CALCULATION OF MULTILOOP AMPLITUDES IN SUPERSTRING PERTURBATION THEORY

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The calculaton of the multiloop superstring amplitudes meets with difficulties. In the well-known scheme based on constructing suitable modular forms [1, 2, 3] the considered amplitudes appear to be depended on a choice of basis of the gravitino zero modes [1, 2]. It is a serious failure of this approach. Besides, for more than three loop amplitudes one is forced to use complicate sets of moduli [3] that prevents the study of the amplitudes obtained. In the (super)conformal approach [4] even for the multi-loop boson string amplitudes it is failed to find [5] factors due to the moduli volume form and the ghost zero modes, as well. If the modular parameterization by superconformal versions of Schottky groups is used [4, 5, 6],

a simple extension of the boson string results [7] allows to obtain [6] contributions to multi-loop amplitudes of those spinor structures where fermion fields appear to be periodical about all the A-cycles. But all the other spinor structures are too complicate to be considered in this way. The approach to (super)string theory developed in [8, 9, 10] is free from the above difficulties.

Unlike [4], this approach employs widely not only the (super)conformal symmetry, but all the local gauge symmetries of the (super)string. The multi-loop amplitudes are calculated from equations that are none other than Ward identities. The above equations are obtained from the condition that the discussed amplitudes are independent of a choice of the gauge fields, for superstrings they being the vierbein and the gravitino field. Hence, multi-loop amplitudes appear to be independent of a choice of basis of the gravitino zero modes. In addition, the factors connected with the moduli volume form and the ghost zero modes are calculated in this scheme by means of a suitable modification of ghost vacuum correlators. Besides, present method employs neither BRST quantization nor the bosonization prescription.

The equations under discussion being differential in moduli determine the partition functions up to constant factors. To calculate all these factors in the terms of a coupling constant, the factorization of the amplitudes is required when two handles move away from each other. This requirement replaces the unitarity condition. Though the vertices are known for a long time, it seems interesting to note that they could be calculated in this way, too. Thus, the amplitudes turn out to be fully determined by the gauge invariance together with the above mentioned "factorization requirement". It is worthwhile to note that the discussed equations are (super)modular invariant. Therefore, one can be sure that superstring amplitudes satisfy, after a suitable summation over spinor structures, restrictions due to the modular invariance though a direct proof of this statement may be quite difficult.

In [9, 10, 11] the discussed scheme has been applied to the closed, critical Ramond-Neveu-Schwarz superstrings, while for spinor structures superconformal versions of Schottky groups being employed. In this modular parameterization the most difficulties arise for those spinor structures where the fermion fields are non-periodical for at least one A-cycle. The method of calculation of the superfield vacuum correlators for the above spinor structures has been proposed in [11, 12, 13]. We calculated also in [11] the contributions to boson emission superstring amplitudes due to all even spinor structures.

We found that the discussed amplitudes have no divergencies due to a degeneracy of any A_s or B_s cycle in accordance with earlier results [6]. But the multiloop amplitudes could have divergences due to the handles moving away from each other, just as they do it in the boson string theory [14]. The absence of these divergences is caused by the "nonrenormalization theorems" [15], but the problem of a validity of the above theorems needs a further study.

Furthermore, the holomorphic structure of the considered amplitudes is found [11, 13] to be different from that assumed before in the scheme [1, 2, 3] based on modular forms. This is caused by a dependence on odd moduli in period matrices associated the complex supermanifolds, which has been ignored in [1, 2, 3]. Moreover, among terms proportional to odd moduli there are terms depending on a spinor structure [11, 13]. These terms arise

because due to the world-sheet supersymmetry the fermions mix with the bosons while going around B_s-cycles. This might be essential for the divergency problem, but this matter requires an additional investigation.

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