NONWANDERING POINT SET FOR JOSEPHSON TYPE EQUATION

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The Josephson equation is a model equation describing the behaviour of the electron wave function phase difference on the different sides of the point Josephson junction [1] and [2]

$$\beta\ddot{\phi} + (1 + \gamma\cos\phi)\dot{\phi} + \sin\phi = k\sin\omega t, \quad \phi \in \mathbf{R}.$$

We will consider it in context of the equation class of the following structure

$$\beta\ddot{\phi} + (1 + \gamma(G(\phi) + \phi G'_{\phi}(\phi))\dot{\phi} + G(\phi) = k\sin\omega t, \quad \phi \in \mathbf{R}, \tag{1}$$

the function $G : \mathbf{R} \to \mathbf{R}$ possesses the following properties:

1) $G(\phi) \in C^1$,

2) $G(-\phi) = -G(\phi)$,

3) $G'_{\phi}(0) = 0, G(0) = 1$,

4) if $\mid \phi \mid < a$, then $G(\phi) > 0$, $G(a) = G(-a) = 0$, if $\mid \phi \mid > a$, then $G(\phi) < 0$.

Particularly, for the Josephson equation [1] $G(\phi) = \sin(\phi)/\phi$.

Below us consider the case of (1) $\beta = 1$ and $\gamma < -1$.

The equation (1) is transformed to the anautonomous system of the second order:

$$\begin{pmatrix} \dot{\phi} \\ \dot{\rho} \end{pmatrix} = \begin{pmatrix} -\gamma G(\phi) & 1 \\ (\gamma - 1)G(\phi) & -1 \end{pmatrix} \begin{pmatrix} \phi \\ \rho \end{pmatrix} + \begin{pmatrix} 0 \\ k\sin\omega t \end{pmatrix}, \tag{2}$$

The flow due to by the vector field of the equation (2) is by definition C^r -map $\varphi : \mathbf{R}^2 \times \mathbf{R}_{\to} \to \mathbf{R}^2$, $(t, [\phi, \rho]) \mapsto \phi_t([\phi, \rho])$. To switch from the flow to the diffeomorphism one needs to consider the map φ_t for some fixed t. Since the vector field is smooth, then according

to the existence and uniqueness of the solutions of the differential equation theorem, the map $\varphi_t: \mathbf{R}^2 \to \mathbf{R}^2$ is a diffeomorphism. In such a way, the iterations of the map

$$(\varphi)^n = \underbrace{\varphi_t \circ \ldots \circ \varphi_t}_{n \text{ time}} = \varphi, \quad n \in \mathbf{Z},$$

form the action of the integer numbers group $\mathbf{Z}$ on $\mathbf{R}_{\rightarrow}$.

Definition 1. *The point x is nonwandering for diffeomorphism $f: \mathbf{M} \to \mathbf{M}$, if for every neighborhood $U(x) \in \mathbf{M}$ and $N > 0$, there is $n > N$ such that $f^n(U) \bigcap U \neq \emptyset$. The nonwandering point set is denoted Ω.*

Definition 2. *The space $\mathcal{X}$ is indecomposable, if it is connected and cannot be represented as the union of two closed connected sets other then $\mathcal{X}$. If $\mathcal{X}$ is a Hausdorff space then $\mathcal{X}$ is the indecomposable continuum.*

Theorem 1. [5] *Let Ω be a nonwandering point set of the diffeomorphism $f: \mathbf{R}^2 \to \mathbf{R}^2$, then there is a two-dimensional non-orintable factorspace of cylinder $\mathcal{S}_q \in \mathbf{R}^3$ by stock q, such that $\Omega \in S_q$, besides, Ω irreducible cuts $\mathcal{S}_q$ to $q+1$ disks.*

Corollary 1. *The nonwandering set $\Omega \in \mathcal{S}_\infty$ of diffeomorphism f is an indecomposable continuum or union of two indecomposable continua.*

Example illustrating the substance of the theorem: we shall construct two dimensional set $\Psi \in \mathbf{R}^3$, the cut of this set is indecomposable continuum and frontier of any number (finite or account) of discs simultaneously. Moreover it is supposed that this example is the model of the nonwandering set Ω of the diffeomorphism $f: \mathbf{R}^2 \to \mathbf{R}^2$.

The first stage of construction of indecomposable continuum is started with consideration of the following map structure: let Σ be the compact space of one-sided sequences $\eta = \{\omega_k\}_{k \leq 0}$, ω_k are symbols from the alphabet $\mathcal{L}_3: \{\alpha_0, \alpha_1, \alpha_2\}$ with a defined ratio of order $\alpha_0 \prec \alpha_1 \prec \alpha_2$.

The map $g: \Sigma \to \Sigma$ which is defined by following image: let the map $inv: \mathcal{L}_3 \to \mathcal{L}_3$, $\alpha_i \mapsto \alpha_{2-i}$, $i = 1, 2, 3$, and let the sequence $\mathbf{Q}_m: \Sigma \to \Sigma$ by the following structure:

$$(\mathbf{Q}_m): \begin{cases} \omega_m \mapsto inv(\omega_m), \\ and \\ \{\omega_k\}_{k \geq m+1} \mapsto \mathbf{Q}_{m+1}\{inv(\omega_k)\}_{k \geq m+1}, \quad \text{if} \quad \omega_m \neq \alpha_2. \end{cases}$$

Let us research the iterations of the map $g: \Sigma \to \Sigma$.

$$g = \lim_{m \to \infty} \mathbf{Q}_m.$$

Lemma 1. *The sequences containing α_1, have a periodic orbits.*

Corollary 1. *The sequences $\eta = \{\omega_k\}_{k \leq 0}$, in which $\omega_k \neq \alpha_1$ form the set C point cutting space Σ.*

Corollary 2. *Let U_i are neighbourhood of the sequence having 2^i-periodic orbits, then frontier of set $\Sigma = \bigcup\limits_{i \leq 1} U_i$ contains the sequence which does not contain the symbol α_1. In other words, the set of periodic point of the map $g: \Sigma \to \Sigma$ every where dense.*

Lemma 2. *The set $C \times S^1$ irreducibly cuts the Ψ.*

Theorem 2. *The set $C \times S^1$ is indecomposable continuum.*

This example is fully analogous to the Smale horsehoe.

About the compact, which representating common frontier at least of three regions of Euclidean space is told that it possesses Brower peculiarity and nothings of its true closed subset already do not separates a subspace of space $\mathbf{R}^3$ [4].

Theorem 1 and the constructing example are indicating that the nonwandering set of equation (2) possesses Brower peculiarity.

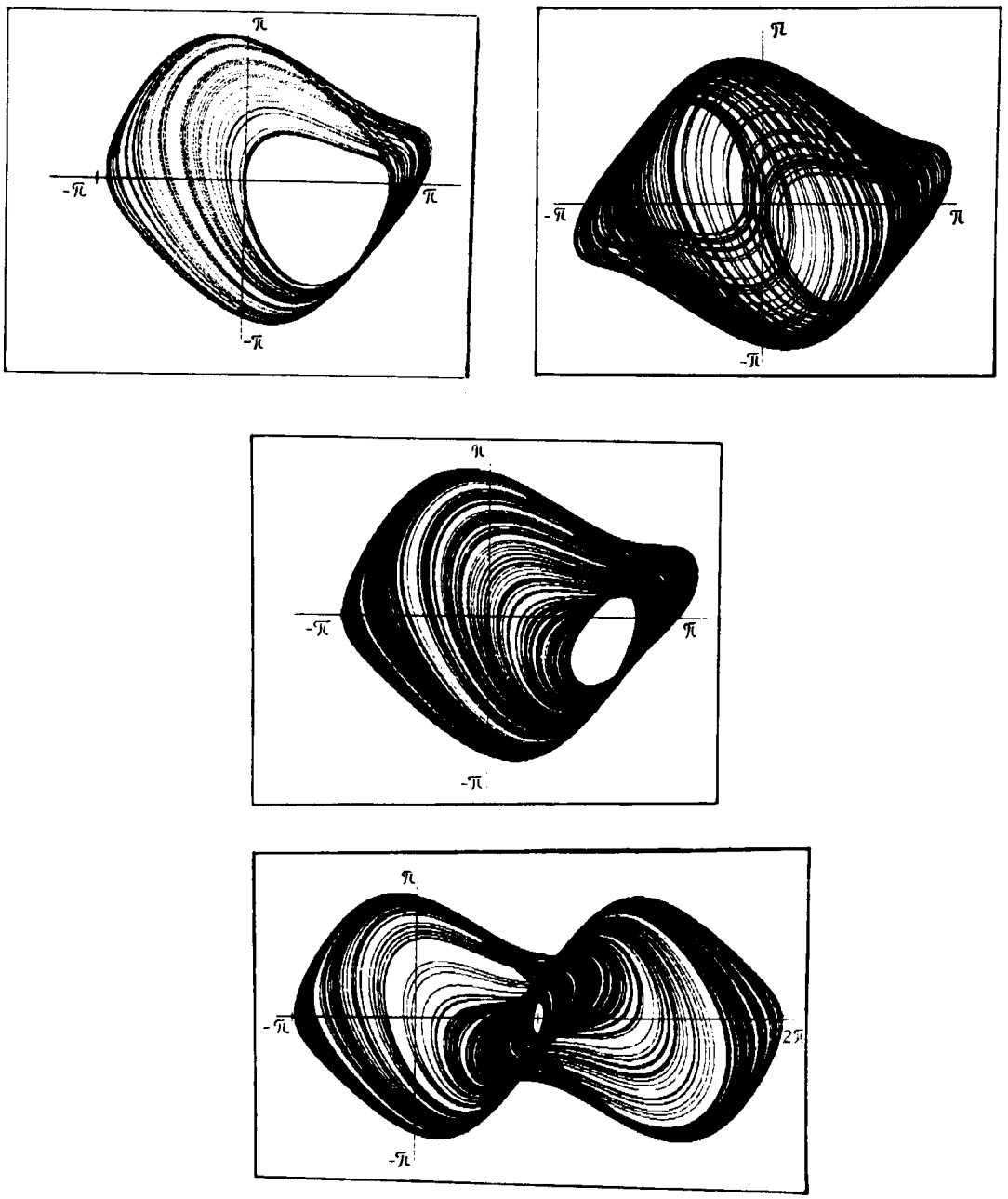

Fig. The phase portraits of the Josephson equation ($G(\phi) = \sin\phi/\phi$) trajectories: $\beta = 1.0$, $\gamma = 1.07$, $k = 2.8$. *a*) $\omega = 0.880$; *b*) $\omega = 0.870$; *c*) $\omega = 0.823$; *d*) $\omega = 0.813$

These figures show that the nonwandering point set may be an indecomposable continuum or union of two indecomposable continua.

REFERENCES

1. *Sanders J.A.* The (Driven) Josephson Equation: an Exersise in Asimptotics. LNM. Asimptotic Analysis 2. 985 (Springer-Verlag), 1983.
2. *Schlup W.* Possible Influence of the Quasiparticle-Pair Interference Term on the Current-Voltage Characteristics of Oxide Barrier Josephson Junctions. Solid State Commun., 1973. V.12. N°7. P.631–634.
3. *Kuratowski K.* Topology. Academic Press, New York and London, 1968. V.2. P.499.
4. *Borsuk K.* Theory of Retracts. Warzawa, 1966. P.184.
5. *Serow D.W.* Nonwandering Point Set for Josephson equation. Proc. Second Annual Seminar: Nonlinear Phenomena in Complex Systems, ed. Kuvshinov V.I. & Serow D.W. Gatchina, 1993.