EFFECTS OF QUARK STATISTICS ON CHARGE FORM FACTOR OF ^{4}He NUCLEUS

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The nucleus wave function is antisymmetric under quark permutations both in any nucleon and among any nucleons. The latter property should cause observable effects. The Pauli exclusion principle is more important at higher nuclear densities, so the most interesting object among the lightest nuclei is the ^{4}He nucleus, which is discussed in this paper. All the details of the calculations are given in /1,2/.

Model. The nucleon is considered as a colourless system of three quarks, each is in the S-state. The colour symmetry group is SU(3) and the spin-isospin symmetry of quarks is SU(4). The helium nucleus is considered as a system of four colourless clusters (nucleons). The space parts of all the wave functions are taken in the Gaussian form. The spin-isospin part of the nuclear wave function is the SU(4)-singlet.

Let the wave function Φ be the product of the nucleonic one and the four quark wave functions of the nucleons. Then Φ has correct permutation properties only under quark permutations in each nucleon and under the permutations of nucleons. This corresponds to common nuclear physics. The correct quark wave function of the nucleus Ψ is equal to $NA\Phi$, where A is a full antisymmetrization operator of twelve quarks and N is a normalization factor. The charge form factor of ^{4}He can be given with the help of the wave function Φ by the formula

$$F_e(\vec{q}) = <\Phi| \sum_{j=1}^{12} exp\{i \vec{q} \vec{r}_j\} B |\Phi> , \qquad (1)$$

where $\vec{q}$ is the momentum transfered to the nucleus and $\vec{r}_j$ is the radius-vector of the jth quark. The operator B in (1) can be chosen as a sum of transpositions of simple permutations of the jth and lth quarks (jl) (the Kth nucleon consists of quarks with numbers 3K-2, 3K-1, 3K)

$$B = e - 54(14) + 243(14)(710) + 648(14)(57) - 216(14)(57)(68) -$$
$$- 144(15)(27)(48) + 486(34)(511)(612)(710) - 648(17)(48)(910) + \qquad (2)$$
$$+ 3888(18)(25)(36)(910) - 3888(16)(57)(910) + 1296(27)(34)(512)(811) + 3888(24)(37)(68)(910).$$

The first term in (2) corresponds to common nuclear physics, the others give exchange corrections to the form factor. For example, the second and fourth terms take into account the quark exchange between two and among three nucleons respectively. The number of permutations in (3) is equal to 15400. In spite of this tremendous number of permutations the full correction to the form factor for $q^2 \leqslant 10$ fm^{-2} is small. The explanation of this fact is as follows: the contribution of quark exchanges among K nucleons is a quantity of the order λ^{k-1}, where λ in the three-dimension version of the model is equal to

$$\lambda = (r_p/r_{He})^3 / (N_C N_{SI}) . \qquad (3)$$

N_C and N_{SI} are the numbers of colour and spin-isospin quark degrees of freedom, r_p and r_{He} are the radii of the nucleon and ^{4}He respectively, and are equal to 0.8 fm and 1.67 fm. It is obvious from (3) that the parameter λ is small mainly due to the existence of twelve colour and spin-isospin degrees of freedom of a quark.

This is the most important reason why the quark statistics correction is so small. The curves 1,2,3 and 4 of figure 1 show the ^{4}He form factor calculated with accuracy up to λ^0, λ, λ^2, λ^3 (form factors F_0, F_1, F_2, F_3). The curves 2a, 3a, 4a, show the ratios F_1/F_0, F_2/F_0, F_3/F_0 as functions of q^2. One can see that the contributions of order λ and λ^2 are rather noticeable in comparison with that of order λ^0 for $q^2 \leqslant 10$ fm^{-2}, but they cancel each other considerably, so the net quark statistics effect is small.

For the real deuteron λ is given by the formula

$$\lambda = (r_p / r_D) / (N_C N_{SI}) . \qquad (4)$$

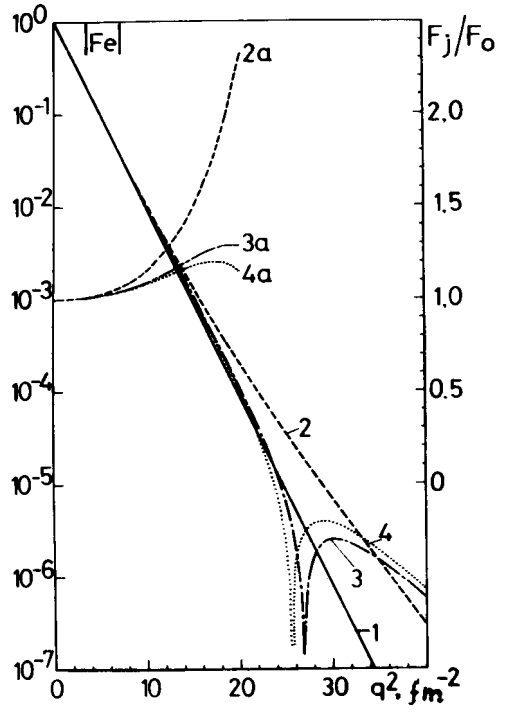

Fig. 1

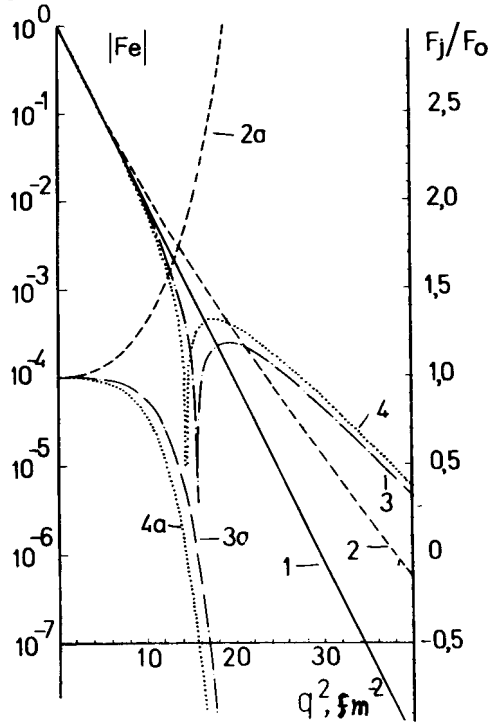

Fig. 2

The first power of the ratio r_p/r_D in (4) is caused by the fact that in a deuteron the range of the NN-force r_{NN} is far shorter than the nuclear radius r_D and for $r>>r_D$ the nucleon wave function has the common form $\exp\{-ær\}/r(æ\sim 1/r_D)$. The dependence (4) appears in the one-dimensional model, so the short range NN-force limit can be studied in this model. The results of a calculation in the one-dimensional model are presented in figure 2, which is completely analogous to figure 1. One can see the exchange effects to be more important, as the parameter λ in this case is bigger than in the three-dimensional model. The one-dimensional model gives an upper limit of quark statistics effects. The Gaussian wave function of the three-dimensional case corresponds to the long range NN-force limit (the oscillator potential). The situation in the real ^{4}He nucleus is between the two models under consideration, as the range of the NN-force is shorter than but comparable to the mean distance between the nucleons in ^{4}He. The quark exchange effects are insufficient to produce the experimental value of the form factor in the region of the second maximum, but they are as important as the effects of meson exchange currents. The last statement follows from the comparison of our curves with the curves obtained in /3/, where meson exchange currents in ^{4}He have been considered.

REFERENCES

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