \times \vec{q})$.

Angular Distribution of Gammma-Quanta

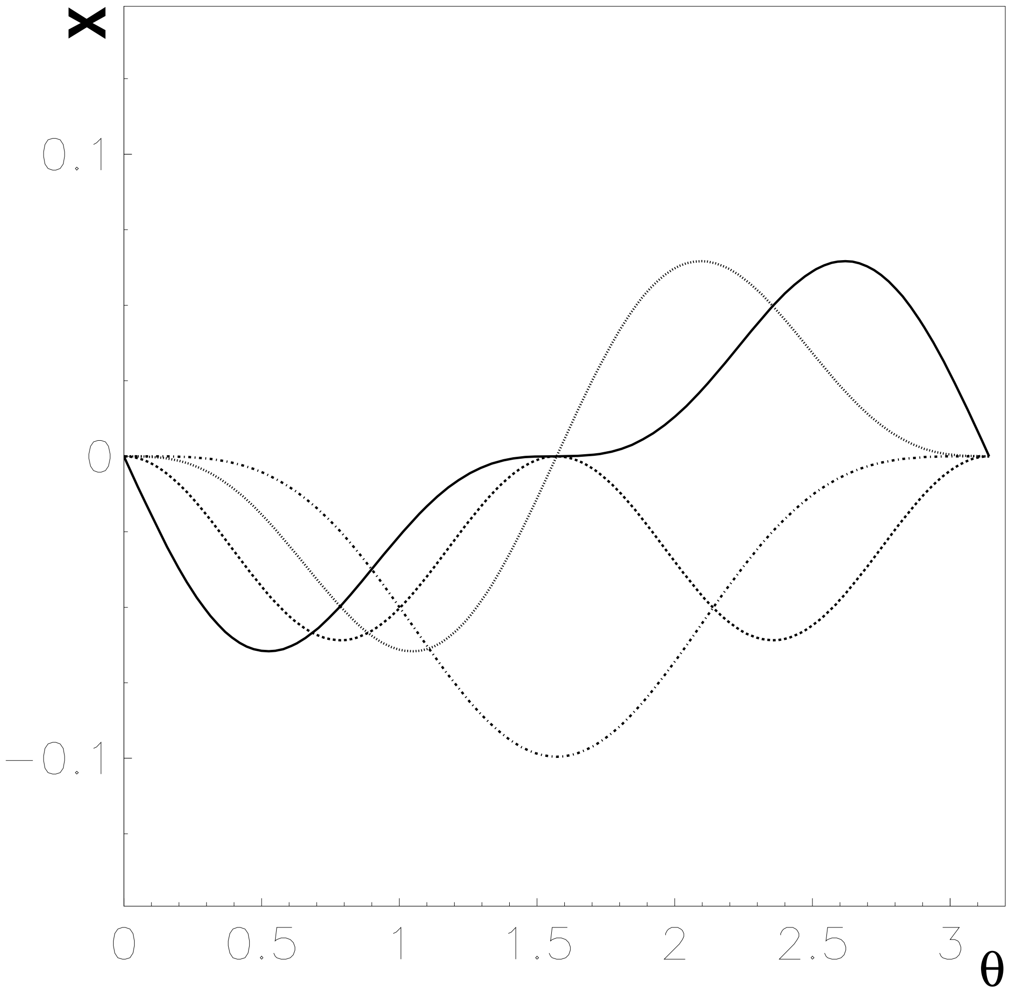
- Gamma-Quanta from Transition $2^+ \to 0^+$



Solid line – $|\vec{Y}^{(1)}_{22}|^2$, dashed line – $|\vec{Y}^{(1)}_{21}|^2$, dotted line – $|\vec{Y}^{(1)}_{20}|^2$.

Angular Distribution of Gammma-Quanta

- Gamma-Quanta from Transition $2^+ \to 0^+$



Solid line: $X = (\vec{Y}^{(1)}_{22} \cdot \vec{Y}^{*(1)}_{21})(\varphi = 0)$, dashed line: $X = (\vec{Y}^{(1)}_{22} \cdot \vec{Y}^{*(1)}_{20})(\varphi = 0)$,
dotted line: $X = (\vec{Y}^{(1)}_{22} \cdot \vec{Y}^{*(1)}_{2-1})(\varphi = 0)$,
dash-dotted line: $X = (\vec{Y}^{(1)}_{22} \cdot \vec{Y}^{*(1)}_{2-2})(\varphi = 0)$.

Experiments with γ-Quanta in the Last Century

- Excitation of 3^- (6,13 MeV) level in ^{16}O by protons; ΔE_γ(Ge(Li))$= 10 \div 15$ KeV
  Yu.M. Goryachev, V.P. Kanavets, I.V. Kirpichnikov, I.I. Levintov, B.V. Morozov, N.A. Nikiforov, A.S. Starostin (ITEP group), Excitation of nuclear levels in O^{16} with high energy protons, Phys. of Atom. Nucl. (Yad. Fiz.) 17 (1973) 910.

Beam energy (GeV)	σ^*, mb
1,0	7,3 $\pm$ 1,1
2,9	8,5 $\pm$ 1,4
6,3	7,8 $\pm$ 1,3

- Radiation of γ-quanta with nuclei excited by high-energy hadrons
  S. I. Manaenkov, Emission of γ-quanta with nuclei excited with high energy hadrons, *JETP Letters* **18** (1973) 535-538.
  S. I. Manaenkov, Excitation of individual levels in ^{16}O with protons at energy 1 GeV, *JETP Letters* **19** (1974) 593-597.
  S. I. Manaenkov: γ-radiation of nuclei excited with high energy hadrons, *Yad. Fiz.* **20** (1974) 677-689.
  S. I. Manaenkov: Study of NN amplitude in nucleon-nucleus scattering experiments at high energies, *Yad. Fiz.* **26** (1977) 302-311.

- γ-Quantum Spectrum of ^{16}O Excited with the 1 GeV Protons

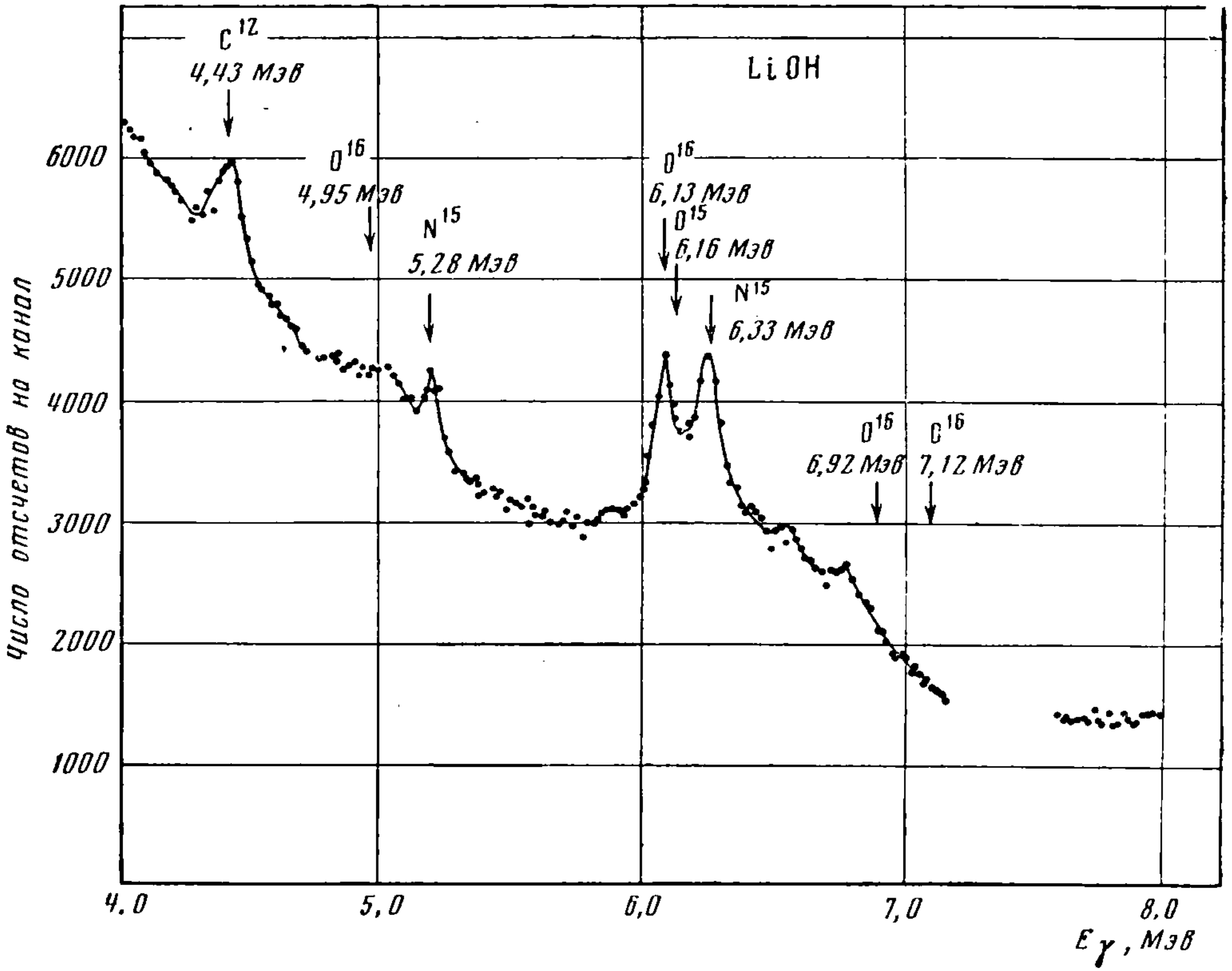


Рис. 2. Спектр γ-излучения, испускаемого LiOH мишенью при облучении протонами с энергией $T_p = 1$ *Гэв*. Угол наблюдения 178° по отношению к протонному пучку. Энергетическая цена канала анализатора $\Delta E = 20$ *кэв*

Experiments with γ-Quanta in the Last Century

- J.L. Groves, L.E. Holloway, L.J. Koester, W.-K. Liu, L.J. Nodulman, D.G. Ravenhall, J.H. Smith (Illinois Univ.), Study of the reaction $\pi^- + ^{12}C \to \pi^- + ^{12}C^*$ (4.44 MeV) at 4.5 GeV/c. Phys. Rev. D15 (1977), 47. $\sigma^* = 1.70 \pm 0.23$ mb.

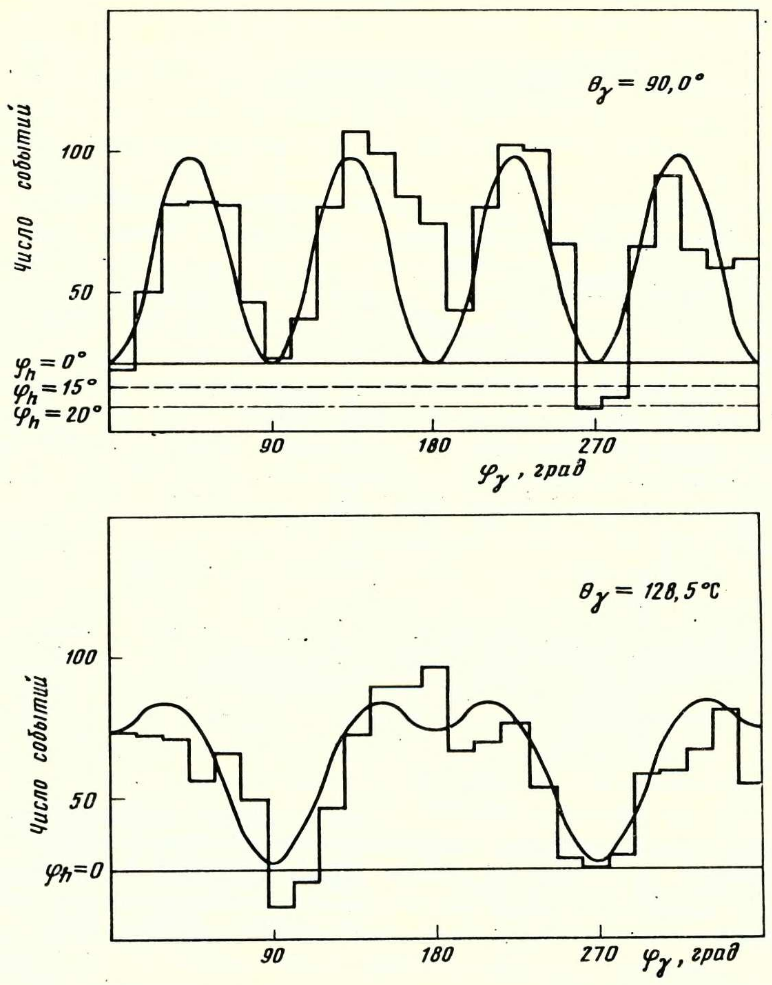


Рис.7. Азимутальное распределение гамма-квантов в реакции $^{12}C(\pi,\pi'\gamma)^{12}C(2^+)$ относительно плоскости реакции при p_π =4,5 Гэв/с; расчет выполнен в приближении одного неупругого соударения; экспериментальные данные – из работы [II].

Summary

- Study of reaction $(p, p'\gamma)$ permits to extract from experimental data on nucleus excitation with intermediate energy hadron beam all moduli of amplitudes $F_M(q)$ and phase difference between all pairs of amplitudes. The ground state of target nucleus is to be non-degenerate energy level (0^+ or 0^-).
- Angular distribution of photons from transition $J^P \to 0^+$ or $J^P \to 0^-$ fixes the total angular momentum of excited level and its parity.
- Behaviour of amplitudes F_M(q) at $q \to 0$ allows to establish the excited level parity.
- Spin-dependent πN amplitude contributes in linear approximation to amplitudes of nucleus excitation $F_M(q)$ for M different from those which are excited with spin-independent πN amplitude.
- Extracted Information from Hadron-Nucleus Scattering Data
  For the pion scattering two types of form factors can be extracted:
  $A_M(q) = \langle \epsilon_f, P, J, M | \sum_{j=1}^{A} \exp\{i(\vec{q} \cdot \vec{r}_j)\} | \epsilon_i, 0^+ \rangle$,
  $B_M(q) = \langle \epsilon_f, P, J, M | \sum_{j=1}^{A} (\vec{\sigma}_j \cdot \vec{n}) \exp\{i(\vec{q} \cdot \vec{r}_j)\} | \epsilon_i, 0^+ \rangle$.
  For proton beam in addition to $A_M(q)$ three form factors can be obtained:
  $B_M^{(m)}(q) = \langle \epsilon_f, P, J, M | \sum_{j=1}^{A} (\vec{\sigma}_j)_m \exp\{i(\vec{q} \cdot \vec{r}_j)\} | \epsilon_i, 0^+ \rangle$, where $m = x,\ y,\ z$.
  This gives possibility to test nucleus models more detailed than in lepton scattering, besides long-range correlations (deformation, alpha-clusterization) can be studied.

- Proof of Selection Rules

$\hat{O}_z$ is reflection of the Z-axis $(z_j \to -z_j)$. Since $\hat{O}_z = \hat{P}\hat{R}_z(\pi)$, where $\hat{P}$ is inversion and $\hat{R}_z(\pi)$ is rotation around Z-axis by the angle $\varphi = \pi$,

then $\hat{R}_z(\pi)|\epsilon_f, P, J, M\rangle = (-1)^M|\epsilon_f, P, J, M\rangle$ and $\hat{P}|\epsilon_f, P, J, M\rangle = P|\epsilon_f, P, J, M\rangle$,

therefore $\hat{O}_z|\epsilon_f, P, J, M\rangle = P(-1)^M|\epsilon_f, P, J, M\rangle$, and $\hat{O}_z|\epsilon_i, 0^+\rangle = |\epsilon_i, 0^+\rangle$.

Since the profile function $\Gamma = \Gamma((\vec{b} - \vec{s}_1)^2, (\vec{b} - \vec{s}_2)^2, ..., (\vec{b} - \vec{s}_A)^2)$ does not depend on z_j $(1 \le j \le A)$ which means $\hat{O}_z\Gamma\hat{O}_z = \Gamma$, then

$\langle\epsilon_f, P, J, M|\Gamma|\epsilon_i, 0^+\rangle = \langle\epsilon_f, P, J, M|\hat{O}_z\Gamma\hat{O}_z|\epsilon_i, 0^+\rangle =$

$= P(-1)^M\langle\epsilon_f, P, J, M|\Gamma|\epsilon_i, 0^+\rangle$.

Therefore the matrix element of profile function and amplitude $F_M(q)$ is nonzero if $P(-1)^M = 1$.

- Proof of Relations $F_{-M}(q) = (-1)^J F_M(q)$

Let us choose the X-axis parallel to $\vec{q}$. Rotation $R_x(\pi)$ around the X-axis

by the angle $\varphi = \pi$ gives $x_j \to x_j$, $y_j \to -y_j$, $z_j \to -z_j$, $b_x \to b_x$, $b_y \to -b_y$.

Since the profile function Γ depends only on $(b_x - x_j)^2 + (b_y - y_j)^2$ $(1 \le j \le A)$ it is invariant under rotation $R_x(\pi)$, $(\vec{q} \cdot \vec{b}) = qb_x$ is also invariant.

The group of profile function invariance is C_{2v}.

Nuclear wave functions are transformed as

$\hat{R}_x(\pi)|\epsilon_f, P, J, M\rangle = (-1)^J|\epsilon_f, P, J, -M\rangle$, $\hat{R}_x(\pi)|\epsilon_i, 0^+\rangle = |\epsilon_i, 0^+\rangle$.

Relation for amplitudes:

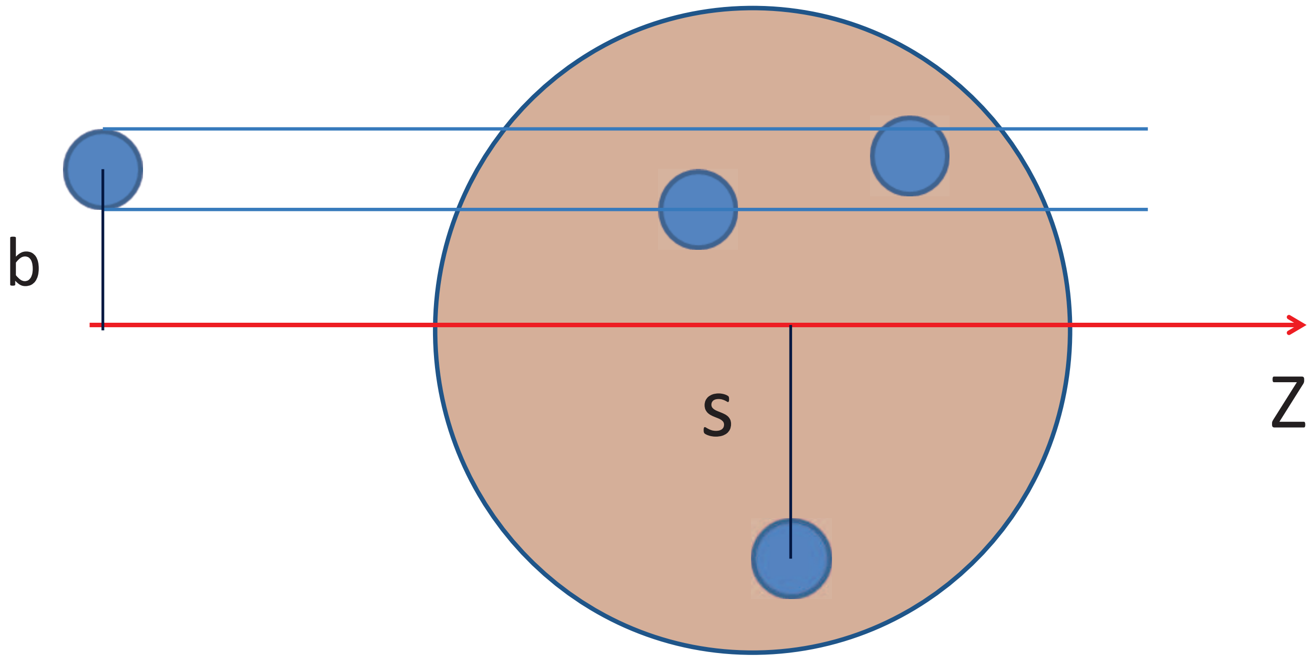
$$F_M = \langle\epsilon_f, P, J, M|\left(\frac{ik}{2\pi}\int e^{i(\vec{q}\cdot\vec{b})}\Gamma d^2b\right)|\epsilon_i, 0^+\rangle =$$

$$= \langle\epsilon_f, P, J, M|R_x(\pi)\left(\frac{ik}{2\pi}\int e^{i(\vec{q}\cdot\vec{b})}\Gamma d^2b\right)R_x(\pi)|\epsilon_i, 0^+\rangle =$$

$$= \langle\epsilon_f, P, J, -M|(-1)^J\left(\frac{ik}{2\pi}\int e^{i(\vec{q}\cdot\vec{b})}\Gamma d^2b\right)|\epsilon_i, 0^+\rangle = (-1)^J F_{-M}$$

Therefore $F_{-M}(q) = (-1)^J F_M(q)$.

- Transverse and longitudinal distances



- Energy Levels of ^{16}O

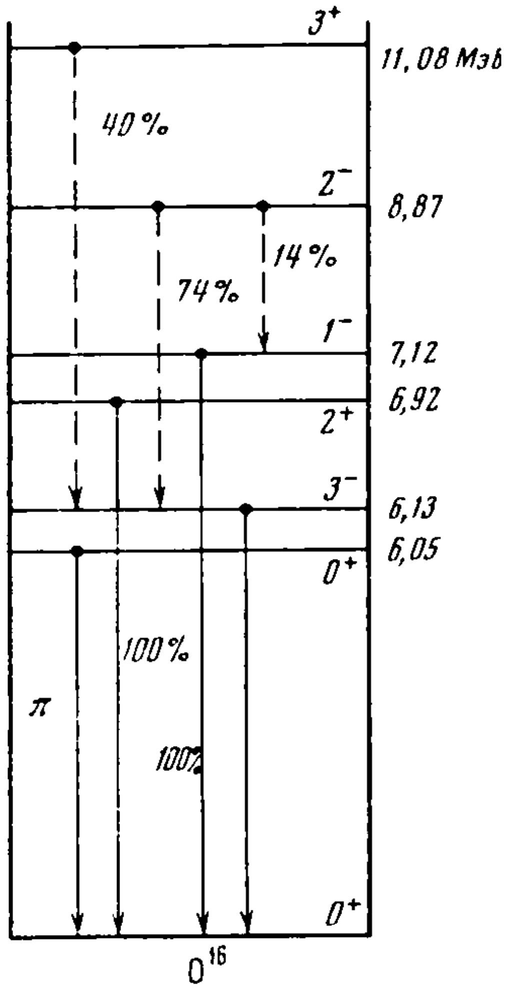


Рис. 4. Схема нижних уровней ядра O¹⁶