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# EXPERIMENTAL DETERMINATION OF THE EXTERNAL BEAM MATRIX ON 1 GEV SYNCHROCYCLOTRON


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The critical question for the beam optics calculations is a correct determination of the input beam parameters. Presented report is devoted to description the procedure of the experimental determination of the extracted beam parameters on 1 GeV synchrocyclotron.

In fields of linear forces the transversal movements in planes x and z are independent and the beam phase space can be characterised by two 2-dimensional symmetrical matrixes $\sigma_x$ and $\sigma_z$, each of which will be transformed along a beam line under the law [1]

$$\sigma(s) = R(s)\ \sigma_0(s)\ R^T(s)\ , \tag{1}$$

where $\sigma_0$ is initial matrix, R(s) is optical matrix of transition. An element of a matrix $\sigma_{11} = < (x - \bar{x})^2 >$ is a mean square beam size, which can be received from profile measurement. From (1) we receive expression

$$\sigma_{11}(s) = \sigma^0_{11}(s)\ R^2_{11}(s) + 2\sigma^0_{12}(s)\ R_{11}(s)\ R_{12}(s) + \sigma^0_{22}(s)\ R^2_{12}(s)\ , \tag{2}$$

which is a parabola equation in a drift space (since $R(s) = 1, R(s) = s$).

To measure the beam profile the profilometers developed at PNPI SC was used [2]. The beam ellipse parameters which are elements of matrix can be determinated by measuring the profiles for three profilometer position. The most accurate measurements can be done in drift space as far as the transfer matrix can be determinated accurately. The procedure is very sensitive to the accuracy in determining the beam profile. This situation is illustrated on Fig.1. When a beam envelope, shows up as a parabola with small curvature, small errors in profile can result in physically incorrect result. Parabola, passed through 3 measurement

points, has curvature of other sign, that means, that $\sigma_{22}^0 < 0$ and $\det\sigma = \varepsilon^2 < 0$. The algorithm can be improved by measuring the profiles for more then three positions and applying a least-square fit (LSM) with minimising functional

$$F = \sum_i [< (x - \bar{x})^2 >_i - \sigma_{11}^0 R_{11}^2(i) - 2\sigma_{12}^0 \; R_{11}(i) \; R_{12}(i) - \sigma_{22}^0 R_{12}^2(i)]^2 \; . \tag{3}$$

However LSM-procedure under limited number of measurements points does not always eliminate of "non-physical" results. The attempt to modify functional by the introduction of penalty functions, which "drive" parameters into the physically allowable region, were made. The penalty functions used here are as follows:

$$f_1 = D \cdot \exp(-\sigma_{22}^0/a) \;\; \text{and} \;\; f_2 = D \cdot \exp(-\det\sigma/\varepsilon_0^2) \; , \tag{4}$$

where $a$ and $\varepsilon_0$ are characteristic areas of changes $\sigma_{22}$ and $\varepsilon$, D is a parameter of the penalty. Functionals $\Phi_1 = F + f_1$ and $\Phi = F + f_2$ were minimised, where $F$ is defined by the formula (3). Fast rise of the exponent under $\det\sigma < 0$ and $\sigma_{22}^0 < 0$ forces process of minimisation to be shifted in the direction of physically allowable parameters. Under $\det\sigma > 0$ and $\sigma_{22}^0 > 0$ the penalty function is being quickly aimed for a zero and main role plays functional $F$. The matrix elements received as a result of this procedure have been in the physically allowable region, but beam envelopes, calculated with their use, do not sufficiently accurate describe experimental data.

It is preferable to do profile measurements nearly of the waist position for both planes. But on the practice for high emittance beam it is very often impossible to produce the waists for both the two planes simultaneously. The best results on determination $\sigma$-matrix were obtained at measurement of beam profiles in a drift space after doublet of lenses in two various modes of operation. In each of operation modes in one plane the envelope had a waist. The series of measurements at different modes operation were processed together taking into account a conservation of a beam emittans. The mathematical formulation of this problem is coming to functional minimisation

$$\begin{aligned} F = & \sum_i [< (x - \bar{x})^2 >_i \; - \sigma_{11}^A - 2\sigma_{12}^A s_i - \sigma_{22}^A s_i^2]^2 + \\ & + \sum_j [< (x - \bar{x})^2 >_j - \sigma_{11}^B - 2\sigma_{12}^B s_j - \sigma_{22}^B s_j^2]^2 + \\ & + \lambda \{ \; [ \; \sigma_{11}^a \; \sigma_{22}^A - (\sigma_{12}^A) \; ] - [ \; \sigma_{11}^B \; \sigma_{22}^B - (\sigma_{12}^B)^2 \; ] \; \} \end{aligned} \tag{5}$$

Indexes A and B concern to different mode operations of lenses, $\lambda$ is a Lagrange multiplier, enabling to take into account a condition

$$\varepsilon^A \;\; = \;\; \varepsilon^B \tag{6}$$

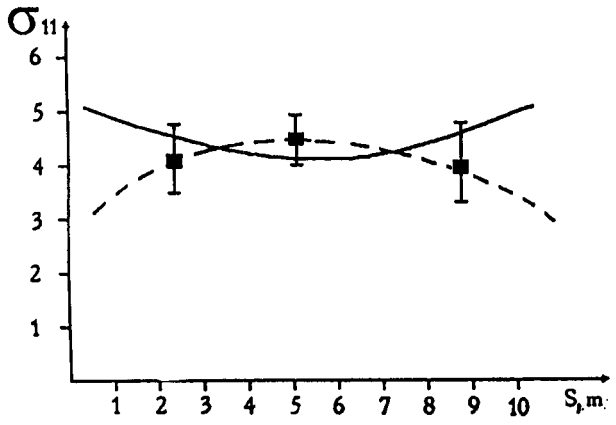


**Fig.1.** Beam envelope in drift space

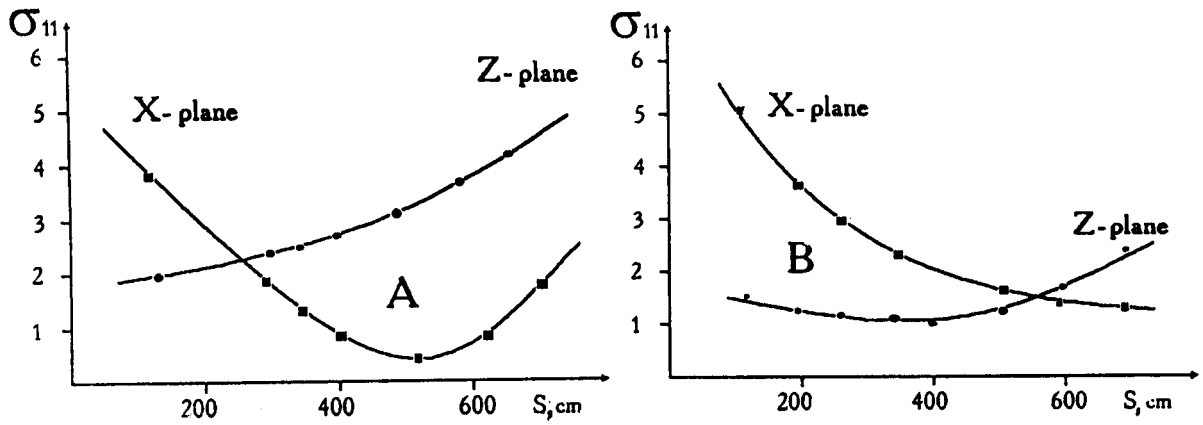


**Fig.2.** Beam envelope under two modes lenses operation

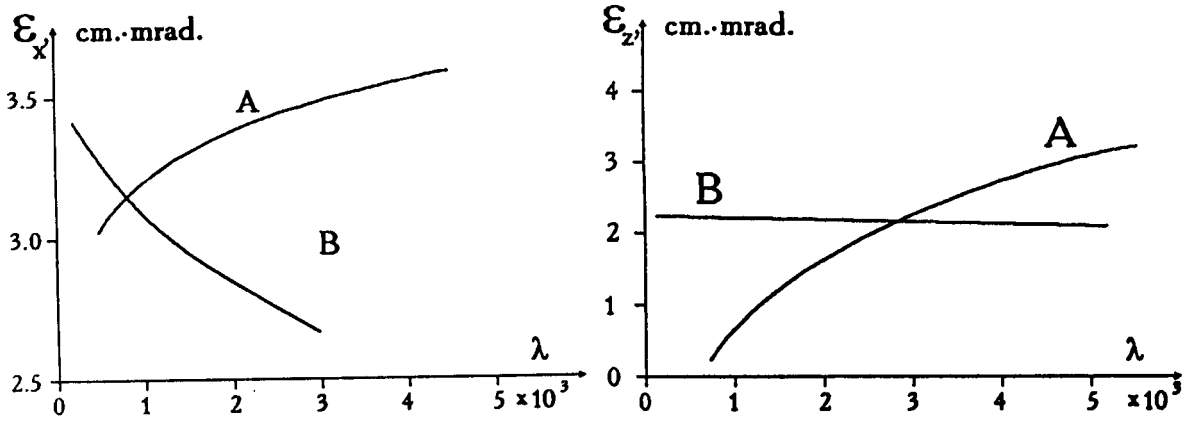


**Fig.3.** Dependence $\varepsilon_x$ and $\varepsilon_z$ on $\lambda$

Minimising (5), we receive two systems of linear equations

$$C_1^K = \sigma_{11}^K A_1^K + 2\sigma_{12}^k A_2^K + \sigma_{22}^k (A_3^K - \lambda)$$
$$C_2^k = \sigma_{11}^K A_2^k + 2\sigma_{12}^K (A_3^K + \lambda) + \sigma_{22}^K A_4^K \qquad (7)$$
$$C_3^K = \sigma_{11}^K (A_3^K - \lambda) + 2\sigma_{12}^K A_4^K + \sigma_{22}^K A_5^K$$

Here $K$ is a index of a lenses mode operation; the factors $C$ and $A$ are equal

$$S = \sum_i < (x - \bar{x})^2 >_i , \quad C_2 = \sum_i < (x - \bar{x})^2 >_i s_i, \quad C_3 = \sum_i < (x - \bar{x})^2 >_i s_i^2 ,$$

$$A_1 = \sum_i i = N, \quad A_2 = \sum_i s_i^2, \quad A_4 = \sum_i s_i^3, \quad A_5 = \sum_i s_i^4, \qquad (8)$$

$N$ is a number of measurement points.

Varying $\lambda$, from the decision of a system of equations (7) we search those significances $\sigma_{ij}^K$, for which the condition (6) is satisfied.

The beam envelope at two modes of the lenses operation are shown on Fig.2. On Fig.3 dependences of calculated emittans $\varepsilon_x(\lambda)$ and $\varepsilon_z(\lambda)$ are submitted. The point of crossing gives those significances $\lambda$ for plane x and z, which satisfies requirement (6). Two sets of matrix elements $\sigma_{ij}^{A,B}$, satisfying to condition (6) and corresponding A or B mode operation are received for each plane. Natural criterion of check was recalculation $\sigma$-matrix on a doublet input, as far as on the doublet input the beam was the same for a A and B mode operation. Results of recalculation under the formula (1) are as follows:

$$\sigma_x^A = \begin{bmatrix} 2.84 & 3.69e-3 \\ 3.69e-3 & 8.23e-6 \end{bmatrix} \qquad \sigma_x^B = \begin{bmatrix} 3.0 & 3.77e-3 \\ 3.77e-3 & 8.0e-6 \end{bmatrix}$$

$$\sigma_z^A = \begin{bmatrix} 4.503 & -1.61e-4 \\ -1.61e-4 & 1.18e-6 \end{bmatrix} \qquad \sigma_z^B = \begin{bmatrix} 4.306 & 3.85e-4 \\ 3.85e-4 & 1.26e-6 \end{bmatrix}$$

Emittanses are the values: $\varepsilon_x = 3.13$ cm·mrad, $\varepsilon_z = 2.3$ cm·mrad. Distinction in values of matrix elements is located within the limits of 5%. The exception is made correlation element $\sigma_{12}^z$.

Having taken the $\sigma$-matrixes as an initial data for the code TRANSPORT [1], we have been calculated an IRIS beam transport line, for which we had the most complete information (a beam sizes and a beam envelopes at various modes operation of magnetic elements). A qualitative agreement between calculation and experimental data were obtained.

The knowledge $\sigma$-matrix of a beam provides a way of optimisation existing and designing new beams transport lines at the PNPI synchrocyclotron.

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