SUPERSYMMETRIC APPROACH IN THEORY OF ANDERSON LOCALIZATION

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Despite great efforts spent in order to understand the critical behaviour of disordered systems in the vicinity of Anderson metal-insulator transition the problem is still far from being solved. The main breakthrough was achieved after introducing non-linear σ-models of corresponding symmetries. However, a symmetry of these models turns out to be always broken, the mean-field approximation is meaningless and a transition into the localized phase emerges only as a result of renormalization group treatment in $d = 2 + \varepsilon$ dimensions. Such an approach does not take into account non-perturbative effects which can play an important role in view of non-compactness of symmetry group, especially for $d \geq 3$ where the localization transition occurs in the region of strong disorder.

In view of shortcomings of a naive mean-field (infinite-range) approximation, it is especially interesting to study the problem of the Bethe lattice which is an infinite tree-like graph with fixed coordination number for all sites commonly interpreted as corresponding to infinite dimensionality. The hierarchical lattice structure makes the problem analytically tractable which allows one to investigate the transition beyond the scope of the perturbation theory.

In the paper [1] we investigate the Bethe lattice version of the Anderson model

$$H = \sum_i \varepsilon_i c_i^+ c_i + \sum_{ij} t_{ij} c_i^+ c_j, \tag{1}$$

where ε_i are assumed to be independent random variables with distribution $\gamma(\varepsilon)$; $t_{ij} = 1$ if sites i,j are the nearest neighbours and zero otherwise. To calculate the averaged two-paricle correlation function

$$K(i,j) = < i \mid (E + i\varepsilon - H)^{-1} \mid j > < j \mid (E - i\varepsilon - H)^{-1} \mid i >, \tag{2}$$

where E is an energy and 2ε plays a role of imaginary frequency, we use the supersymmetric method of averaging and define at each site i an 8-component supervector Φ_i. The correlator can be expressed through a function $G(\Phi)$ satisfying the self-consistency equation

$$G(\Phi) = \int [\, d\Psi \,] \int du\gamma(u) \exp\{ -i\Phi^+ L\Psi + \frac{i}{2} \Psi^+ [\, (E-u)\, L + i\varepsilon \,] \Psi \} \, G^m(\Psi), \tag{3}$$

where L is the diagonal matrix with elements $\{1,1,1,1,-1,-1,-1,-1\}$, m is a lattice connectivity. In the limit $\varepsilon \to 0$ eq. (3) is invariant under transformation $G(\Phi) \to G(T\Phi)$ where a matrix T satisfies the condition $T^+LT = L$ determining the graded group UOSP (2,2|2,2). If $G(\Phi)\mid_{\varepsilon \to 0}$ possesses this invariance, i.e. depends on $\Phi^+ L\Phi$ only, the symmetry is not spontaneously broken which means [1] the localization of eigenstates. In contrast, if a dependence of the function $G(\Phi)$ on both $\Phi^+ L\Phi$ an $\Phi^+\Phi$ keeps when $\varepsilon \to 0$, the symmetry is spontaneously broken which corresponds to the delocalized phase.

Thus, within the scope of the supersymmetric approach the localization transition manifests itself as a spontaneous breaking of UOSP (2,2|2,2) invariance, the function $G(\Phi) = g(\Phi^+ L\Phi, \Phi^+\Phi)$ playing a role of an order parameter. This orderparameter function has a clear physical meaning being connected with a distribution of one-particle Green functions.

Analyzing eq. (3) in the vicinity of the transition point E_c it is possible to find the critical behaviour of the model, namely:

a) the diffusion constant $D \alpha \mid E_c - E \mid^{-3/2} \exp\{ -const \mid E_c - E \mid^{-1/2} \}$;

b) the localization lenght $\xi \alpha \mid E - E_c \mid^{-\nu}$ with $\nu = 1$;

c) the inverse participation ratio $P = \lim_{\varepsilon \to 0} \varepsilon K(i,i)/\pi\rho(E)$ has a jump at the transition point.

Let us note that the analogous critical behaviour was found previously for supermatrix σ-models on the Bethe lattice.

In the papers [2—4] the ensemble of sparse random matrices combining features of both infinite-range and infinite-dimensional models is studied. It is shown that this model also undergoes the localization transition descibed by a spontaneous symmetry breaking, corresponding order parameter being a function. The transition leads to a change of level correlation which is absent in the localized phase and acquires the Dyson form in the region of delocalized states.

The exactly soluble models (1—4) may serve as a convenient starting point for a derivation of effective field-theoretical formulation of Anderson Localization describing the localization transition already in the saddle-point approximation [5]. In this approximation a critical behaviour coincides with that obtained for the Bethe lattice model (with $\nu = 1/2$ instead of $\nu = 1$). Calculation of corrections to these mean-field results is the prospect of a future work.

References

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