COVARIANTLY HOMOGENEOUS CLASSICAL SOLUTIONS IN NONLINEAR σ-MODELS

A. P. Bukhvostov and L. N. Lipatov

The nonlinear σ-models are of great interest both as a phenomenological tool for description of low energy interactions of hadrons and strings, and as a toy field theory to clear up the effects of asymptotic freedom, nontrivial topology, supersymmetry, and other attributes of modern theories of elementary particles.

Most familiar are two types of σ-models which we consider here. First are O(N) models which contain N real fields n^a (a=1,...,N) related by the constraint $(n^a)^2=1$. Second are SU(N)×SU(N) models where the field variables are the elements of unitary unimodular N×N matrix U_{ij} ($UU^+=$ $=1$, $\det U=1$).

We have got a new set of solutions of classical equations for these models and have calculated the functional integral over fluctuations near these solutions /1/.

The discussed models are invariant under some global isotopic transformations:

$$n^a \to O^{ab} n^b, \qquad U \to V_1 U V_2, \tag{1}$$

where O, V_1, V_2 are constant matrices (O orthogonal, V_1 and V_2 unitary unimodular). It is therefore natural to generalize the concept of a constant field, requiring that its change at the spatial translations could be compensated by the isotopic rotation (1):

$$n^a(x+\Delta) = O^{ab}(\Delta) n^b(x), \quad U(x+\Delta) = V_1(\Delta) U(x) V_2(\Delta). \tag{2}$$

Such fields we call covariantly homogeneous. They realize the embedding of the group of space shifts to the isotopic group O(N) or, respectively, SU(N)×SU(N).

It follows from (2) that covariantly homogeneous field $n^a(x)$, U(x) must have the form of a superposition of plane wave exponents $e^{ip_\mu x_\mu}$. Substituting such ansatz in the equations of motion, one can provide their fulfillment by an appropriate choice of the coefficients and parameters p_μ.

When calculating the functional integral over fluctuations near covariantly homogeneous solutions in the Euclidean space (i. e., at positive squares of vectors), a difficulty arises in connection with the instability of these solutions. (It is just the same defect that plagues the constant fields in QCD). By considering the example of 2-dimensional O(N) σ-model with $N \to \infty$ we show that the quasiclassical approach to the construction of the vacuum in the Euclidean theory does not yield in this case the true result.

In the Minkowski space the effective action is real and has an extremum at some $p^2 < 0$. The corresponding state can be treated as a coherent system of massive particles.

As an example, the effective action for the chiral SU(2)×SU(2) π-meson theory is /1/

$$S_{eff} = \int d^4x \{p^2/2g + p^4/32\pi^2 [1/3 ln(-p^2/\mu_1^2) + \frac{2-c^2}{3} ln(-p^2/\mu_2^2) - \\ - 1/2 + (1-c^2/3) ln(1+|c|) - |c| + c^2/9]\}, \tag{3}$$

where

$$p^2 = 1/2\, tr(\partial_\mu U) \cdot (\partial_\mu U^+), \quad c^2 = 2(1 - \frac{1}{p^2}[tr(\partial_\mu U)(\partial_\nu U^+)]^2), \tag{4}$$

and μ_1, μ_2 are the mass parameters arising at the renormalization. Here the ground state remains massless but at some values of p^2 and c there is an additional extremum which may show itself as a massive particle (excited π-meson).

REFERENCE

1. *Bukhvostov A. P. and Lipatov L. N.* Covariantly homogeneous fields in nonlinear σ-models. Preprint MPI-PAE/Pth 61/89.