ON THE Z_2 STRUCTURE OF THE CONFIGURATION SPACE IN NON-ABELIAN YANG-MILLS THEORIES

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We argue for the presence of a $\mathbf{Z}_2$ topological structure in the space of static gauge-Higgs field configurations of $SU(2n)$ and $SO(2n)$ Yang-Mills theories. We rigorously prove the existence of a $\mathbf{Z}_2$ homotopy group of mappings from the 2-dim. projective sphere $\mathbf{R}P^2$ into $SU(2n)/\mathbf{Z}_2$ and $SO(2n)/\mathbf{Z}_2$ Lie groups respectively. Consequently the symmetric phase of these theories admits infinite surfaces of odd-parity static and unstable gauge field configurations which divide into two disconnected sectors with integer Chern-Simons numbers n and $n + 1/2$ respectively. Such a $\mathbf{Z}_2$ structure persists in the Higgs phase of the above theories and accounts for the existence of $CS = 1/2$ odd-parity saddle point solutions to the field equations which correspond to spontaneous symmetry breaking mass scales.

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$2d$ GRAVITY STRESS TENSOR AND MULTI-LOOP AMPLITUDES IN THE STRING THEORY

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It is well known [1] that in non-critical dimensions, the conform invariance of the string theory is destroyed by the local conform anomaly. In the widely spread scheme [1–3] the above anomaly is canceled by the $2d$ gravity field contribution. The conform anomaly being canceled, one expects for the interaction amplitudes to be conform invariant. That is really right for tree and one-loop amplitudes [4]. But we argue [5] that the cancellation of the local conform anomaly does not yet provide the conform invariance of multi-loop amplitudes. We show [5] that in the considered scheme the vacuum value of the $2d$ gravity stress tensor on the higher genus $n \geq 2$ Riemann surfaces contains a singularity in the fundamental region on the complex plane where the Riemann surfaces are mapped. Intuitively, it seems that a singularity of the vacuum value of the stress tensor destroys the modular invariance of the theory. Such is indeed the case and can be proved in a rigorous manner. In turn, the destroying of the modular invariance means breaking of the conformal invariance. So in this scheme the conform invariance appears to be destroyed though the local conform anomaly absents.

The above singularity in the $2d$ gravity field stress tensor is due to of the existence of the nonzero vacuum expectation value $< \Phi >$ of $2d$ gravity field Φ appearing to be singular. The nonzero value of $< \Phi >$ arise because the $2d$ gravity field stress tensor

$$T_\Phi = -\frac{1}{2}\partial_z\Phi\partial_z\Phi + f\partial_z^2\Phi \quad \text{with} \quad f^2 = \frac{25-d}{12} \tag{1}$$

includes the terms linear in Φ. Here d is the dimension of the target space. Just this linear in Φ term provides the cancellation of the conformal anomaly in the theory. At the same time, owing to the above term, on the closed Riemann surfaces of a genus-$n > 1$ the discussed Φ field is necessarily receives an additional term under twists about B-cycles. Indeed, one can obtain from (1) that under the $z \to g_s(z)$ transformation associated with 2π-twist about B_s-cycle the Φ field changes as

$$< \Phi(g_s(z), \overline{g_s(z)}) > = < \Phi(z, \bar{z}) > + f \ln |\frac{dg_s(z)}{dz}|^2 . \qquad (2)$$

Only in the genus-1 case when the genus-1 Riemann surfaces are mapped into parallelograms the additional term in this expression disappears because in this case $g(z) = z + \omega$, ω being independent of z. So in the discussed case $< \Phi >= const$ that does not contribute to the vacuum value of the stress tensor (1). In particular, one can choose $< \Phi >= 0$. But except this case, the above additional term presents, and, therefore, the vacuum expectation value $< \Phi >$ of the Φ field is unequal to zero. In addition, $< \Phi >$ depends on z contributing to the vacuum value of the stress tensor (1). Relation (2) together with the condition that $< \Phi >$ is nonsingular at finite values of z determines the desired $< \Phi >$ up to the constant term that does not contributes to the stress tensor, as it was already noted above. Using the Schottky modular parameterization one can calculate $< \Phi >$ for an arbitrary genus. In this case one can show that at $z \to \infty$

$$< \Phi'(z) > = \frac{d < \Phi(z) >}{dz} \to -\frac{2fn}{z} \quad \text{at} \quad z \to \infty \qquad (3)$$

where n is the genus, and, therefore,

$$< T_\Phi(z) > \to -\frac{2f^2 n(n-1)}{z^2} \quad \text{at} \quad z \to \infty . \qquad (4)$$

It is worthwhile to note that every non-singular 1-form decreases at $z \to \infty$ not slower than $const \cdot z^{-2}$. So one concludes that $< \Phi'(z) >$ is singular at $z \to \infty$. Furthermore, every non-singular 2-form decreases at $z \to \infty$ not slower than $const \cdot z^{-4}$. So, for $n \geq 2$, $< T_\Phi(z) >$ appears to be singular at $z \to \infty$. It follows from (3) that for the Schottky parametarization to be used, $< \Phi > \neq 0$ even in the $n = 1$ case. But one can see from (4) that in this case the stress tensor (1) appears to be nonsingular. That is why in the discussed Schottky parametarization the genus-1 integration measure over the moduli space can be constructed to be modular covariant. The above integration measure is consistent with that calculated [4] in the parameterization when the genus-1 Riemann surfaces are mapped into parallelograms. For the higher genus, the discussed singularity (4) in the stress tensor prevent a construction of the modular invariant integration measures over the moduli space. One could construct the $< \Phi(z) >$ value to be non-singular at $z \to \infty$. In this case $< \Phi(z) >$ appears to be singular at finite values of z. In any case, for $n \geq 2$ the vacuum value $< T_\Phi(z) >$ of the stress tensor T_Φ turns out to be singular in the fundamental region on the complex z-plane where the Riemann surfaces are mapped. For the discussed theory to be self-consistent the above singularity should be cancelled some other contributions to the stress tensor. Perhaps, one can not be convinced in full degree that the discussed cancellation does not arises in the considered theory. Indeed, the Lagrangian of the discussed theory contains the Liouville term that complicates the investigation of this theory. But a number of authors [6] believes

that the above theory can be reduced to a free field one. In this case one can verifies by the explicit calculation that there are no other singular contributions to the stress tensor and, therefore, the cancellation of the discussed singularity does not arise. So it appears impossible to construct the multi-loop integration measures, which would be consistent with the requirement of the modular invariance. Therefore, it appears impossible to construct multi-loop amplitudes. This statement concerns the continual theory [3], but not the results obtained by continual limit of matrix models because it is not clear what is a continual theory, which the above results correspond to.

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NEW APPROACH IN THE MICROSCOPIC FERMI SYSTEMS THEORY

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We present a review of new approach to the microscopic theory of Fermi systems. This approach is based on functional relations between the ground state energy of the system and its linear response function. We have derived an exact functional equation for the effective interaction between particles in medium in terms of the vacuum interaction potential. We have shown that this exact equation can be transformed into a chain of coupled integral equations for the interaction itself and its variational derivatives which is equivalent to an analogous set of coupled equations of the $\exp(-S)$-theory or a system of equations of the Green function approach. We have described a mathematical procedure of solving the equation obtained which includes the calculation of the variational derivatives of the response functions and the Green functions with respect to the density.

The considered approach is closely related to the well-known density functional theory which is based on the remarkable theorem proved by Hohenberg and Kohn, stating that the ground state energy of a many-body system located in an external field is a unique functional of the particle density. In order to apply the Hohenberg-Kohn theorem, one must be able to construct the density functional. Unfortunately, the Hohenberg-Kohn theorem only states the existence of the functional, and so far the density functional theory is unable to derive by itself its key notion — the functional of the density. The considered approach